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MAGNETOSTRICTION OSCILLATORS

BY GEORGE W. PIERCE

Presented January 11, 1928. Received February 15, 1928.

Outline.—The present paper describes a newly-discovered method of using magnetostriction to produce and control electrical and mechanical frequencies of oscillations in a range of frequencies extending from a few hundred cycles per second to more than three hundred thousand cycles per second. The method involves the interaction of the mechanical vibrations of a magnetostrictive rod and the electric oscillations of an electric circuit in such a way that the electric currents in the circuit stimulate the rod to longitudinal vibration by magnetostriction and the vibrations of the rod react by magnetostriction on the electric circuit to maintain constancy of frequency.

The constancy of frequency obtained compares favorably with that obtained with the piezo-electric crystal oscillators. For ease of construction and operation the magnetostriction oscillator has a great advantage over the piezo-electric oscillator, particularly in that the construction and adjustment of the magnetostriction vibrators is so simple that large numbers of standards of frequencies all operable with the same electrical circuits may be had at small expense.

The magnetostriction oscillators supply a particularly pressing need in the range of frequencies below twenty-five thousand cycles per second in which range crystal-control is impractical on account of the expense of obtaining sufficiently large crystal vibrators. In the range between twenty-five thousand cycles per second and three hundred thousand cycles per second the magnetostriction oscillators and the crystal oscillators have a common field of usefulness. At frequencies greater that three hundred thousand cycles per second, the magnetostriction oscillators (although active up to two million cycles per second) are feeble with the present arrangement of apparatus. Their harmonics may, however, be employed up to frequencies of several millions per second.

This account contains also methods for calibration of the vibrators and their use in the calibration of wavemeters and frequency meters, data on the velocity of sound in various metallic alloys, data on the elastic constants of metals, including their temperature coefficients, description of methods of sound production, and a theoretical investigation of sound propagation in a viscous magnetostrictive medium.

Circuits and Mounting of Magnetostriction Oscillator for Controlling Oscillations in a Vacuum-Tube Circuit.—As shown in Fig. 1, two coils are employed, one, L_1, in the plate circuit and the other, L_2, in the grid circuit of a high-mu vacuum tube. The magnetostriction rod, *R*, which we may call the vibrator, is placed axially within the coils, and rests centrally on a support between the coils, or if desired is held in a clamp at the support. In the diagram the right-hand coil is in the plate circuit in series with a B-battery. The left-hand coil is in the grid circuit. A variable air condenser *C* is connected between the plate and grid, so as to be across both coils. Sometimes the condenser *C* is connected across only one of the coils.

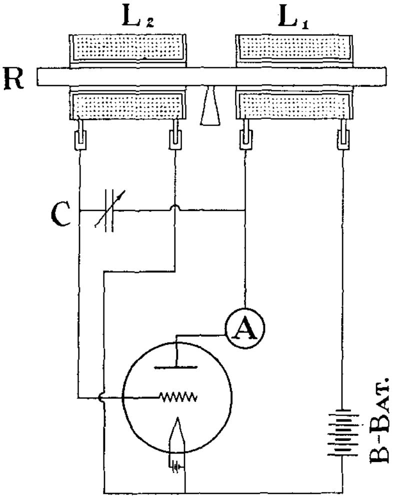

FIGURE 1. Oscillator Circuit.

By means of plugs and sockets the double-coil unit may be replaced by others of different inductances. With a given coil unit many different rods may be used by merely pulling out one and putting in another. The rod is free within the coils, which may have a clearance as large as the diameter of the rod itself. The rod is magnetized permanently or by the plate current, or by a permanent magnet placed near it.

A direct current milammeter at *A* serves to indicate the plate current and by its change with change of *C* to indicate the presence of oscillations.

Reversed Coil Note.—One fact is to be noted that for the best operation of the apparatus as a constant-frequency device, the coils are so wound and oriented that a steady current flowing from the filament to the plate and a steady current, if there were one, flowing from the filament to the grid produce magnetic fields in the same direction along the axis of the coils. This sense of the winding of the coils with respect to each other is opposite to that of the familiar electric oscillator circuits; but it should be noted that such an arrangement as that here shown, with certain values of circuit constants, will oscillate electrically without the rod or with the rod restrained.

Operation of the Magnetostrictive Rod in Rendering the System Oscillatory.—A system such as is shown in Fig. 1, may be entirely non-oscillatory when the rod is restrained from vibration by being held or when the condenser *C* has a value far removed from the value required to give the circuit a period near the period of vibration of the rod. In such a case when the rod is released and the condenser has, or is made to have, the proper value, the rod and circuit fall into oscillation with a frequency which is essentially the frequency of the rod, and the frequency remains practically unchanged even when the condenser is varied over a large range or removed altogether. With proper choice of the coils the condenser is unnecessary.

Numerical data as to the constancy of frequency will be given below.

The existence of the oscillations is evidenced by the sound emitted by the vibrating rod, if its frequency is within the audible range; or, whether audible or not, the vibration is evidenced by the change of direct-current indication of the plate milammeter. When the rod is allowed to vibrate the plate current changes to double or triple the value it has when the rod is restrained.

Operation of the Magnetostrictive Rod in Stabilizing the Frequency of an Electrically Oscillatory System.—If we consider the circuit of Fig. 1, and especially if we note that the electrical feed-back between grid coil and plate coil is the reverse of that usually employed in producing electrical oscillations, we may understand that with a given choice of condenser, coil-winding and coil-spacing, the system may or may not be electrically oscillatory when the magnetostrictive rod is restrained from vibration.

In the preceding paragraph note is made of the action with a non-oscillatory electrical system, which is the preferable mode of operation of the magnetostrictive system when employed with low frequency vibrations of say five hundred to three thousand cycles per second.

At higher frequencies ranging from three thousand to three hundred thousand cycles per second, it is more convenient, and just as reliable, to allow the system to be electrically oscillatory even when the rod is restrained, and to employ the magnetostrictive rod merely to stabilize an already existing frequency of the electrical system when the latter is independently adjusted to a value near resonance with the period of mechanical vibration of the rod. This is done as follows: Any given rod, say one of frequency of twenty thousand, is placed in the coils. The plate milammeter is observed as the condenser C is varied. At a certain value of C, the plate current suddenly jumps to a large value (from one and one-half to three times its previous value). This indicates the incidence of rod-vibrations. The condenser may now be adjusted through a considerable range with no material change of the electrical or mechanical frequency of the system. It may be left oscillatory for days without changing frequency although the electrical characteristics may undergo large changes in that time.

General Note on Materials of the Magnetostrictive Rods.—This research, which has now extended over some years, has concerned itself largely with the study of materials for the vibrators. The qualities required are large magnetostrictive effects, and constancy of mechanical frequency in spite of changes of temperature and of intensity of magnetization, coupled with constancy of frequency in spite of changes of condenser settings, vacuum tube characteristics, plate currents and filament currents.

For these purposes pure iron and irons with various carbon contents are relatively useless, as having too small a magnetostrictive effect.

Pure nickel on the other hand is a good vibrator with, however, some lack of stabilizing power in that detuning slightly affects the frequency.

Alloys of nickel and iron in certain proportions are good vibrators, especially those having about 36% nickel and 64% iron (that is of about the constitution of *invar* and *stoic* metal). This alloy has, however, a large temperature coefficient of frequency.

Alloys of chromium, nickel and iron are good vibrators, and for many purposed commercial *Nichrome* is one of the best materials easily available.

An alloy of nickel and copper known as Monel metal, containing

68% Ni, 28% Cu, and small percentages of Fe, Si, Mn, and C, is a very powerful oscillator. It usually has too small a residual magnetism to oscillate without an auxiliary polarizing means.

Alloys of cobalt and iron are strong vibrators.

Tubes of nickel make good oscillators for qualitative work and demonstration purposes and for sources of sound but are somewhat lacking in constancy as frequency-stabilizers. Such tubes of nickel wholly or partly filled with lead or type metal permit the easy construction of low-frequency vibrators because the velocity of sound in lead is small and gives a low prequency of longitudinal vibration without excessive lengths of the rods.

By using a tube of material that has a negative temperature coefficient of frequency (such as nickel), in combination with a tight-fitting internal core of metal that has a positive temperature coefficient of frequency, (such as stoic metal), I have made composite vibrators of frequency practically independent of temperature. Other composite vibrators are described below.

All of the vibrators are made more powerful by annealing.

Mechanical Vibrations Longitudinal.—In most of the applications of the magnetostrictive vibrator to the production and stabilization of frequency I have employed the longitudinal vibrations of the rod either in its fundamental mode or some harmonic mode of longitudinal vibration. In the designation of constants in this paper, unless otherwise stated, reference is to the fundamental mode of longitudinal vibration. It is usually not worth while to use harmonic modes of vibration of the rods since new rods of any desired frequency can be made and calibrated with such ease that the harmonic mechanical vibrations are interesting only in studies of the mechanics of the rod itself. Investigations of this character are now under way.

By sending a current lengthwise through the rod so as to give a circumferential magnetic field and at the same time applying a current to the solenoid it is theoretically possible to produce torsional oscillations in the rod and these may be used to stabilize frequencies. I have found evidence of these torsional vibrations, but because of the complication of apparatus involved, these torsional oscillations are at present left out of the account.

Vibrators Magnetically Polarized.—The rods are polarized by a steady state of magnetization upon which the oscillatory magnetizing forces are superposed, so that the resulting state is an increase of magnetization when the oscillatory current is in one direction and a decrease of magnetization when the oscillatory current is in the other direction. These two states are accompanied by an increase (or decrease) of length and a decrease (or increase) of length respectively. If the rod were not thus polarized the change of length would be the same for each half cycle of the current and would result in a mechanical vibration of double periodicity.

The polarization of the rods is usually attained by permanently magnetizing them in advance by a solenoid traversed by a strong direct current. With most substances the remanent magnetism is sufficiently strong to keep the rods sensitive as oscillators through years of normal use as standards of frequency in spite of any demagnetizing effects of the oscillations. The plate current of the tubes also assists in the polarization provided the rods, if permanently magnetized, are always inserted with their poles in the proper direction, as may be indicated on the coil mounting.

In cases of metals, like Monel metal, which have very small residual magnetism, I find that a small horse-shoe permanent magnetic placed in the vicinity of the rod is sufficient to give the required polarization, even when the small horse-shoe permanent magnet is 15 centimeters from the rod. With large oscillatory currents the polarization should be correspondingly large so that reversals of magnetization do not occur.

First Step in Explaining Action of Magnetostrictive Rod.—Along with the description of the apparatus occasional paragraphs are introduced explanatory of the theory of operation, with a final mathematical treatment of the system. Magnetostriction is the distortion of a body (in this case the lengthening or shortening of the rod) when magnetized. A rod of nickel when magnetized shortens by about one one-millionth of its length for a magnetizing field of one gauss. This is the observed fact when the magnetization is produced by a steady magnetizing force. This shortening is thus very small, for the reason that the shortening must take place against the enormous elastic force of the body. On the other hand, when the specimen is magnetized by a force that increases and decreases in an oscillatory manner at a period resonant with the period of the body the shortening and lengthening may be more than one hundred times as great as that obtained with the same magnitude of constant current applied to the specimen. In the resonant oscillatory case the contraction and expansion are no longer opposed by the elastic force of the body but only by its viscosity. We may call the decrease or increase of length under magnetization the *direct effect.*

There is also an *inverse effect.* When a magnetized bar is stretched mechanically its state of magnetization is changed.

We may now apply these facts to the system of Fig. 1. Any small fortuitous change of current through the plate coil L_1 changes the magnetization of the rod and causes it to be deformed (lengthened or shortened). This deformation is propagated along the rod to its left-hand end and exists temporarily as a deformation within the coil L_2. The deformation changes the state of magnetization and consequently induces an electromotive force in L_2. This acts on the grid and produces an amplified current change in the plate circuit and in L_1. The oscillating currents in the system thus build up to a large amplitude with a frequency determined by the frequency of longitudinal mechanical vibration of the rod.

Sample Exhibit of Rods.—Plate I is from a photograph of several rod vibrators. *G* is a set of *Stoic* metal vibrators with a range of frequencies extending from ten thousand to thirty thousand cycles per second at intervals of one thousand cycles per second. These are described in Table 1. *H* is a set of rods of *Nichrome* for each thousand cycles extending in range from twenty-six thousand to fifty-seven thousand cycles (see Table II). *F* is a rod of Nichrome with a frequency of 117,705 cycles per second. *D* shows a nickel tube filled with type metal. On account of the low velocity of sound in the type metal the specimen has a frequency of two thousand cycles per second, while a solid nickel rod or a nickel tube of the same length would have a frequency of nearly four thousand. *E* is a part of a similar tube, which is 94.35 cm. long, and is filled with lead. It has a frequency of one thousand cycles per second, which can be adjusted over a range of some seven cycles by clamp weights that may be moved nearer together or farther apart to get a higher or lower frequency. By using these movable weights this rod or a harmonic of it can be brought to beat zero with other standards. *C* is a solid oscillator of stainless steel (about 11 per cent chromium). It has a frequency of 23,640 cycles per second, and is here shown to call attention to the fact that rods of large diameters do not fail to vibrate even at high frequencies. In fact the specimen *A* is a thin disc between corks in a glass tube and while 1.5 cm. in diameter is only about 2 millimeters long and has a period of about a million and a half cycles per second. It is, however, a very feeble oscillator and evidences its oscillations only by transient clicks heard in a telephone in the plate circuit as the tuning condenser passes through resonance.

At *B* is shown a Nichrome oscillator of frequency 102,000 cycles per second in a sealed glass tube in vacuo, in which it is loose. In the picture gravitation has pulled the oscillator to the lower end of the inclined glass tube. In operation the glass tube is placed axially in a horizontal position in the coils of the oscillator, and the rod is then centrally supported within the glass tube and within the coils. The mounting in a sealed glass tube has the advantage of permanence and avoids heating of the rod by the body temperature when it is handled. The absence of air or gas in the tube prevents the production of stationary sound waves in the tube, which might slightly affect the period of the rod, but this effect is very small.

A great variety of rod sizes are in use in various researches now under way at this laboratory.

Lengths and Frequencies of Rods.—Table I contains data obtained with a set of *Stoic* metal rods. These are the rods shown at *G* in the photograph. The frequency values given in the second column are accurate to 1/100 of 1%. The lengths are not accurately known because the ends of the rods are not accurately true. The errors in the measurements of these lengths are of the order of 0.1 mm. which amounts to more than 1/10 of 1% for the shorter rods.

It is seen that the length of the rods times their frequencies give a constant to the degree of precision to which the lengths are measured. This constant, when doubled, gives the velocity of sound in the metal, which is

$$v_{\text{stoic}} = 4160 \pm 2 \text{ m/sec at } 20^\circ \text{ C}.$$

All of these rods have roughly the same diameter, and the fact that there is no progressive change in the values of the last column indicates that there is no appreciable end-correction for the velocity determination.

The importance of this result for the present purpose is that a whole series of vibrators can be precomputed as to length with sufficient accuracy to make their final adjustment to required frequency very simple. A rod can be predetermined and cut in a lathe to 0.01 cm. so that the final frequency adjustment to a standard value requires very little grinding. The end is ground off to raise the frequency. If too much is ground off at the end, it can be corrected by grinding away a little from the girth near the center of the rod.

Table II contains a similar set of observations of frequency for different lengths of Nichrome. Velocities for a large number of alloys are given below.

TABLE I.

Length and Frequency of Stoic Metal Rods at 20° C. Diameter 0.79 cm.

Specimen No.	Frequency Cycles/sec.	Length Cm.	Length in M. × Frequency
10	10001	20.815	2081.7
11	11005	18.91	2081.0
12	11985	17.36	2080.6
13	12990	16.01	2079.7
14	14000	14.85	2079.4
15	14981	13.87	2077.8
16	16009	13.00	2081.2
17	17006	12.24	2081.5
18	18015	11.55	2080.7
19	19013	10.94	2080.0
20	20003	10.40	2080.3
21	21007	9.90	2079.7
22	22008	9.44	2077.5
23	22970	9.035	2075.2
24	24009	8.65	2076.8
25	24992	8.33	2081.8
26	25979	8.01	2080.9
27	26981	7.70	2077.5
28	27966	7.44	2080.5
29	29000	7.17	2079.2
30	29981	6.93	2077.7
		Average	2079.6 ± 2

$v_{\text{stoic}} = 2 \times \text{Length} \times \text{Frequency} = 4160 \pm 2$ meters/sec. at 20° C.

Experimental Data Illustrating the Manner in which the Magnetostrictive Rod Functions in Controlling Frequency.—Illustrative of the action of the rod as a frequency-controlling element the experimental curves of Fig. 2 are given. These are from observations taken with a rod (No. 17) of Nichrome in the coils and circuit of

TABLE II.

Length and Frequency of Nichrome Rods at 23° C. Diameter .96 Cm.

Specimen No.	Frequency Cycles/sec.	Length Cm.	Length in M. × Frequency
26	25999	9.58	2490.7
27	27000	9.21	2484.0
28	28000	8.87	2486.4
29	29005	8.56	2482.8
30	29992	8.27	2480.3
31	31005	8.02	2486.6
32	32011	7.77	2487.2
33	33006	7.55	2491.9
34	34011	7.32	2489.6
35	35002	7.13	2495.6
36	36002	6.88	(2476.9)*
37	37004	6.73	2490.3
38	38004	6.55	2489.2
39	39005	6.38	2488.5
40	40004	6.23	2492.2
41	41007	6.06	2485.0
42	42000	5.93	2490.6
43	43005	5.79	2490.0
44	44009	5.66	2490.9
45	45008	5.53	2488.9
46	46004	5.42	2493.4
47	47009	5.30	2491.4
48	48011	5.20	2496.5
49	49008	5.09	2494.5
50	49996	4.99	2494.8
51	51040	4.87	2485.6
52	52000	4.78	2485.6
53	53062	4.69	2488.6
54	54027	4.61	2490.6
55	55014	4.53	2492.1
56	56042	4.45	2493.9
57	57015	4.37	2491.5
		Average	2490.3

v_{nichrome} = 4981 m/sec. at 23° C.

* Inconsistent on account of grinding at girth for adjustment.

Fig. 1. In the lower part of the Fig. 2 electric wave length λ, read on a precision wavemeter, are plotted against condenser divisions of the condenser *C* of Fig. 1. Readings were taken first with the rod restrained by holding it with the hand. A plot of λ versus *C* for the rod thus restrained is the curve ABCDE of Fig. 2.

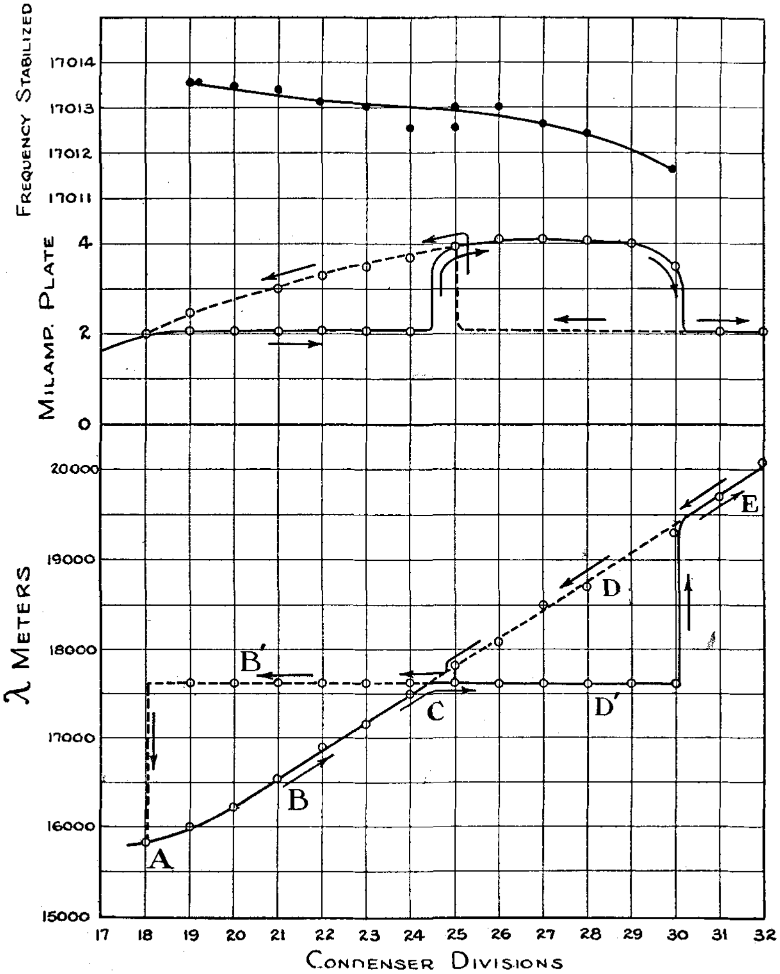

Figure 2. Illustrating Stabilization of Frequency.

Next readings were taken with the rod free, starting with small values of the condenser C. These readings plotted give the curve ABCD′E, with a constant λ along the element D′ of the path.

As a third operation, with the rod free the condenser was started at large values of condenser and gave the curve EDB′A, with constant λ along the element B′ of the path.

As a fourth operation it is noted that when we have once got the rod vibrating by either of the above operations we can pass back and forth along the whole of B′ and D′ (i. e. from 18 divisions to 30 divisions of the condenser) and the wavelength remains essentially constant.

The wave-meter measurements are not sensitive enough to detect any change of frequency along this path B′D′. To measure this quantity resort was had to beats between the oscillating system with rod No. 17 under investigation and a second oscillator with magnetostriction-rod control. Since the total change of No. 17 was of the order of one cycle per second in seventeen thousand, the second oscillator, known as No. 117.7, was chosen to have a frequency about seven times that of No. 17, and beats were observed between the seventh harmonic of No. 17 and the fundamental of No. 117.7. This means that one cycle per second change in No. 17 gives seven cycles per second change in the beat note, which can be measured to about one cycle per second.

With this device the actual frequencies of the system stabilized by rod No. 17 against condenser settings are plotted as the top curve of Fig. 2. This curve shows that a change of the condenser from 19 to 30 divisions changes the frequency of the system by only 1.5 parts in seventeen thousand. This effect is discussed further in the next section.

In this experiment I have changed the condenser through a wide range of values, so as to make the change of frequency as much as possible, but when used as a constant frequency apparatus the condenser is brought always to the value which makes the plate current a maximum so that repeated settings of the apparatus may be made with a much higher degree of precision that the 1.5 cycles in seventeen thousand cycles per second.

Let us look next at the middle curves of this Fig. 2, which exhibits the plot of plate current against condenser settings, when the rod is free to vibrate. The arrows indicate the sense of a change given to the condenser of the oscillator circuit. When the condenser, on being increased, reaches 24.5 divisions the rod begins to vibrate and the plate direct current jumps from two to four milamperes, and remains about the same for further increases of the condenser up to 30.1 divisions when the rod ceases to vibrate. Now a reduction of the condenser leaves the plate current two milamperes till $C = 25$ divisions is reached when the plate current jumps again to four milamperes and then gradually decreases as the condenser is changed further down to eighteen divisions, when the rod is found to have again ceased to vibrate. In fact with this particular set of apparatus constants the system ceases also to vibrate electrically at C equal to or less than eighteen divisions. This accounts for the smooth fall of plate current to its constant two milampere value. With more clearance in the coil between the rod and the coil or with other electrical or mechanical constants we may get an abrupt change of plate current at the left hand part of the curves as well as at the right hand part. With less clearance, or by putting a separate condenser about each of the coils of Fig. 1, the apparatus can be so constructed that all oscillations cease whenever the rod is restrained from vibrating.

The Apparent Reactance of the Coil Containing the Tuned Magnetostriction Rod a Function of the Frequency.—A crude view of the above result is that in the range of frequency-stabilization the system oscillates as a condenser-inductance circuit with an inductance that varies enormously with a small change of frequency. If we regard the wavelength-condenser curves of the lower part of Fig. 2 as having the equation

$$\lambda_d = 2\,\pi\,c\,\sqrt{L_d\,(C + C_o)} \text{ for the damped curve}$$

and

$$\lambda_f = 2\,\pi\,c\,\sqrt{L_f\,(C + C_o)} \text{ for the free curve}$$

we obtain

$$\left(\frac{\lambda_f}{\lambda_d}\right)^2 = \frac{L_f}{L_d}.$$

The values of this ratio are plotted against frequency in the curve of Fig. 3, from which it appears that the inductance of the coil and free rod, in the range of its stabilization frequencies, changes by about 50% for a frequency change of 1.5 in 17013; that is, for a frequency change of less than 1/100 of 1%.

Though the correct understanding of the device requires mathematical discussion, such as is given below, we may here regard the constancy of frequency, with large variation of condenser, as due to the fact that an increase of C makes the frequency lower. This is accompanied by a shift of L along the curve of Fig. 3 to a smaller value so that the product of $L \times C$, and consequently the frequency, is almost the same as before the increase of C and, in fact, with this particular magnetostrictive material (Nichrome) the frequency changes only 1/100 of 1% for a change of 50% in the value of C. The potency of the rod in controlling the frequency is seen by noting that if we omit the rod or restrain it from vibrating this change of 50% in C causes 25% change in frequency instead of the 1/100 of 1% with the rod present and free.

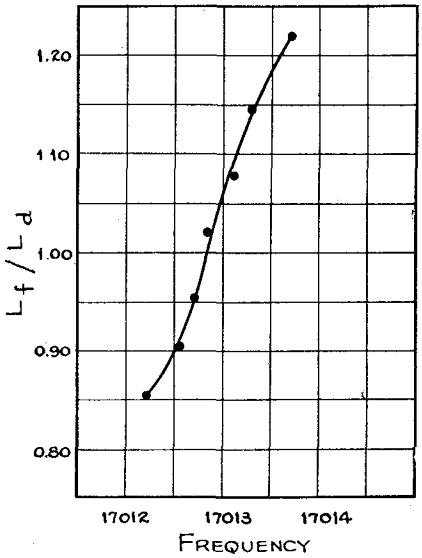

FIGURE 3. L_f = Inductance Free; L_f = Inductance Damped; plotted against Frequency.

Frequency Essentially Independent of Vacuum-Tube Voltages and Characteristics.—I have made numerous experiments that show that the very large change of plate voltage from 135 volts to 67 volts changes the frequency of oscillation of the magnetostriction controlled system by only about one in thirty thousand. Voltage change of the filament battery from a condition of practically zero emission to the point of destruction of the filament, or a change of the inductance of driving coils over the whole range of oscillatory operativeness or a change of the tubes to tubes of widely different characteristics affects the frequency within about the same limits.

These various notes as to constancy apply to annealed Nichrome rods. Some of the other materials and particularly certain tubular vibrators are not quite so constant in these particulars, but are still very serviceable when less precision is required.

Temperature Coefficient of the Nichrome Vibrators.—The temperature coefficient of the frequency of the Nichrome vibrator is

$$\frac{\Delta f}{f\Delta\theta} = -\ .000107/\text{deg. C.}$$

where f = frequency, Δf = increase of frequency for a change of temperature of $\Delta\,\theta$ degrees Centigrade. This equation means that the frequency decreases about 1/93 of 1% per degree Centigrade.

The temperature coefficient of other materials is given in a later section.

Comparison of Constancy with that of the Piezo-Electric Crystal Vibrators.—For the purpose of comparison of the magnetostriction oscillators with the piezo-electric crystal method of frequency control to which I have contributed in previous researches I have made extreme tests on some of my best piezo-electric crystal frequency standards. One is a crystal, No. 28, mounted in vacuo in an accurately designed and carefully machined holder. When operated under constant conditions its absolute frequency has been repeatedly measured and shows variations below 1/500 of 1%. However, care must be taken to keep the circuits constant with this crystal oscillator as the data of Table III show.

TABLE III.

Piezo-Electric Crystal Oscillator

Conditions		Frequency
Tube	Plate Volts	
199	45	28072
Daven 20	67	28064
Daven 20	112	28061
Daven 20	135	28058
201A	90	28056
201A	112	28054
201A	135	28052

It is seen that extreme variations of tubes and plate voltage changes the frequency of this crystal oscillator by 7/100 of 1%.

This crystal also changes frequency to about the same extent with change of the inductance and distributed capacity of the plate coil. Table III compared with previous sections of this account shows that for equally violent change of the circuits, tubes and voltages with the piezo crystal oscillator and the magnetostriction oscillator the magnetostriction oscillator is much more constant as to frequency than is the piezo crystal oscillator.

It should be noted however that in the matter of temperature coefficient of frequency the particular Nichrome magnetostriction oscillator with 1/93 of 1% per degree Centigrade is more subject to temperature changes than the piezo-electric oscillators with 1/200 to 1/1000 of 1% per degree Centigrade.

Below I give, however, descriptions of other magnetostrictive materials with temperature coefficients of about the same order as that of the piezo-electric crystal oscillators. The temperature effect in no case is permanent and the frequency comes back to its calibrated value when the temperature returns to normal for the calibration. A knowledge of the temperature permits the application of a correction to the rated calibration.

Magnetostriction Oscillator with Universal One-Stage Amplification.—In order to increase the output power one or more stages of amplification may be employed. Fig. 4 shows a very simple and useful arrangement for a system to be operated at frequencies extending all over the audio and radio frequency ranges. The oscillator tube shown at the left has the connections of Fig. 1. The grid of the amplifier tube at the right is connected through a condenser C_1 to the plate of the oscillator tube, and is provided with a leak resistance. The plate circuit of the amplifier tube contains a "speaker filter" unit consisting of an audio choke which is in series with the B-Battery and which communicates with the output terminals through a condenser C_2 of two microfarads.

The constants given in the legend to Fig. 4 are so chosen that the amplifier operates with considerable efficiency at both radio and audio frequency. In the latter case a loud speaker connected to the output terminals gives volume enough to be heard throughout a large lecture room. This is the case also with radio oscillations upon which are superposed other radio oscillations to produce audio frequency beats.

A commercial apparatus of this character made for me by the

General Radio Company of Cambridge is shown in Plate II, and at the left in Plate III. The coil units, containing rods, on top of the boxes are plugged into jacks and can be removed and interchanged. The upper cover of the box opens.

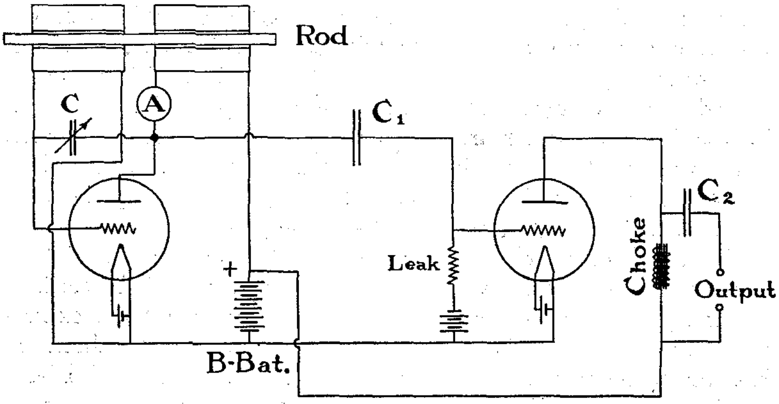

FIGURE 4. Magnetostriction Oscillator with One-stage Amplification. C_1 = 0.1 Microfarad, C_2 = 2 Microfarads, Leak = .05 Megohms, A = Milammeters.

Twin Magnetostriction Oscillator.—For calibration purposes we need two oscillators like that of Fig. 4, one of which is maintained constant by a standard magnetostriction rod, and the other of which is at a frequency to be compared with the rod. The two oscillators are then caused to interact and their beat frequency is determined or reduced to zero. After much experimenting I find that the best way to get the interaction is to connect their output terminals together through a small condenser and then to plug the telephones or other indicator in at one or the other of the output terminals. If the rod is left out of one of the coil units that half of the twin oscillator may be used as an ordinary variable electric oscillator.

I have found so much use for this arrangement that I have had both of these oscillators with common B-Batteries and common A-Battery terminals mounted in a single apparatus shown as the middle unit of Plate III.

The "speaker filters" of the two output circuits are connected with a switch so that if desired, either or both may be cut out and the B-Battery and plate of the amplifiers be connected directly to the output terminals. This is useful in running a synchronous-motor clock, which requires a polarizing current, as the output apparatus. Such a clock made also by the General Radio Company from their own designs and kindly put at my disposal is shown in Plate II, and was used in timing some of the low-frequency rods (at about 1000).

Absolute Frequency Calibration by Magnetostriction Oscillators and the General Radio Synchronous-Motor Clock.—Use is made of the adjustable Magnetostriction Rod of about 1000 cycles per second, which is shown in Plate II mounted on top of the box containing the electric oscillator circuits and the one-stage universal amplifier. Used with the "Speaker filter" cut out this drives the motor clock synchronously with the oscillations of the rod. The amplifier tube is a U.X. 171, operating on 130 volts in the plate.

The synchronous clock has been provided with circuit-making contacts that close for a very short time each second of the clock.

For a standard of time with which to compare this motor clock Professor Stetson of the Harvard Student's Astronomical Laboratory has kindly permitted me to lead wires from his standard Riefler Clock which beats siderial seconds with an error not greater than two-tenths of a second per week. All relay contacting members are obviated in this circuit by the device of placing an inductance coil near the field coil of his relay, so that the local current from the time wheel of the clock through his relay field acts inductively on my circuit and supplies second impulses without the inconstancy of any contact other than that of the time wheel of the clock. These second impulses arriving at this laboratory are amplified by a one-stage amplifier, to the output of which is connected a loud-speaker through the contacting points of the motor driven clock. When the standard Riefler clock and the motor clock make contacts at the same time the ticks are heard in the loud speaker. By reading the motor clock for several successive coincidences accurate timing is obtained of the motor clock and of the magnetostriction driving rod.

For timing any other suitable magnetostriction rod, (say one of 102,000 vibrations per second) the latter is used to drive and control a second oscillator mounted on a second box like that of Plate II, or else the twin system is used with both of the rods. The 1,000 cycle rod is now adjusted by moving the weights out or in along the rod to points where zero beats is obtained between the 102nd harmonic of the 1,000 cycle rod and the fundamental of the 102,000 rod. When this adjustment is nearly perfect the remaining small adjustment is made by changing the tuning condenser of the 1,000 cycle system. In this way I have succeeded in reducing the beat frequency to one cycle in five seconds, which means an error of this adjustment of the harmonic of the low frequency to the fundamental of the high frequency of two parts in a million. Such accuracy is ordinarily not required, so that more tolerance is usually given to this adjustment.

It is interesting to note that the beats may be observed on a D. C. milammeter as well as in a telephone. As a means of obtaining beats on the milammeter the circuits of Fig. 5 are recommended.

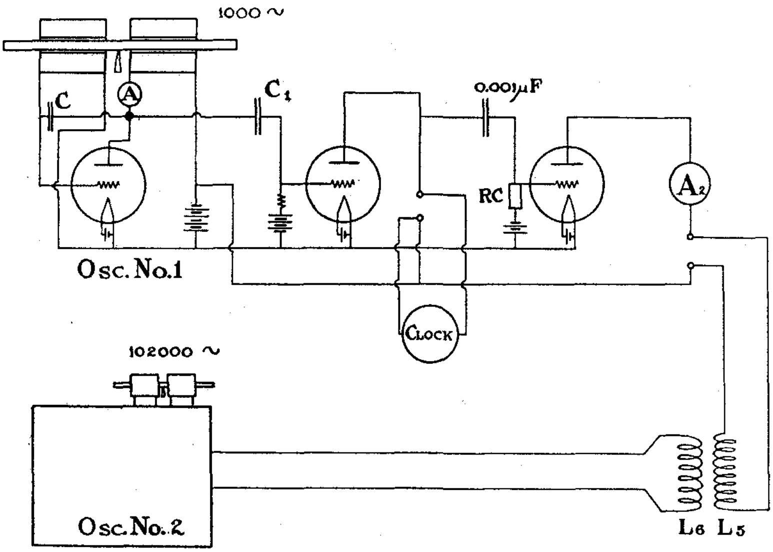

Figure 5. Circuits for Absolute Calibration of Frequency.

In Fig. 5 Oscillator No. 2 has the circuits of Fig. 4 with a coil L_6 connected to its output terminals. Oscillator No. 1 has the same circuits for the two left-hand tubes with, however, the motor-clock in place at the choke of the second tube. Paralleling the clock a radio-amplifying unit is employed for further amplifying the higher harmonics of Oscillator No. 1 and for suppressing its 1,000 cycle note and low pitch harmonics. The connection to the third tube is through a condenser of 0.001 microfarad, which has high impedance to these low notes. The customary grid-leak of this third tube is replaced by a radio choke R C of about 0.1 henry, which has a small impedance for the low notes and a high impedance for the higher harmonics. The output circuit leads to L_5 inductively connected with L_6 so that the oscillations of the two systems are superposed in their output circuits, and their beats when nearly of zero frequency are observed on the D. C. milammeter A_2.

In this way timing of the motor-clock gives a direct standardization of the 102,000 cycle rod as well as of the 1,000 cycle rod.

In a similar way any of the rods may be calibrated directly in one step. I have used harmonics of the 1,000 cycle rod up to its 234th.

To calibrate at the same time directly any higher frequency a double step is required.

Examples of Absolute Calibration of Some of the Rods.—In calibrating Nichrome Rod No. 30, the Rod No. 1 (of about 1,000) was adjusted to give zero beats of its 30th harmonic with Rod No. 30. Coincidences of the Rod-Clock seconds with the Riefler clock siderial seconds occurred as follows:

Coincidences Read on Rod Clock			Time Between Coincidences in Rod Clock Seconds
hr.	min.	sec.	
3	14	31	396
3	21	17	395
3	27	42	399
3	34	21	

For computation we use

$$1 \text{ solar true second} = 1.00273 \text{ siderial seconds.}$$

$$1 \text{ rod second} = 1 + \frac{1}{397} = 1.00252 \text{ siderial seconds.}$$

$$\therefore \quad 1 \text{ solar second} = \frac{1.00273}{1.00252} = 1.00021 \text{ rod seconds.}$$

The motor clock operated by the rod has its hands so geared to its motor that its time is correct when the motor is driven by a current of 1,000 cycles per second, so we have for the frequencies of the rods:

$$f_{\text{no. }1} = 1000.21, f_{\text{no. }30} = 30{,}006.3 \text{ at } 20^\circ.4 \text{ C.}$$

Two other independent determinations of this rod No. 30 give 30,007.2

and 30,006.9 at 20°.2 C. The average of the three values is 30,006.8, with an average error of ½ cycle. Each of these calibrations had a timing period of about 20 minutes; by prolonging the time any required greater accuracy can be obtained.

The following values, Table IV, taken at random from my notebook have been obtained in the absolute calibration of a number of standards, and illustrate what can be done with even very careless work provided the standard second is good and is applied sharply to the apparatus.

TABLE IV.

Sample Absolute Calibration of Standard

Standard No.	Rod No. 30 at 20°.3 C.	Rod No. 102 at 20°.6 C.	Rod No. 117.7	Crystal No. 5884
Cycles per Second	30006.3 30006.9 30007.2	102078 102085 102084 102084 102083 102084	117718 at 21°.9 C. 117679 at 24°.5 C. 117705 at 23°.0 C.	5884707 5884722
Average	30006.8	102083	117701 at 23°.C.	5884719

Referring to the values for Rod No. 117.7 the three values were taken at different temperatures and if reduced to a common temperature of, say 23°, these values become 117704, 117689, 117705 averaging 117701 with an average error of 2 cycles per second. With this correction for temperature the values for No. 117.7 have about the same consistency as the other values in the table, and the variations in the table amounting to about 1/500 of 1% are due to inaccuracy arising from the paucity of the coincidence observations.

The measurements of the frequency of Crystal No. 5884 of about 5884 kilocycles were obtained by using the rod of 1000 cycles adjusted to 997.28 cycles so that its 118th harmonic beats zero with rod No. 117.7 and at the same time beats were measured between the 50th harmonic of No. 117.7 and the fundamental of Crystal No. 5884. The latter number of beats per second was found to be 772, so we have $f_{5884} = 50\ (118 \times 997.28) + 772 = 5{,}884{,}722$. This double operation steps up the 1000 cycle frequency to its 5900th harmonic.

It is interesting to note that this Crystal No. 5884 can also be used at all of its harmonics up to its 30th harmonic to measure frequencies up 150 million cycles.

Mr. M. T. Dow, Dr. Frederick Drake and I are using such a calibration in the precision measurement of the velocity of electric wave.

By the methods here described the several rods of Plate I have been calibrated and intercompared. In the intercomparison the differences between fundamentals or harmonics of various oscillators may be conveniently measured by a bridge-type of audiofrequency meter such as is described in the next section.

Audiofrequency Meter.—I here call attention to a bridge-type of audiofrequency meter involving well-known circuits[1] that I have found useful for measuring audio-frequencies. A photographic view of this apparatus is shown as the right-hand unit of Plate III. The circuits are shown in Fig. 6 and comprise a bridge of three resistance arms and one arm of a variable L in series with fixed C. The two resistances R R are equal. The resistance R_1 plus ΔR is equal to

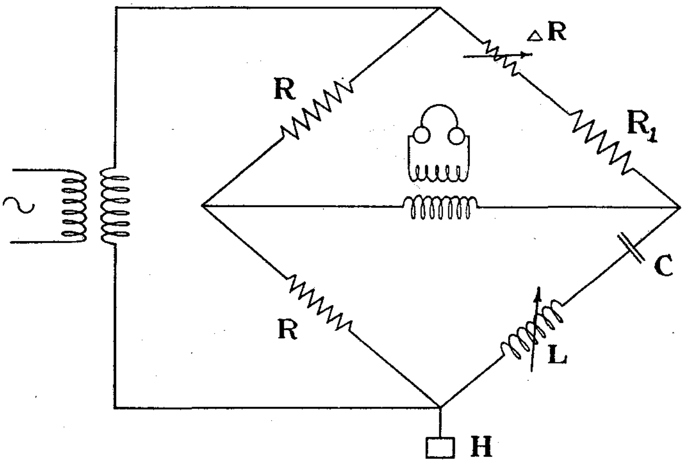

FIGURE 6. Circuits of Audiofrequency Meter.

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[1] Although this instrument has been in use here for many years it was apparently previously described and used as a frequency meter by *Heydweiller and Hagemeister: Verh. d. D. Phys. Ges.* 18, p. 52, 1916. For other references see *Banneitz: Taschenbuch der drahtlosen Tel. und Tel.*

the resistance of the branch $L\ C$ which is chiefly the resistance of L. Since L is of copper wire it has a temperature coefficient. This is compensated by the small adjustable resistance ΔR, which when necessary is independently adjusted when the bridge is to be balanced. The chief variable element is L. At any given frequency, f, within the range of the apparatus the balance is obtained when

$$2\pi f = \frac{1}{\sqrt{LC}}$$

Frequencies are read on a direct-reading scale calibrated in frequency attached to the dial of L. One important point is that for repeatable readings with an accuracy of 1 part in 1000 the variable effects of body-capacity must be eliminated. I have found it advisable to provide the inductance L with a metal knob so that when one takes hold of this knob to adjust L he connects his body to the point H in Fig. 6.

The inductance L at full scale is about 200 milhenries. The capacity C is a three-step mica condenser, having the three values 0.04, 0.2, and 1.0 microfarad, which gives the instrument a three-fold scale extending from 365 to 5000 cycles per second. The switch in the center of the photographic view of the instrument is for switching to different values of the condenser, and automatically switches the pointer to the proper direct-reading scale. The scale was drawn by hand on a bakelite disc from experimentally determined frequency readings and was then engraved on the disc.

In the instrument are included an input transformer, for connecting directly into the input circuit of a vacuum-tube oscillator, and an output transformer connecting to head telephones or a loud-speaker.

Absolute Calibration of Audiofrequency Meter of Fig. 6 by Nichrome Rod No. 30 with f = 30,000.—An electric oscillator with frequencies variable to cover the range of the Audiometer is made to beat against various harmonics of the fixed 30,000-cycle frequency of the rod No. 30. Practically, in the present calibration, this was done by using the Twin Oscillator of Plate III (shown at the center) with the magnetostriction rod in its coil assembly plugged into one set of jacks on the instrument and with various coil-pairs without magnetostriction core plugged into the other set of jacks. One pair of the output terminals of the Twin Oscillator were now connected to the input posts of the audiofrequency meter to be calibrated. A telephone receiver was at the output posts of the audiofrequency meter.

The procedure is as follows: Adjust the variable electric oscillator to beat zero with some harmonic of the magnetostriction rod, then adjust the audiofrequency meter for silence in the telephone, and read the scale of the audiofrequency meter. Proceed *ad lib.* with other harmonics.

Table V gives the results. The value of the harmonic used is given in the column marked n. For example $n = 3/50$ means a beat zero between the 50th harmonic of the 30,000 cycles rod and the 3rd harmonic of the electric oscillator, to which the audiometer was set, giving $f = 1800$ cycles per second. This is tabulated in the second column.

The observations in this Table V were made hurriedly without any repetition of readings. Only the more striking harmonics (simple ratios) were used, and a large number of other values were passed over without being read, as there was no need of obtaining any more dense distribution of the calibration points. The accuracy of reading the scale of the instrument was only about 1 part in 1000 in the more open parts of the scale. Checks on the identity of the harmonics can be obtained by using other magnetostriction rods

Absolute Calibration of a Wavemeter by Nichrome Rod No. 30.— To show a further use of the magnetostriction standards this same rod of 30,000 cycles per second ($\lambda = 10{,}000$ meters) was used to obtain the values of Table VI in calibration of a wave-meter ranging from 233 to 2224 meters. The procedure was the same as in the preceding section, except that the electric oscillator coils were of small inductance so as to give high frequency. The telephone connected directly at the output of the Twin Oscillator, served to evidence audible beats, which were adjusted to a zero-beat-frequency.

The wavemeter was placed near the electric oscillator coil and had its coil shortcircuited, while setting the oscillator to beat zero. Then the shorting switch of the wavemeter was opened, and the condenser of the wavemeter was adjusted. As resonance was approached beats were again heard in the telephone. Changing the condenser of the wavemeter in one direction, we hear these beats rise in frequency, then drop to zero, then appear again at a high frequency and gradually change to zero. The drop of the beats to zero at the setting between the two high frequency beat values is the adjustment of the wavemeter to resonance.

In Table VI n indicates the harmonics employed. This is here tabulated as $1/n$, since we are concerned with wavelengths. The wavelength λ of the wavemeter is obtained by taking $1/n$ times the electric wavelength (10,000) of the 30,000 cycle rod.

TABLE V.

Calibration of Audiofrequency Meter vs. Nichrome Rod No. 30.

Scale of Instrument	*f* by No. 30	*n*	Scale of Instrument	*f* by No. 30	*n*
872	870	2/69	1760	1767	1/17
897	896	2/67	1800	1800	3/50
909	909	1/33	1818	1818	2/33
922	923	2/65	1871	1875	1/16
937	938	1/32	1909	1915	3/47
952	952	2/63	1935	1935	2/31
956	957	3/94	2000	2000	1/15
968	968	1/31	2068	2069	2/29
978	978	3/92	2140	2143	1/14
982	984	2/61	2223	2222	2/27
1090	1091	2/55	2305	2308	1/13
1130	1132	2/52	2395	2400	2/25
1181	1176	2/51	2500	2500	1/12
1228	1224	2/49	2605	2609	2/23
1252	1250	1/24	2722	2727	1/11
1278	1277	2/47	2999	3000	1/10
1330	1333	2/45	3165	3158	2/19
1390	1395	2/43	3330	3333	1/9
1462	1463	2/41	3535	3529	2/17
1498	1500	1/20	3740	3750	1/8
1543	1538	2/39	4000	4000	2/15
1581	1579	1/19	4285	4285	1/7
1603	1607	1/23	4478	4500	3/20
1620	1622	2/37	4595	4615	2/13
1660	1666	1/18	4710	4737	3/19
1692	1696	3/53	5010	5000	1/6
1710	1714	2/35			

In this section and the preceding section it is seen that the 30,000-cycle rod is used in frequency calibrations extending from 870 cycles to 1,200,000 cycles. The possible range is vastly greater than this.

TABLE VI.

Calibration of Wavemeter LC_1 vs. Nichrome Rod No. 30. Readings are in Meters Wavelength.

Scale of Instrument	λ by No. 30	1/*n*	Scale of Instrument	λ by No. 30	1/*n*
2224	2222	2/9	866	869	2/23
2175	2174	5/23	832	825	1/12
2144	2143	3/4	799	800	2/25
2106	2105	4/19	768	769	1/13
2085	2084	5/24	712	714	1/14
2070	2068	6/29	662	666	1/15
2000	2000	1/5	621	625	1/16
1940	1944	7/36	574	578	1/17
1924	1923	5/26	553	555	1/18
1892	1892	7/37	523	526	1/19
1870	1875	3/16	514	513	2/39
1848	1852	5/27	502	500	1/20
1815	1818	2/11	475	476	1/21
1790	1786	5/28	454	455	1/22
1712	1714	6/35	435	435	1/23
1668	1667	1/6	416	417	1/24
1573	1579	3/19	401	400	1/25
1535	1538	2/13	383	385	1/26
1510	1515	5/33	368	370	1/27
1500	1500	3/20	356	357	1/28
1480	1481	4/27	344	345	1/29
1462	1463	6/41	333	333	1/30
1426	1429	1/7	322	323	1/31
1388	1389	5/36	312	313	1/32
1378	1379	4/29	303	303	1/33
1362	1364	3/22	295	294	1/34
1350	1352	5/37	285	286	1/35
1332	1333	2/15	276	278	1/36
1303	1304	3/23	270	270	1/37
1251	1250	1/8	263	263	1/38
1201	1200	3/25	257	256	1/39
1178	1176	2/17	251	250	1/40
1111	1111	1/9	245	243	1/41
1050	1052	2/19	238	238	1/42
999	1000	1/10	233	233	1/43
952	952	2/21			

TABLE VII.

Experimental Results for Alloys.

v = velocity of sound, g = temperature coefficient of frequency, ρ = density, E = Young's Modulus, a = coefficient of expansion, h = temperature coefficient of velocity, b = temperature coefficient of elasticity.

Material	v m/sec	$g \times 10^6$	ρ g/cc	$E \times 10^{11}$ Dyne/sq .cm.	$a \times 10^6$	$h \times 10^6$	$b \times 10^6$
Iron	5074	−171	7.688	19.79	12	−159	−354
Nickel	4937	−132	8.803	21.46	12	−120	−276
Stoic	4161	+224	8.02	13.89	1.5	+226	+446
Nichrome	4981	−107	8.269	20.52	12	− 97	−226
Monel	4549	−151	8.854	18.32	14	−137	−316
Stainless Steel	5430	−136	7.720	22.76	10	−135	−282
Stainless-Iron	5133	−130	7.743	20.40	10	−130	−270
Nickel-Iron % Ni							
0	5074	−171	7.688	19.79	13.3	−158	−355
10	4919	−164	7.812	18.90	9	−155	−337
20	4582	−159	7.818	16.41	8	−151	−326
30	4527	+135			9	+144	+261
30	4241*	+ 77*	7.950*	14.29*	4*	+ 81*	+150*
32	4540	−101			3	− 98	−205
34		+182			2	+184	+362
36	4161	+224	8.020	13.89	1.5	+226	+446
40	4075	+218	8.042	13.36	5	+223	+431
50	4352	− 64	7.901	14.96	10	− 54	−138
60	non oscillatory						
70	non oscillatory						
80	4908	−124	8.521	20.52	12	−112	−260
90	4990		8.666				
100	4937	−132	8.803	21.46	12	−120	−276
Chrom-Iron % Cr.							
10	5290	−153	7.619	21.32	10	−143	−316
20	5448	− 90	8.028	23.82	10	− 80	−190
30	5392	−102	7.659	22.27	10	− 92	−214
40	4329	− 91	7.514	21.33	10	− 81	−192

* After dipping in liquid air.

TABLE VII—*Continued.*

Material			v m/sec	g $\times 10^6$	ρ g/cc	E $\times 10^{11}$ Dyne / sq. cm.	a $\times 10^6$	h $\times 10^6$	b $\times 10^6$
Si	Cr	Fe %							
5	5	90	5166	−118	7.559	20.17			
5	10	85	5285	−111	7.581	21.17			
5	15	80	5192	−112	7.482	20.17			
5	20	75	4806	−144	7.488	17.30			
5	25	70	5473	−140	7.405	22.18			
5	30	65	5387	−133	7.414	21.52			
5.16	4.53	90		− 82					
Carbon-Steel % C									
.8			5233	−110	7.849	21.49	11	− 90	−231
1.0			5209	−137	7.827	21.24	11	−126	−285
1.5			5217	−123	7.866	21.41	11	−112	−257

In practice I no longer use directly the wavemeter or the audiofrequency meter for accurately measuring wavelengths or frequencies, but instead beat everything against the rods or piezo-crystals, and use the audiofrequency meter for measuring differences of frequencies, and the wavemeter for identifying harmonics.

Study of Frequency, Sound-Velocity, and Temperature Coefficients of Frequency of Various Alloys.—I have investigated a large number of alloys for their magnetostrictive properties, and, by measuring the frequency of oscillation of a known length of specimen, have determined the velocity of a longitudinal sound wave in these various materials. These are recorded in Column II, Table VII. By making these measurements at various temperatures I have obtained the temperature coefficient of frequency (Column III). By the formula

$$v = \sqrt{\frac{\text{Young's modulus}}{\text{Density}}}$$

I have computed Young's modulus (Column V) from measured values of velocity and density (Column IV). This method gives high precision, and is interesting in that the value of the modulus is obtained for very small applied forces. Additional data of Table VII are described below.

Other alloys are under investigation and will be reported in a later publication. Many of the specimens of the table contain a small quantity of manganese (about .3%) added to make the specimen malleable.

Curves showing Results for Velocity and Temperature Coefficients of Frequency with Alloys of Various Compositions.—Fig. 7 shows velocity of sound in *km/sec* and also temperature coefficient of frequency of vibration with rods of nickel-steel plotted against percentages of nickel of the rods. It is seen that the velocity has a minimum and the temperature coefficient has a maximum at 36% to 40% nickel. This is at about the composition of Invar (36% Nickel). Fig. 8 contains corresponding curves for chromium-steel of various percentages of chromium; Fig. 9, similar results for various percentages of chromium in chromium-steels which contain also 5% silicon. In Fig. 8 the velocity is a maximum and the temperature coefficient of frequency is a maximum at 20% to 25% chromium. These results, Fig. 7 and Fig. 8, suggests as a possible law *that in a*

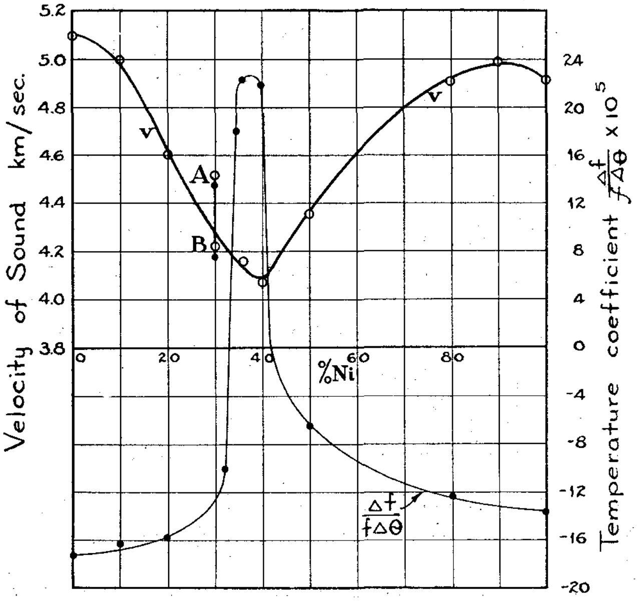

FIGURE 7. Velocity and Temperature Coefficient of Frequency of Nickel-Iron Alloys of Various Percentages of Nickel.

binary metallic alloy the temperature coefficient of frequency is a maximum or minimum at the composition at which the velocity is a maximum or minimum, or the reverse. Having found a means of extending the methods of the present research to non-magnetic alloys also I have assigned to a research student the problem of testing this apparent law for a wide variety of alloys.

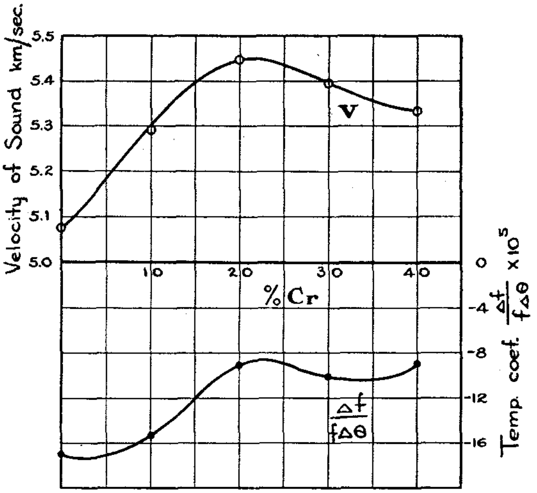

FIGURE 8. Velocity and Temperature Coefficient of Frequency of Chromium-Iron Alloys with Various Percentages of Chromium.

The curves of Fig. 9 show the effects of adding a third component, 5% silicon, to the chromium-iron series. This added silicon introduces a large minimum of velocity at 20% chromium, 5% silicon, and 75% iron. Indications of corresponding maxima or minima of the temperature coefficient of frequencies at compositions giving a maxima or minima of velocity are also apparent in these curves, but the compositions employed were not selected closely enough together to make the result certain.

An interesting result is the largeness of the change of velocity with change of composition.

The points marked *A* and *B* of Fig. 7 were obtained with a nickel-steel of 30% nickel. The values at *A* were obtained with the annealed specimen, and the interesting fact was noted that when the specimen was heated to about 50° C. it became non-magnetic and ceased to oscillate. When it was cooled down to 20° C. it again became magnetic and oscillatory, but reversibly lost or gained its magnetism as the 50° C. point was passed. This fact for about this composition has been previously observed by Hopkinson (see Ewing: Magnetic Induction in Iron and other Metals, Van Nostrand, 1900). Acting on the facts there given regarding effects of cooling, I had my specimen dipped into liquid air. After coming back to room temperature it gave the points marked *B* in Fig. 7. It had become a new substance and no longer lost its magnetism with change of temperature over the range up to 100° C.

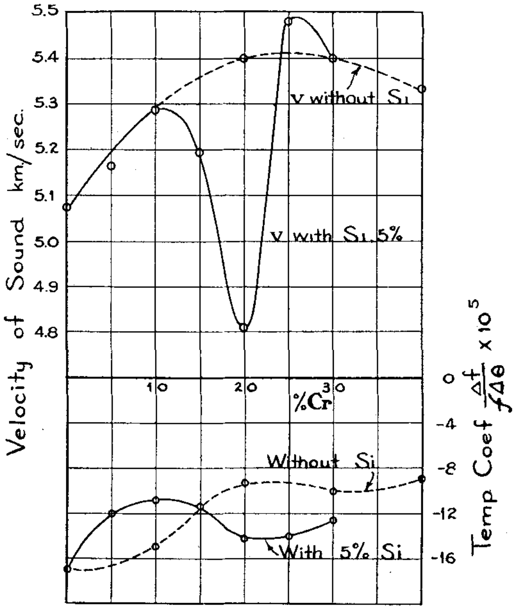

FIGURE 9. Effect of Adition of 5% Silicon to Chromium-Iron Alloys of Various Percentages of Chromium (Alscissae) and Rest Iron.

Temperature Coefficients of Elasticity and of Velocity.—Simple relations exist among various temperature coefficients, so that other coefficients may be obtained from the experimental data of Table VII combined with known quantities taken from the work of previous investigators.

Given $\Delta\theta$ = increment of temperature,

$$a = \frac{\Delta l}{l\Delta\theta} = \text{coefficient of linear expansion,}$$

$$b = \frac{\Delta E}{E\Delta\theta} = \text{coefficient of Young's modulus } E,$$

$$g = \frac{\Delta f}{f\Delta\theta} = \text{coefficient of frequency of longitudinal vibration,}$$

$$h = \frac{\Delta v}{v\Delta\theta} = \text{coefficient of velocity of longitudinal wave,}$$

and

$$2lf = v = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{EV}{W}}. \qquad (1)$$

In this equation (1)

l = length of a specimen, which has
f = frequency
ρ = density,
E = Young's modulus,
W = weight,
V = volume.

If the specimen satisfying equation (1) is raised 1° C. in temperature W does not change and we obtain

$$2lf(1 + a)(1 + g) = v(1 + h) = \sqrt{\frac{EV}{W}(1 + b)(1 + 3a)}$$

Dividing this by (1) and performing the indicated operations for small values of the coefficients we obtain

$$1 + a + g = 1 + h = 1 + \frac{b}{2} + \frac{3a}{2}$$

$$\therefore \qquad h = a + g, \text{ and } b = 2g - a. \qquad (2)$$

Equation (2) enables us to compute the temperature coefficient of elasticity (b) and the temperature coefficient of velocity (h) from the measured values of the temperature coefficient of frequency (g) and values of temperature coefficient of expansion (a). Since the latter are small in comparison with g they are given sufficiently accurately in ordinary tables of constants.

Values of a taken from *Bureau of Standards Circular No. 58* and from *International Critical Tables* are entered as Column VI of Table VII, and computed values of h and b are entered in Columns VII and VIII.

Figure 10 is a plot of E from my data on sound and also a plot of the corresponding values (Curve B) from the *Bureau of Standards Circular No. 58*. The large differences between these curves may be due to the fact that the values given in the Bureau Circular are obtained at large stresses while my values are for the very small stresses of sound waves. On the other hand, small differences in composition, as the .3% manganese in my specimens, or different heat treatment, may account for the differences. It should be further noted that my specimens were magnetized.

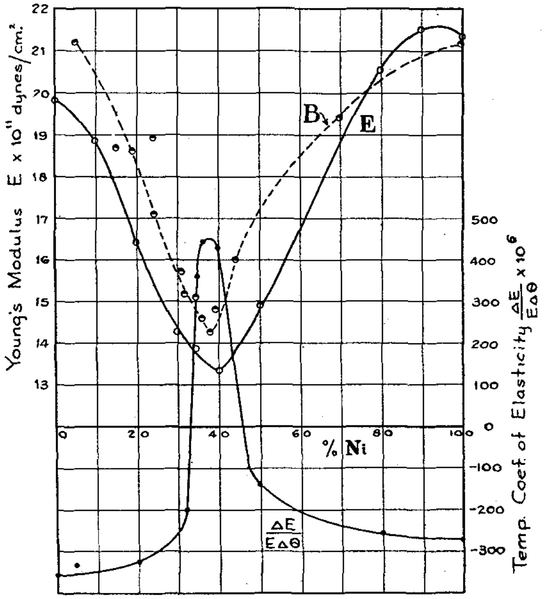

FIGURE 10. Young's Modulus and Temperature Coefficient of Elasticity of Nickel-Iron Alloys with Various Percentages of Nickel.

Methods of Obtaining Magnetostriction Vibrators of Zero or Very Small Temperature Coefficients of Frequency.—In Table VII the temperature coefficients of frequency (g) of magnetostriction vibrators of various alloys are given in column III. In particular it is seen that the nickel-iron alloys vary from

$$g = -171 \times 10^{-6} \text{ to } g = +224 \times 10^{-6}$$

and that this coefficient is zero for alloys of 33% ni and 47% ni. Unfortunately these alloys are difficult to produce with exact properties because the curve of temperature coefficient is very steep at these points. Furthermore, these materials are only very weak magnetostriction oscillators. With sufficient care in their production and heat treatment they may prove highly serviceable as oscillators of zero temperature coefficients. A further study of alloys of this class is under way.

In combinations of three metals I have found that a vanishing temperature coefficient is usually associated with non-magnetism or non-magnetostriction.

Consequently I have investigated the alternative method of composite vibrators made of different metals so arranged that the positive coefficient of one specimen counteracts the negative coefficient of another specimen, which have the merit of simplicity of construction and can be made of materials giving a strong resultant vibration.

Two kinds of composite vibrators have been employed with success:

(*a*) A central rod of negative coefficient with a rod of positive coefficient soldered end to end at each end of the central rod. This I shall call *longitudinally composite.*

(*b*) A tube of coefficient of one sign containing a tight-fitting core rod of coefficient of the opposite sign. This I call *concentrically composite.*

A number of vibrators of both these types have been made. For standards the concentrically composite type have more power in controlling frequency and are preferred. Two such vibrators consisting of nickel tubes with stoic metal cores driven into them at a tight press-fit were found to have coefficients

$$\frac{\Delta f}{f\Delta\theta} = -\frac{1}{50000} \text{ and } -\frac{1}{65000}$$

respectively. Smaller values of this coefficient can be obtained by better choice of relative diameters of the components.

It should be noted that the cause of the change of any of these vibrators with change of temperature is the temperature coefficient of elasticity of the materials and not the change of length or change of density, both of which are relatively small.

Elementary Theory of Dynamic Magnetostriction.—As a simplification of the more exact treatment of the subject, such as is given in Appendix I, there is here presented a discussion based on the assumption that the rod acts mechanically as a pair of masses separated by an elastic member and damped by friction proportional to velocity, so as to satisfy the equation

(1) $$f' = m\ddot{x} + r\dot{x} + \varepsilon\, Sx/l.$$

In this equation the dots indicate time derivatives, and r, m, and ε are equivalent values of frictional resistance, mass, and Young's modulus respectively,

S = Area of cross section

l = half length of the rod

x = increase of length of the half rod-length due to the force f'.

The force f' is a periodic magnetostrictive force due to a periodic magnetic induction B of comparatively small amplitude superposed on a fixed induction B_o, which is used to polarize the specimen.

Law I of Magnetostriction.—As a first law of magnetostriction we shall assume that the magnetostrictive force f' caused by the periodic induction B is proportional to B and to the area of cross section S; that is,

(2) $$f' = aBS,$$

where a (positive for some substances and negative for others) is a coefficient depending on B_o but otherwise assumed constant.

We shall regard B as made up of two parts B_i and B'

(3) $$B = B_i + B',$$

where B_i = induction due to the periodic current i,

B' = induction due to the reverse magnetostriction effect (Law II).

Law II of Magnetostriction.—When the rod is stretched an amount $2x$ with an original rod-length $2l$ there is produced an induction B' proportional to x/l; that is,

(4) $$B' = a'x/l.$$

Solution for Time-Derivative of x.—The substitution of (2), (3), and (4) into (1) gives

$$aSB_i = m\ddot{x} + r\dot{x} + \{\varepsilon - aa'\}x/l, \tag{5}$$

of which the solution for the time-derivative of x on the assumption that B_i involves the time only in a factor of the form $e^{j\omega t}$ is

$$\dot{x} = \frac{aSB_i}{r + j\{m\omega - \varepsilon'S/l\omega\}} \tag{6}$$

where as an abbreviation

$$\varepsilon' = \varepsilon - aa'. \tag{7}$$

Electrical Equations.—We shall next write the electrical equations. The alternating current i is related to the impressed *e. m. f.* e by the equation

$$e = Ri + \frac{d}{dt}\{\text{Flux linkage of } (B_i + B')\}. \tag{8}$$

The first two terms of the right-hand side of this equation give the electromotive force if the rod were damped so that it did not move. The remaining term gives the electromotive force due to the motion, which we shall call *motional e. m. f.* and shall designate by e', so

$$e' = \frac{d\phi}{dt}, \text{ where} \tag{9}$$

$$e' = \text{damped } e.\,m.\,f.,$$

$$\phi = \text{Flux linkage of } B'.$$

If now we write

$$\phi = a''B' \quad \text{and} \tag{10}$$

$$B_i = L'i, \tag{11}$$

where a'' and L' are constants depending on the size, shape, and number of turns of the coil and on the permeability of the specimen, and combine the equations (4), (6), (9), (10), and (11), we obtain

$$e' = z'i, \text{ where} \tag{12}$$

$$z' = \frac{aa'a''L'S/l}{r + j\{m\omega - \varepsilon'S/l\omega\}}. \tag{13}$$

In equation (13) z' is the complex motional impedance of the system. ω is the angular velocity of the current. a, a', a'' and L' are coefficients defined by (2), (4), (10), and (11) respectively.

This theory and the discussion of the next section is essentially of the same character as that given by Professor Kennelly and me for the impedance of telephone receivers (*Proc. Am. Acad.*, Vol. 48, No. 6, 1912, and *Electrical World*, September, 1912.)

Impedance Circle Diagrams.—If we let

R' = motional resistance,

X' = motional reactance

of the rod-coil unit, and as further abbreviations, write

(14) $A = aa'a''L'S/l$ and

(15) $Z_m^2 = r^2 + \{m\omega - \varepsilon'S/l\omega\}^2,$

we find that equation (13) after rationalization gives

(16) $$R' = \frac{Ar}{Z_m^2}$$

(17) $$X' = \frac{-A\{m\omega - \varepsilon'S/l\omega\}}{Z_m^2}$$

whence

$$R'^2 + X'^2 = A^2/Z_m^2.$$

The substitution of (16) so as to eliminate Z_m gives

(18) $$R'^2 + X'^2 = AR'/r.$$

This is the equation to a circle, passing through $R' = 0$ and $X' = 0$, and having a diameter A/r, and is represented as the left-hand circle of Fig. 11. Each value of ω gives a point of the circle. The circle is thus the locus of X' plotted against R', or, otherwise stated, it is a vectorial plot of z' as ω varies.

Impedance Circle-Diagrams With Diameter Dipping.—In the preceding section the diameter of the Impedance Circle lies along the axis of motional resistance. Experimentally this is found to be not the fact. The diameter of the circle usually dips by an angle which we shall call β.

This result can be obtained theoretically if we let one or more of the quantities a, a', a'', L' be a complex quantity independent of ω so that

$$Ae^{-j\beta} = aa'a''L'S/l. \tag{17}$$

This is equivalent to multiplying every vector z' by $e^{-j\beta}$ which gives the whole diagram the form of the right-hand circle of Fig. 11.

As to the physical significance of complex values of a, a', a'', or L', which may be inferred from the experimentally determined values of impedance circles, it seems probable that a and a' are the complex quantities and that their complexity is due largely to hysteresis effects and eddy-currents in the magnetostrictive core as the dip angle largely disappears when the core is a thin nickel tube split longitudinally.

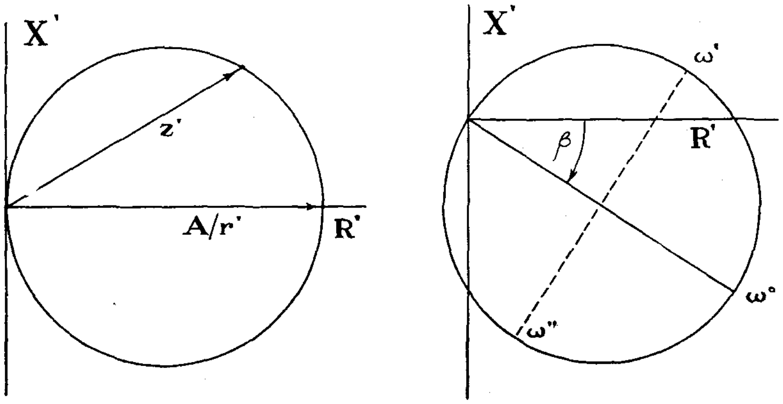

Figure 11. Impedance Circle Diagrams.

An extensive experimental study of motional impedance in the case of magnetostriction has been made at this laboratory by Mr. K. C. Black at my suggestion and is contained in his thesis for the Doctorate of Philosophy deposited in manuscript in the Harvard Library. He is publishing a short account of this investigation in a paper following this in the Proceedings of the American Academy.

The results are very remarkable in showing that the magnetostrictive system has a very high efficiency as a converter of electric energy into mechanical motion.

Magnetostrictive Sources of Sound.—I have made a number of applications of magnetostriction to the production of sound and ultra sound over a wide range of frequencies. These devices are outside of the scope of the present paper. One application, which is to the measurement of the velocity of sound at various frequencies, is being employed by some of my graduate students in filling in the gaps of the work I previously published on Proc. Am. Acad. Vol. 60, No. 5, 1925, in which a piezo-electric crystal oscillator was the radiator of the sound, and the reaction of the reflected sound on the source was the means for indicating nodes and loops of the standing wave system. The frequencies used in those experiments extended from 40,000 to 1,400,000 cycles per second. The use of the magnetostriction vibrators in the same general arrangement of apparatus permits the use of frequencies below the 40,000 cycle value, and also, since rods are easily cut to any predetermined frequency, permits filling in certain significant frequencies that were lacking in the previous investigations. Also various gases are being employed in the experiments.

Another sound-velocity application consists in putting various solid non-magnetostrictive rods on the end of a magnetostrictive driving rod, and by means of frequency measurements determining sound-velocity and elastic properties of non-magnetic and other non-magnetostrictive materials. These new investigations are well under way.

Arrangements for Obtaining High Frequencies.—In the production high-frequency stabilization by magnetostriction rods I have used three methods. (*a*) One is by the use of very short cylinders, placed within the plate coil with their axes parallel to the magnetic field of this coil; (*b*) another is by flat strips or sheets of the magnetostrictive material placed between the plate and grid coils; and (*c*) by what I may call *beaded rods*. The latter method is probably the best. An illustrative example, shown in Fig. 12, consists of a rod 1.26 cm. in diameter and 9 cm. long with grooves turned in it to a depth of half the radius. The grooves are 1 cm. wide and separated by one centimeter, so that the vibrator is in the form of five cylindrical beads, each one centimeter long held in succession one centimeter apart by cylindrical rods of half the diameter of the beads.

When this beaded rod has been annealed and is placed in a coil-pair having 200 turns on each coil, and so positioned that one bead is in one coil and the next bead is in another coil or else one bead between coils, it shows strong stabilization of frequency 295,480 cycles per second. The material is stainless steel with velocity of 5430 meters per second, so the frequency is that of a linear vibrator

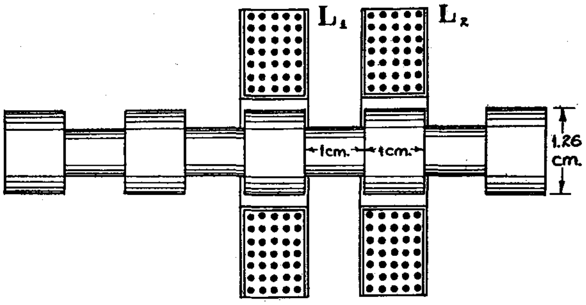
L_1
L_2
1cm.
1cm.
1.26 cm.

FIGURE 12. Beaded Rods.

of length $543000 \div 2 \times 295480 = .918$ cm., which is about 10% less than the computed value of one of the beads standing alone and regarded as a linear vibrator. There are other frequencies of the system particularly one at 19888 cycles per cecond.

The theory of such a beaded vibrator has been pretty well worked out but to save space is not here given.

For lack of space also the general vacuum-tube theory is here omitted.

In the accompanying appendix a more detailed theory of sound propagation in the magnetostriction rod is presented.

APPENDIX I.

Theory of Dynamic Magnetostriction, Including Theory of Propagation of Sound in a Viscous Magnetostrictive Medium.— Referring to Fig. 13, let us consider a rod of length $2l$ wound with a solenoid of N turns per centimeter.

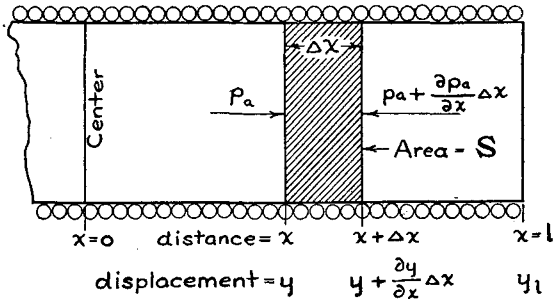
Δx
Center
p_a
$p_a + \frac{\partial p_a}{\partial x}\Delta x$
Area - S
$x=0$
distance = x
$x+\Delta x$
$x=l$
displacement = y
$y + \frac{\partial y}{\partial x}\Delta x$
y_l

FIGURE 13.

Let S = area of cross section of the rod. Let x = the distance from the center to any section of the rod, and let us consider a volume of length Δx extending from x to $x + \Delta x$.

Let p_a and y = external pressure and displacement at x of the section Δx, and

$$p_a + \frac{\partial p_a}{\partial x}\Delta x \text{ and } y + \frac{\partial y}{\partial x}\Delta x = \text{the corresponding quantities at } x + \Delta x,$$

and let us note that the pressure at x and $x + \Delta x$ are both estimated inward upon the section Δx, while the displacements are both in the x-direction.

Before analysing the external pressure p_a into its components, we may equate the force in the x-direction to the mass × acceleration of the elemental volume Δx, obtaining

$$(1) \qquad S\left[p_a - \left\{p_a + \frac{\partial p_a}{\partial x}\Delta x\right\}\right] = S\Delta x\rho\frac{\partial^2 y}{\partial t^2},$$

in which ρ = density of the material of the volume element.

Performing the operations of (1), dividing by $S\Delta x$ and taking the limit as Δx approaches zero, we obtain

$$(2) \qquad -\frac{\partial p_a}{\partial x} = \rho\frac{\partial^2 y}{\partial t^2}.$$

Let us now analyse p_a into its constituents; namely, let

(3) $p_a = p_\varepsilon + p_G + p_m$, where

p_ε = pressure due to elasticity,

p_G = pressure due to viscosity,

p_m = pressure due to magnetostriction.

These various p's are due to the pressures of expansion of an element to the left of that shown in the figure, and are given by the equations

$$(4) \qquad p_\varepsilon = -\varepsilon\left\{\frac{\text{increase of volume}}{\text{volume}}\right\} = -\varepsilon\frac{\partial y}{\partial x},$$

$$(5) \qquad p_G = -G\frac{\partial}{\partial t}\left\{\frac{\text{increase of volume}}{\text{volume}}\right\} = -G\frac{\partial}{\partial t}\left(\frac{\partial y}{\partial x}\right),$$

(6) $p_m = aB$, in which

ε = Young's modulus in dynes per sq. cm.

G = a viscosity coefficient in *c. g. s.* units,

B = magnetic induction at x, producing p_m,

a = a coefficient relating expansive pressure of magnetostriction in dynes per sq. cm. to magnetic induction B.

The coefficients ε and G are positive, a is positive for some magnetostrictive materials and negative for others. Also a is not exactly constant but varies with B as well as with the material. We shall, however, need to consider it a constant where B is a small induction added to an already existent constant large induction.

The substitution of (4), (5), and (6) into (2) and (3) gives

$$\rho \frac{\partial^2 y}{\partial t^2} = \varepsilon \frac{\partial^2 y}{\partial x^2} + G \frac{\partial}{\partial t} \frac{\partial^2 y}{\partial x^2} - a \frac{\partial B}{\partial x}, \tag{7}$$

and

$$p_a = - \varepsilon \frac{\partial y}{\partial x} - G \frac{\partial}{\partial t} \frac{\partial y}{\partial x} + aB. \tag{8}$$

We shall now further analyse B. We shall suppose the existence of a steady magnetic induction B_o, which will polarize the system and determine the size of the coefficient a, but will not contribute to p_m, which we shall suppose to be due to an additional periodic B. This additional B is made up of a quantity due to the variable current i in the windings plus an additional quantity B' called into play by magnetostriction as a result of distortion; that is

(9) $B = B_i + B'$ where
B_i = magnetic induction due to i,
B' = magnetic induction due to expansion of the element Δx and is a function of x.

This B' we shall assume proportional to increase of length of the element divided by the original length of the element; that is,

$$B' = a' \frac{\partial y}{\partial x}. \tag{10}$$

In substituting (9) and (10) into (7) and (8) we shall regard

$$\frac{\partial B_i}{\partial x} = 0. \tag{11}$$

The substitution then gives

(12) $$\rho\frac{\partial^2 y}{\partial t^2} = \varepsilon'\frac{\partial^2 y}{\partial x^2} + G\frac{\partial}{\partial t}\frac{\partial^2 y}{\partial x^2}$$

and

(13) $$p_a = -\varepsilon'\frac{\partial y}{\partial x} - G\frac{\partial}{\partial t}\frac{\partial y}{\partial x} + aB_i,$$

where as an abbreviation

(14) $$\varepsilon' = \varepsilon - aa'.$$

Equations (12) and (13) are to be treated as simultaneous.

In solving (12) we shall here restrict the treatment to the case in which the time t enters in y only in a factor of the form $e^{j\omega t}$, where ω is the angular velocity of the current i. This reduces (12) to the form

(15) $$-\rho\omega^2 = (\varepsilon' + jG\omega)\frac{\partial^2 y}{\partial x^2},$$

which gives

(16) $$y = e^{j\omega t}\,\{Ae^{kx} + A'e^{-kx}\}$$

with

(17) $$k = +j\omega\sqrt{\frac{\rho}{\varepsilon' + jG\omega}}.$$

We shall next stipulate that the center of the rod has zero motion; that is, $y = 0$ at $x = 0$. This gives

(18) $$y = A\,e^{j\omega t}\sinh kx.$$

Now to determine A, we introduce the further boundary condition that $p_a = 0$ at $x = l$, so (13) gives

(19) $$0 = -(\varepsilon' + jG\omega)\frac{\partial y}{\partial x} + aB' \quad \text{at } x = l.$$

Replacing y in this equation by its value from (18) we obtain

$$Ae^{j\omega t} = \frac{aB'}{k(\varepsilon' + jG\omega)\cosh kl},$$

and consequently by (18)

(20) $$y = P\sinh kx,$$

where as an abbreviation

$$P = \frac{aB'}{k(\varepsilon' + jG\omega)\cosh kl}. \tag{21}$$

Let us now turn to a consideration of the electrical system in which there is a variable current i flowing. The variable *e. m. f.* at the terminals of the coil is

$$e = Ri + \frac{\partial}{\partial t}\{\text{flux linkage of } (B_i + B')\}. \tag{22}$$

The first two terms of the right-hand side of this equation give e damped; that is, the value of e if the rod were restrained from vibrating. The last term is the contribution of the motion, and may be called the *motional e. m. f.*, so

$$e' = \frac{\partial \varphi}{\partial t}, \quad \text{where} \tag{23}$$

e' = motional *e. m. f.*,

φ = flux linkage of B' with the circuit.

To obtain φ, let us note that per unit area of S at any element Δx, distant x from the center, magnetic induction of amount $B' - (B' + \Delta x \partial B'/\partial x)$ leaks out between the turns of wire and these return to the core at distance minus x from the center, and thus link with $2Nx$ turns giving

$$\Delta\varphi = -\,2NSx\frac{\partial B'}{\partial x}\Delta x,$$

where N = number of turns of wire per centimeter.

To get the total linkage we must integrate this from $x = 0$ to $x = l$, and must add $2NSlB_l'$ to account for leakage from the end, obtaining

$$\varphi = 2NS\left\{-\int_0^l x\frac{\partial B'}{\partial x}dx + lB_l'\right\}. \tag{24}$$

We may first integrate this by parts by putting

$$u = x,\ dv = \frac{\partial B'}{\partial x}dx, \ \text{obtaining}$$

$$\varphi = 2NSl\int_0^l B'\,dx.$$

We may now complete the integral by replacing B' by its value from (10) and obtain

(25) $\varphi = 2a' NS\, y_l,$ where

$y_l =$ value of y at l.

By (20) and (21) this gives

$$\varphi = 2a' NSP \sinh kl$$

$$= \frac{2aa' NSB_i}{k(\varepsilon' + jG\omega)} \tanh kl.$$

Now returning to the motional *e. m f.* equation (23), and noting that the time derivative is $j\omega$ times the quantity, and then replacing $j\omega/k$ by its value from (17), we obtain

$$e' = \frac{2aa' NSB_i}{\sqrt{\rho\{\varepsilon' + Gj\omega\}}} \tanh kl. \tag{26}$$

If now

$L =$ self inductance of the coil with rod damped, we may write $Li =$ flux linkage of Bi, which is $2NSlB_i$, so

$$B_i = \frac{Li}{2NSl}, \tag{27}$$

which substituted into (26) gives

$$e' = Qi \tanh kl. \tag{28}$$

In this equation I have introduced as an abbreviation

$$Q = \frac{aa' L}{l\sqrt{\rho\{\varepsilon' + jG\omega\}}}. \tag{29}$$

Motional Impedance.—If we let $z' =$ complex motional impedance of the system, and compare (28) we have

$$z' = Q \tanh kl. \tag{30}$$

Equation (30) gives the complex motional impedance of the system, Q and k are defined by (29) and (17) respectively.

To investigate the manner in which z' varies with ω, we may note that while Q involves ω, the term in which the ω appears is comparatively small, and since the whole motional action is confined to a very small range of ω, we may regard Q as a constant.

Let us now write

$$k = \alpha + j\beta, \text{ where} \tag{31}$$

$$\alpha = \omega\sqrt{\frac{\rho}{\varepsilon'(1+h^2)}}\,g(h), \tag{32}$$

$$\beta = \omega\sqrt{\frac{\rho}{\varepsilon'(1+h^2)}}\,f(h), \tag{33}$$

$$f(h) = \sqrt{\frac{\sqrt{1+h^2}+1}{2}}, \tag{34}$$

$$g(h) = \sqrt{\frac{\sqrt{1+h^2}-1}{2}}, \tag{35}$$

$$h = G\omega/\varepsilon'. \tag{36}$$

Regarding the functions $f(h)$ and $g(h)$ see G. W. Pierce: *A Table and Method of Computing Electric Wave Propagation, Transmission Line Phenomena, Optical Refraction, and Inverse Hyperbolic Functions of a Complex Variable, Proc. Am. Acad., Vol. 57. No. 7, 1922,* where a table of the functions f and g is given for a large range of values of h.

Introducing the value (31) for k into (39) and breaking z' into its resistance and reactance components, R' and X', we have

$$z' = R' + jX' = Q\frac{\sinh(\alpha + j\beta)\,l}{\cosh(\alpha + j\beta)\,l}$$

which after expansion and rationalization becomes

$$R' + jX' = Q\frac{\sinh al\cosh al + j\sin\beta l\cos\beta l}{\sinh^2 al + \cos^2\beta l}.$$

From this we may obtain

$$R'^2 + X'^2 = 2R_oR', \qquad \text{where} \tag{37}$$

$$2R_o = Q\frac{\sinh^2 al + \sin^2\beta l}{\sinh al\cosh al}. \tag{38}$$

Equation (37) is in the form of the equation of a circle of radius R_o. Equation (38) shows, however, that R_o involves the variable parameter ω so (37) is not a true circle. Nevertheless, in the case of the magnetostrictive system, the resonance is so sharp that the whole phenomenon of motional reactance is exhibited within a frequency range of a fraction of one percent of the resonant frequency; so the variation of the coefficient R_o is inappreciable to the limit of the errors of the experimental measurements that Dr. Black has made and is publishing in a succeeding number of these proceedings.

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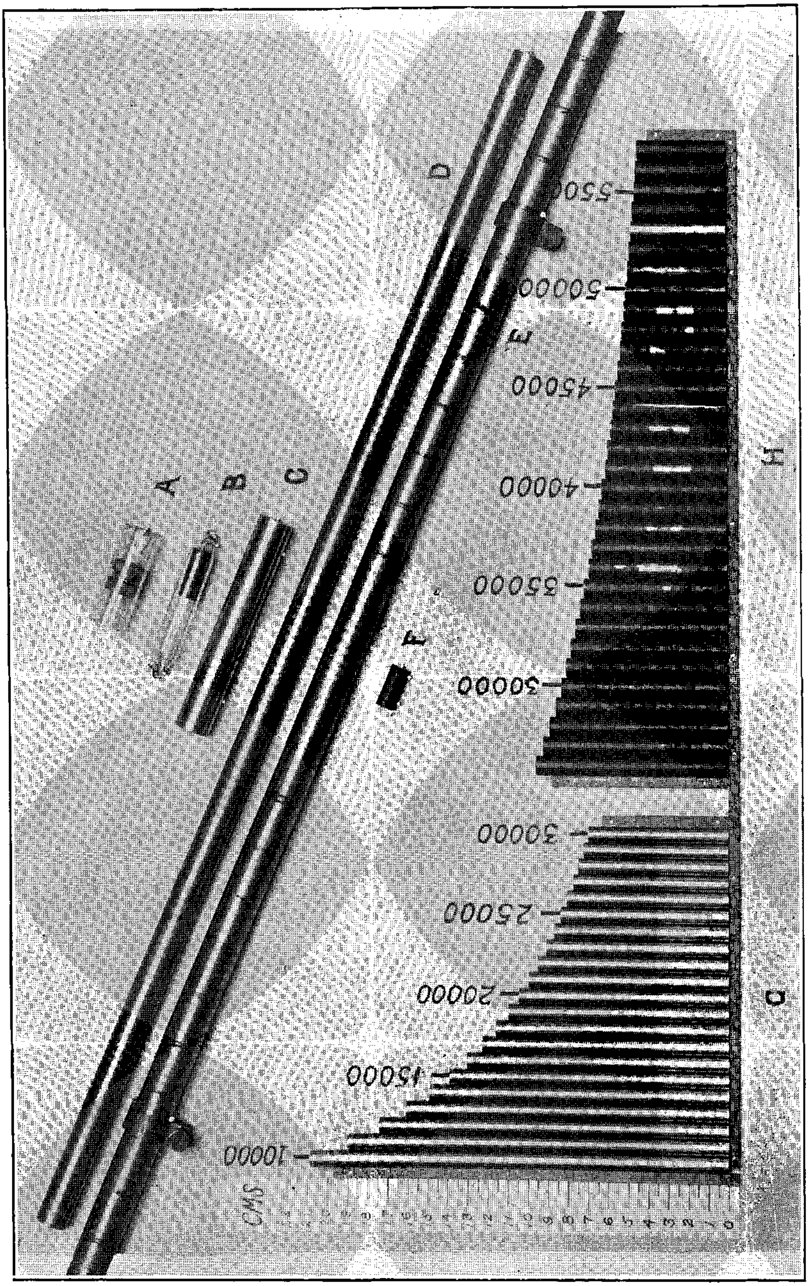
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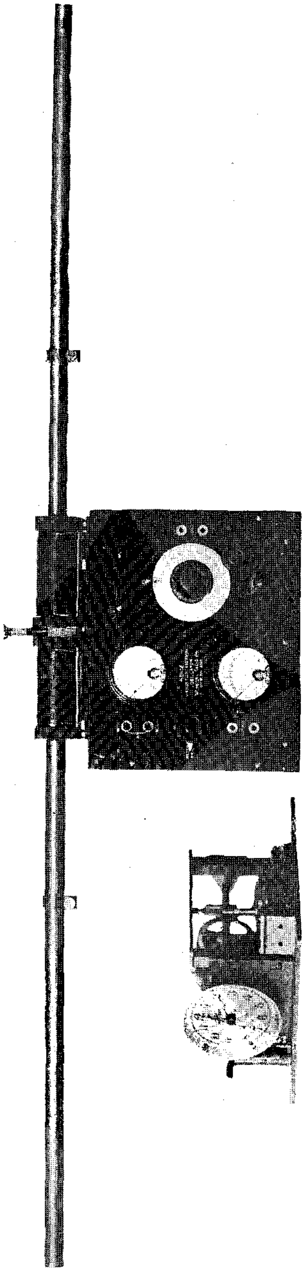

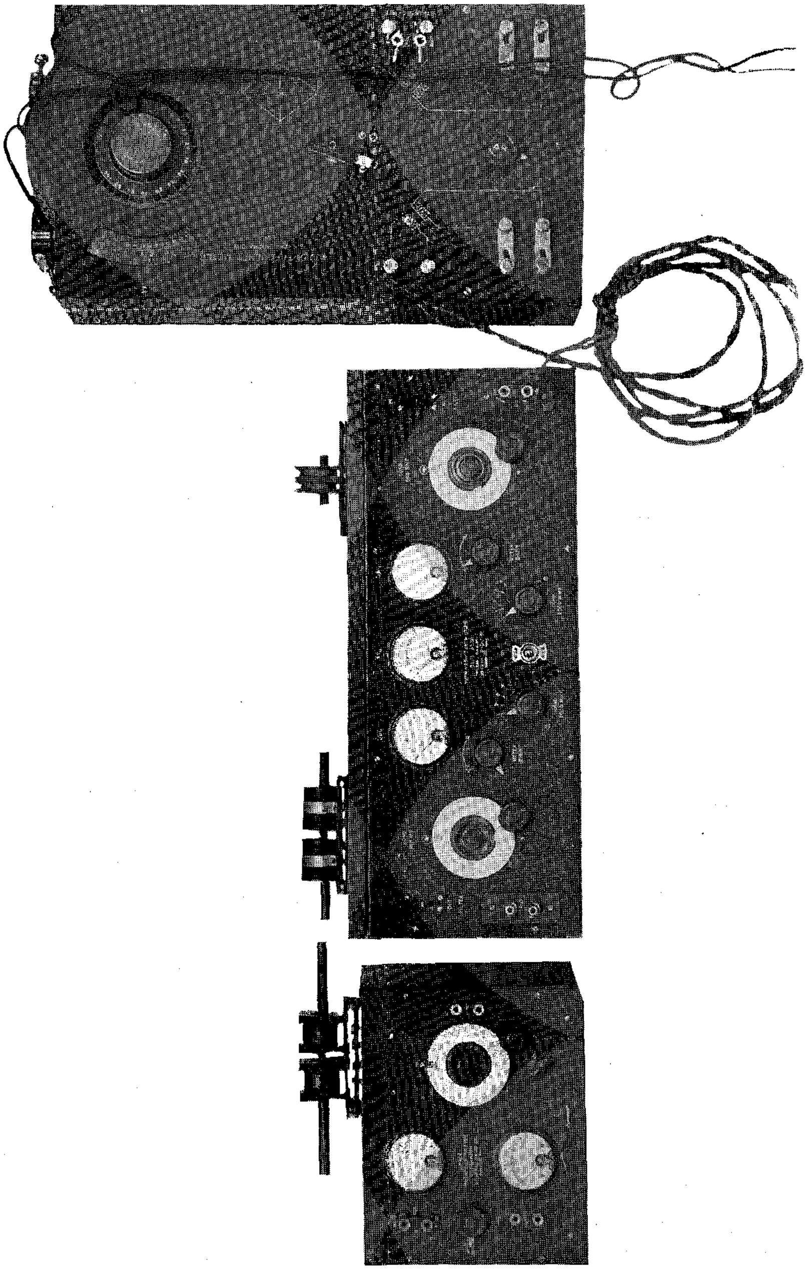

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Vol. 63. No. 2.—April, 1928.

A DYNAMIC STUDY OF MAGNETOSTRICTION.

By K. Charlton Black.

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(Continued from page 3 of cover.)

VOLUME 63.

1. **Pierce, George W.**—Magnetostriction Oscillators. An Application of Magnetostriction to the Control of Frequency of Audio and Radio Electric Oscillations, to the Production of Sound, and to the Measurement of the Elastic Constants of Metals. pp. 1–47. 3 pls. April, 1928. $0.90
2. **Black, Charlton K.**—A Dynamic Study of Magnetostriction. pp. 49–66. April, 1928. $0.45.

Proceedings of the American Academy of Arts and Sciences.

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A DYNAMIC STUDY OF MAGNETOSTRICTION.

By K. Charlton Black.

A DYNAMIC STUDY OF MAGNETOSTRICTION

By K. Charlton Black

Presented Jan. 11, 1928 by G. W. Pierce. Received Feb. 15, 1928.

In a recent paper by Professor G. W. Pierce[1] are described various effects which occur when a magnetic rod has superimposed upon it an alternating axial magnetic field of a frequency in the neighborhood of the natural mechanical frequency of the rod. Rods under the influence of magnetostriction can be made to play a part somewhat analogous to quartz crystals under the influence of piezoelectricity.

If a ferromagnetic rod or tube is clamped so that it lies along the axis of a coil, the combination becomes an electrical vibration instrument, and may be treated in the same way that telephone receivers are treated by Kennelly and Pierce,[2] or by Kennelly in his book on vibration instruments.[3] The present paper describes the measurement of the dynamic electrical characteristics of various magnetostrictive rods and tubes, also the magnitude of vibration and other measurements that could be made conveniently at the same time.

Method of Measurement.—The motional resistance and reactance are measured by connecting the coil containing the rod into an alternating current bridge, as in the reference[2] mentioned above. In this particular experiment, the bridge was designed to pass a direct current as well as the alternating current through the coil, thus it was possible to maintain a known field on the rod at any time. The coil is a long solenoid to insure a uniform magnetic field. The coil is 120 cm. long, has an inside diameter of two inches, and at its center developes a field of 32.5 gauss per ampere. The rods are all about 90 cm. long, and their natural frequencies lie between 2300 and 3000 cycles per second. In all measurements the superimposed alternating field is small compared to the direct or polarizing field.

In measurement of amplitude of vibration, an extremely light phosphor bronze strip upon which is mounted a small mirror is fastened from the end of the rod to the stationary frame to which the center of the rod is clamped. The longitudinal vibration of the rod thus gives an angular motion of the mirror. A point source of light reflected from the mirror is observed in a telescope with a micrometer eyepiece. By observing the motion of the spot of light in the telescope, and by measuring certain lengths (mirror arm etc.) the actual vibration amplitude can be calculated.

---

[1] G. W. Pierce—Magnetostriction Oscillators, These Proceedings, Vol. 63, No. 1, 1928.

[2] Kennelly and Pierce—Proc. Am. Acad., Vol. 48, No. 6, p. 113.

[3] Kennelly—Electrical Vibration Instruments.

Effect of motion on resistance and reactance of solenoid with axial magnetostrictive rod.—Below are shown two curves typical of the electrical characteristics of the circuit near the resonance frequency of the rod.

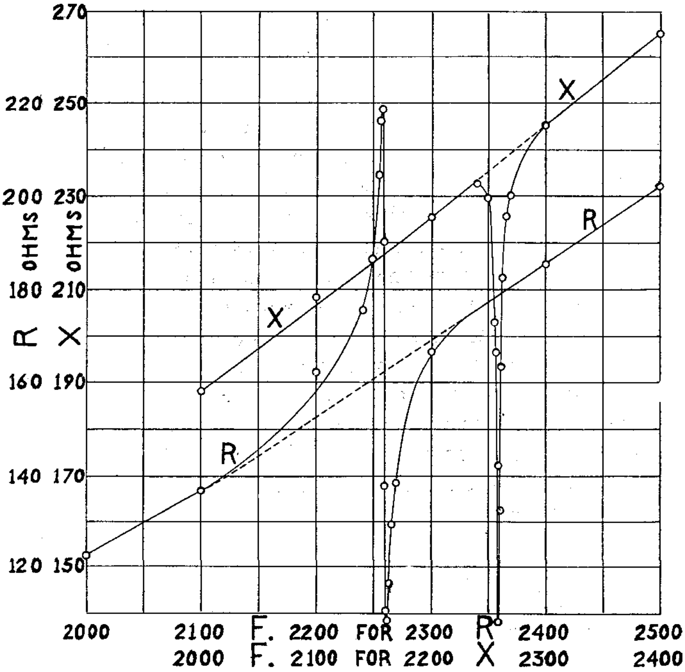

Figure 1.

Resistance *R* and reactance *X* plotted against frequency.

Stoic Metal Rod # 1 (length 92.43 cm. diameter .631 cm.).

The dotted part of the curves indicates damped values.

To avoid confusion, the frequency scales of the curves are displaced 100 cycles.

The large departure of *R* and *X* with the rod free (indicated by continuous curve) from the damped values (indicated by dotted curve) is due to the motion of the rod as the frequency of the oscillations approaches and passes the natural frequency of the longitudinal vibration of the rod. The values obtained by subtracting R and X of the damped system from the R and X of the free system have been called by Kennelly and Pierce motional values. The motional R (for the experimental conditions of the particular case shown above) ranges from -50 to $+50$ ohms approximately, and the motional reactance ranges from 0 to -100 ohms.

Results of measurements on rods.—Readings were taken on ten specimens in all. Three of these ten are of *stoic* metal (about the composition of invar steel), six are of commercial Grade A *nickel,* and one is of *stainless steel.* The different specimens of stoic and

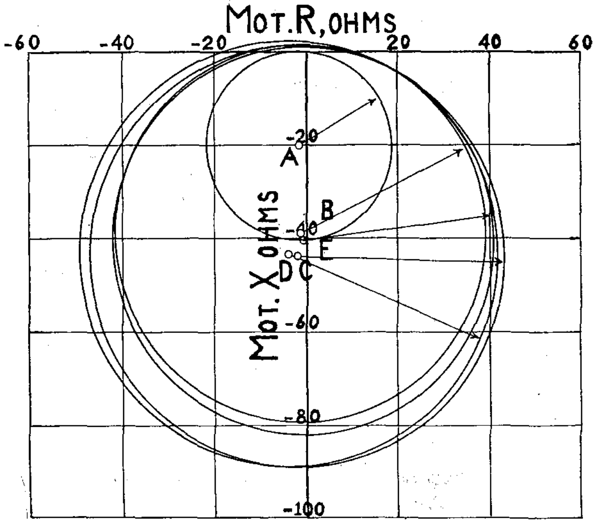

FIGURE 2.

Motional R plotted against motional X.
Stoic Metal Rod # 2 (length 94.38 cm. diameter .471 cm.).
Alternating field = .325 gauss.

Curve	*Polarizing field* *gauss*	*Max. motional impedance* *ohms*
A	4.75	40.2
B	8.49	80.8
C	15.18	91.2
D	20.32	91.6
E	27.63	82.8

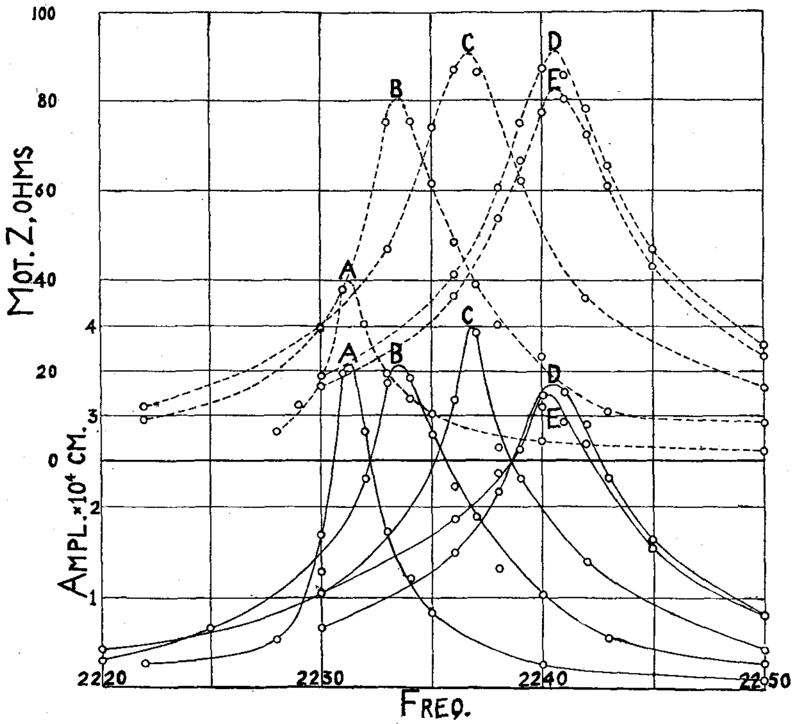

FIGURE 3.

Motional impedance plotted against frequency, dotted curves.
Amplitude plotted against frequency, smooth curves.
Stoic Metal Rod # 2.
Alternating and polarizing fields as in Figure 2.

of nickel are of different diameters, and it was hoped that the law of variation of motional impedance with diameter could be deduced, but owing to the fact that different specimens had different heat treatments, the results (other than that motional impedance increases with increasing diameter) are inconclusive.

One set of curves for each kind of metal is given. (See figures 2 to 7.) Each set has three types of curves.

Type 1.—Graphs of motional resistance against motional reactance. (cf. Kennelly and Pierce, also Pierce, references 2 and 1). These graphs are called circle diagrams.

Type 2.—Graphs of real motional impedance (motional Z) against frequency.

Type 3.—Graphs of amplitude of vibration of one end of the rod against frequency.

The failure of some circles to pass through the origin is due to a slight uncertainty in the determination of the damped values of R and X.

It can be seen at once that the motional impedance circles are very dependent on the polarizing field H. Curves showing the variation of maximum motional Z with H are shown later.

The motional impedance of rods is independent of small variations in the intensity of the alternating field. In measurements on tubes a dependence did appear.

A comparison was made between the amplitude of vibration and the motional impedance. In a majority of cases the motional impedance is found to be proportional to the amplitude. Variations

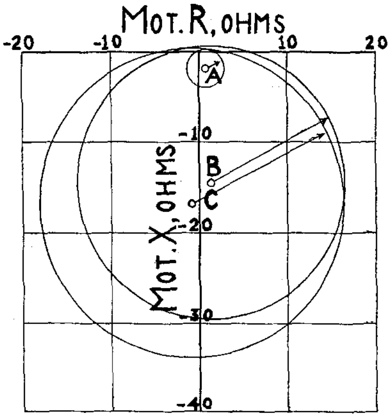

Figure 4.

Motional R plotted against motional X.
Nickel Rod # 6 (length 90.00 cm. diameter .790 cm.).
Alternating field = .325 gauss.

Curve	*Polarizing field* gauss	*Max. motional impedance* ohms
A	14.7	4.4
B	21.1	30.6
C	27.7	34.4

from proportionality of from two to ten per cent did appear, but the first power law more nearly fits the facts than any other simple relation. For small values of amplitude and motional impedance the errors are too great to allow the relation to be checked, but for large values the proportionality of the two quantities seems fairly certain. To the limit of accuracy of the experiment the maximum amplitude and maximum motional impedance occur at the same frequency.

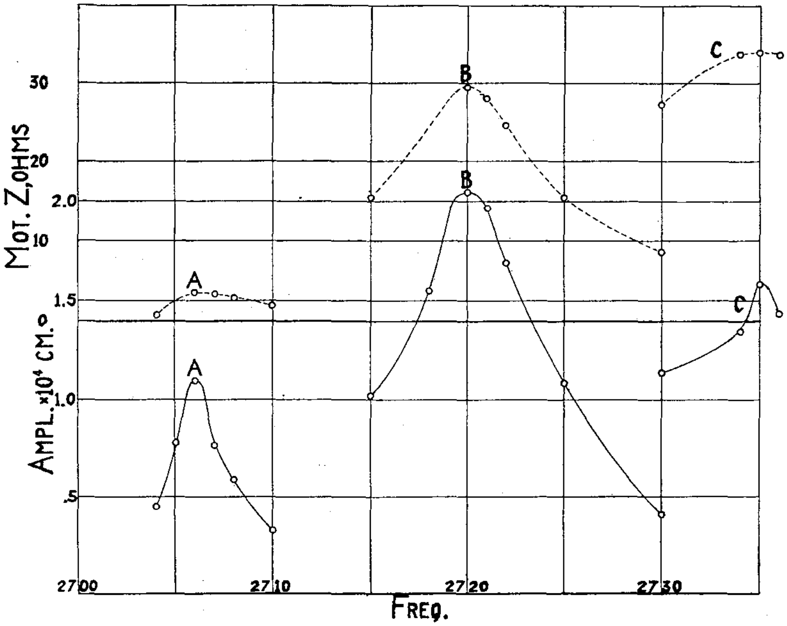

Figure 5.

Motional impedance plotted against frequency, dotted curves.
Amplitude plotted against frequency, smooth curves.
Nickel Rod # 6.
Alternating and polarizing fields as in Figure 4.

It is noticeable that the largest circle for stoic metal is about 90 ohms in diameter, for stainless steel about 70 ohms, and for nickel rod about 35 ohms. This gives a typical indication of the relative intensities of magnetostrictive reaction of these metals on any circuit. The reaction can be increased by suitable heat treatment.

Depression angle.—The depression angle is the angle between the diameter drawn from the origin and the positive motional resistance axis. As shown in a reference already mentioned,[2] this angle represents twice the lag of the magnetic flux behind the current in the coil. For all rods the depression angle is about 90 degrees, and for any one rod it stays essentially constant. This shows that for all rods studied the flux lags approximately 45 degrees behind the current.

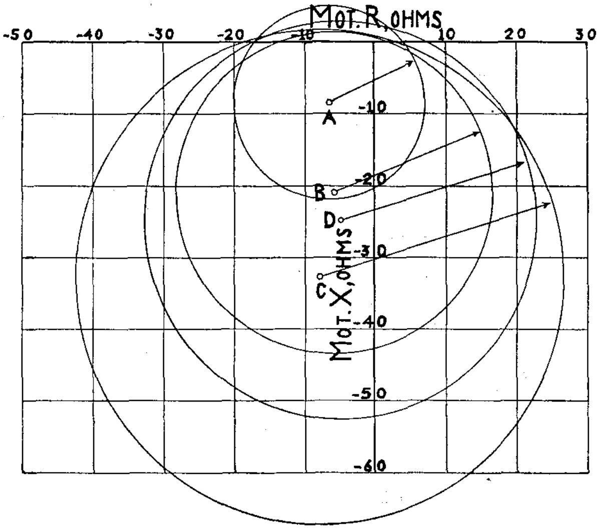

FIGURE 6.

Motional R plotted against motional X.
Stainless Steel Rod # 1 (length 90.00 cm. diameter .691 cm.).
Alternating field = .325 gauss.

Curve	*Polarizing field* *gauss*	*Max. motional impedance* *ohms*
A	10.6	26.8
B	15.0	49.8
C	19.6	69.6
D	25.8	55.2

Decrement.—The decrement Δ of the system may also be calculated by methods described in reference[2]. The expression giving the decrement is

$$\Delta = 2\pi \frac{(f_1-f_2)}{2}$$

where f_1 and f_2 are those frequencies for which the motional impedance is $\frac{1}{\sqrt{2}}$ times its maximum value. Owing to the extreme sharpness of the resonance curves accurate determinations of Δ are quite difficult. In all cases except that of stainless steel the decrement increases with increasing field. Stainless steel shows a decrease which may be regarded as abnormal. For the Stoic Metal Rod # 2 described above, the decrement per second varies from 10 to 30 as the field increases from 5 to 28 gauss. Nickel rods show decrements of the same order of magnitude. As the field on the stainless steel rod rises from 10 to 26 gauss, the decrement drops from 17 to 5. The logarithmic decrement per cycle (d) may be obtained by dividing Δ by f. The d for stoic metal changes from .00045 to .00134 as the

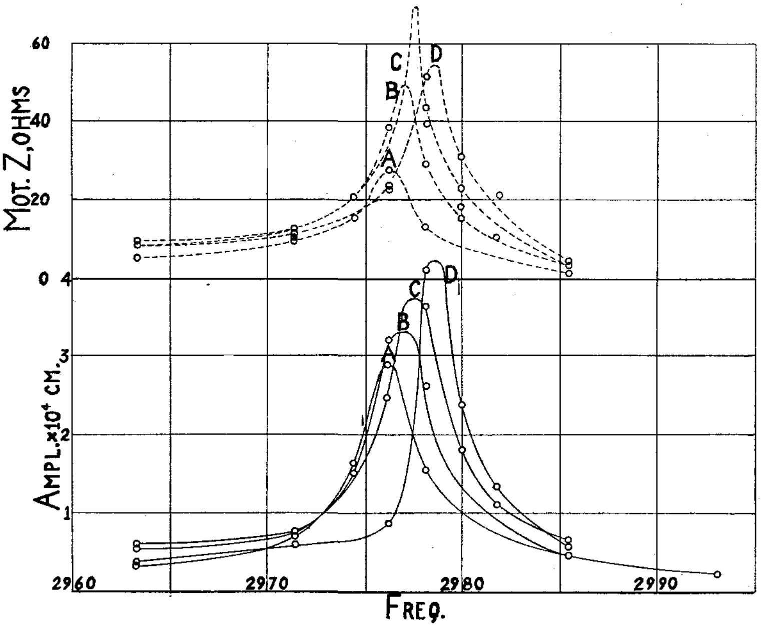

Figure 7.

Motional impedance plotted against frequency, dotted curves.
Amplitude plotted against frequency, smooth curves.
Stainless Steel Rod # 1.
Alternating and polarizing fields as in Figure 6.

field goes from 5 to 28, and for stainless steel the *d* changes from .00057 to .00017 as the field goes from 17 to 5. These decrements are very small compared to the decrements of ordinary electric circuits. If we should tune the coil used in these measurements to a frequency of 2350 cycles, the circuit would have a decrement per cycle of about 1.10.

Variation of motional impedance and amplitude of vibration with polarizing field.—Below are given sets of curves to show the maximum motional impedance and the amplitude of vibration at the resonance frequency of the rod as a function of *H*. In all cases the alternating field is the same, the field due to an alternating current of 10 milliamperes. One group shows the results for three stoic rods of different diameters, the other group of curves shows the results for the three rods whose circle diagrams are given above.

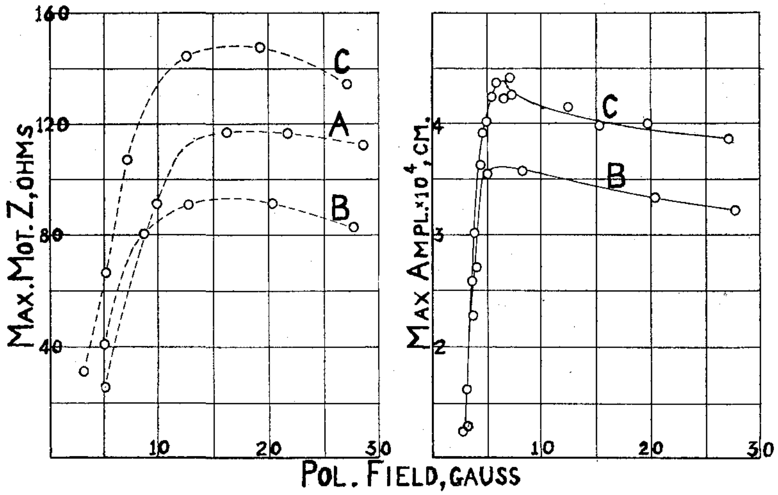

FIGURE 8.

Maximum motional impedance plotted against polarizing field, dotted curves.

Maximum amplitude plotted against polarizing field, smooth curves.

Alternating field = .325 gauss.

Curve	*Rod*	*Length cm.*	*Diameter cm.*
A	Stoic Metal # 1	92.43	.631
B	Stoic Metal # 2	94.38	.471
C	Stoic Metal # 3	86.60	.790

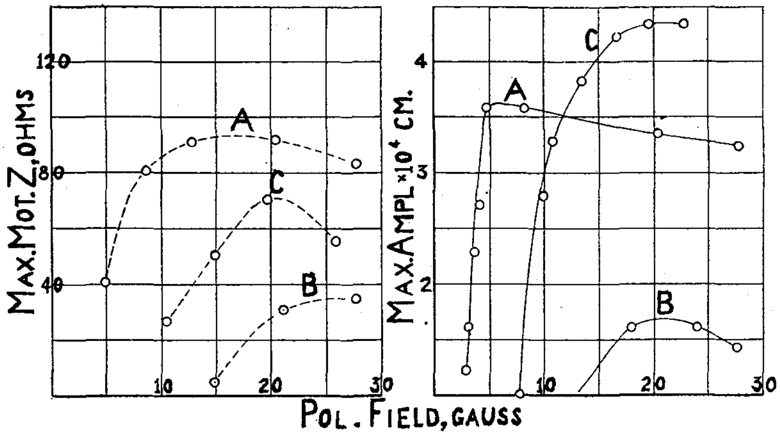

Figure 9.

Maximum motional impedance plotted against polarizing field, dotted curves.

Maximum amplitude plotted against polarizing field, smooth curves.

Alternating field = .325 gauss.

Curve	*Rod*	*Length cm.*	*Diameter cm.*
A	Stoic Metal # 2	94.38	.471
B	Nickel # 6	90.00	.790
C	Stainless Steel # 1	90.00	.691

Comparison with effects of static magnetostriction.—We can compare the effects of alternating and static magnetostriction in the following way. From the static magnetostriction curves, not shown here, we can tell what the change in length of a specimen would be if we increased the polarizing field from H to $H + \Delta H$. If we polarize the specimen at a field H and superimpose upon it an alternating field of the same frequency as the natural frequency of the specimen, and of intensity ΔH, we can measure the amplitude of vibration by the method I have already described. A comparison of the static increment in length with the amplitude shows the relative magnitude of the mechanical reactions. Such a comparison is given below.

TABLE No. 1

STOIC METAL ROD # 3

H gauss	static Δl for $H = .325$	dynamic Δl for $H = .325$	$\frac{\Delta l \text{ dynamic}}{\Delta l \text{ static}}$
2.59	$.0325 \times 10^{-4}$	1.22×10^{-4}	37.5
3.89	$.0298 \times 10^{-4}$	3.92×10^{-4}	131.5
5.70	$.0278 \times 10^{-4}$	4.42×10^{-4}	159.0
10.62	$.0271 \times 10^{-4}$	4.15×10^{-4}	153.0
17.09	$.0234 \times 10^{-4}$	4.01×10^{-4}	171.0
24.71	$.0177 \times 10^{-4}$	3.88×10^{-4}	219.0
STAINLESS STEEL ROD # 1			
10.0	$.0739 \times 10^{-4}$	3.10×10^{-4}	41.9
15.0	$.0739 \times 10^{-4}$	4.10×10^{-4}	55.5
20.0	$.0739 \times 10^{-4}$	4.34×10^{-4}	58.5

It can be seen that the dynamic effect is from about 30 to 200 times as great as the static.

Variation of the natural frequency as a function of the polarizing field.—A knowledge of the rules governing the changes in the natural frequencies of magnetostrictive rods is of great importance, since the natural frequency can be used in the calculation of the velocity of sound in the metal, the elastic coefficient, etc.

It is found that the natural frequency is a slight function of the applied polarizing field in most cases. This effect is studied by G. W. Pierce[1] at higher frequencies.

The curves of figure 10 show the natural frequency vs. H (of the polarizing field) for the three specimens whose motional impedance circles are given above.

The dotted lines in each case represent natural frequencies determined when the polarizing field is cut off, and the rod is left with only its residual magnetism.

Despite the fact that stoic metal and stainless steel expand and nickel contracts for polarizing fields up to 30 gauss, the natural period increases with increasing H in every case. This was true for every specimen studied, no maxima were observed.

Split nickel tubes.—Since it appeared possible that the electromagnetic field did not penetrate deeply into the rods, it was logical to try some experiments on tubes, in which the field may penetrate a greater percentage of the volume. The tube used was split longitudinally to reduce eddy currents.

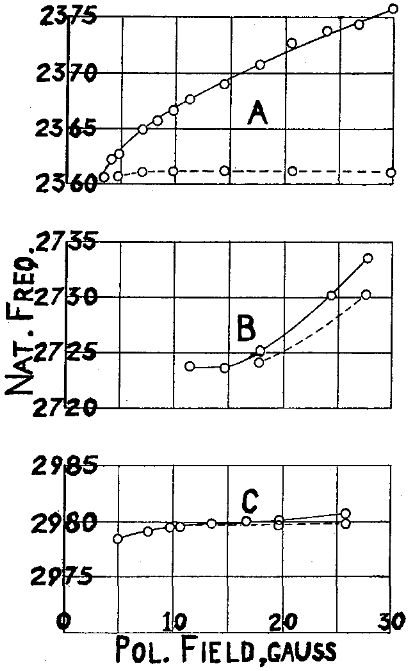

FIGURE 10.

Natural frequency plotted against polarizing field.
Dotted curves represent cases of residual magnetism.
Alternating field = .325 gauss.

Curve	*Rod*
A	Stoic Metal # 1
B	Nickel # 6
C	Stainless Steel # 1

In figures 11 and 12 are shown a set of curves for this nickel tube. Amplitudes of vibration were not measured in this particular case.

Several noteworthy facts can be seen at once. First, the depression angle here, unlike that for rods, is small. Second, the size of the circles is much greater than in the case of rods, especially nickel rods. Third, the decrements are larger in a tube than in a rod.

The effect of the variation of the intensity of the alternating field is illustrated in figure 13.

The four circles were obtained for values of alternating current ranging from .0079 to .0253 amperes. It can be seen that the circle diagrams are almost identical. The change in the diameter of the circles is from 201 to 191. This is a 5% change in motional impedance for a 320% change in alternating field intensity. As I have already mentioned, with rods the change in motional impedance for a similar change of intensity of alternating field was inappreciable.

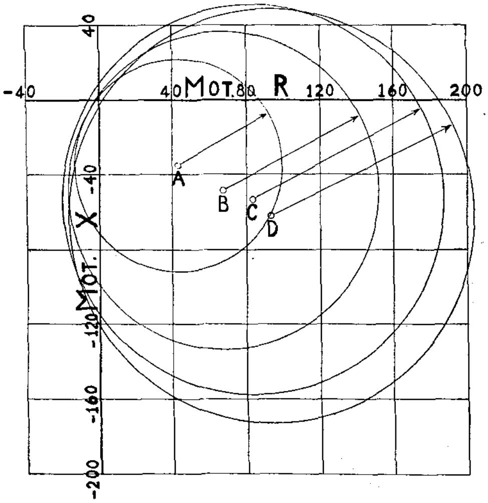

FIGURE 11.

Motional *R* plotted against motional *X*.

Nickel Tube # 1 (length, 90.00 cm., outer diameter 1.27 cm., thickness .064 cm.).

Alternating field = .325 gauss.

Curve	*Polarizing field* *gauss*	*Max. motional impedance* *ohms*
A	9.3	112.6
B	13.9	169.8
C	18.9	208.0
D	23.3	220.0

Nickel tubes as sound sources.—Both the rods and tubes are strong sound sources when used as oscillators, but of the two the tubes are the more powerful. The sound energy comes from part of the motional electric power dissipated in the circuit. One noticeable

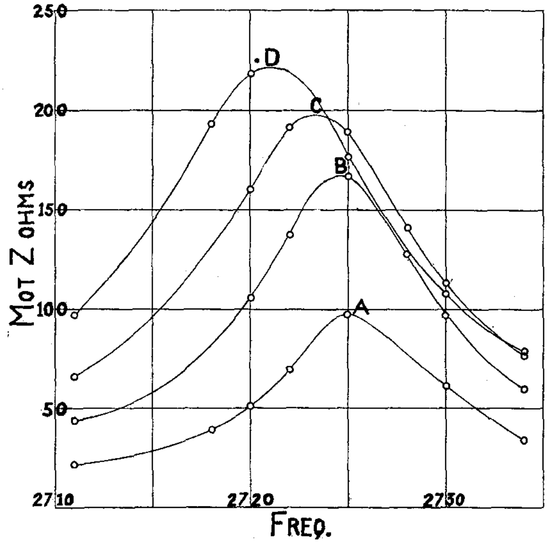

Figure 12.

Motional impedance plotted against frequency.
Nickel Tube # 1.
Alternating and polarizing fields as in Figure 11.

difference between rods and tubes is that at resonance considerable motional power is dissipated in the tubes, while rods at resonance have practically no motional resistance.

In order to increase the sound output, discs or cans may be fastened to the ends of the tube. Also much more electrical reaction may be obtained if the coils are compact ones about the center part of the tube. I did not use such a system for the preceding measurements because with short coils it is impossible to maintain a uniform field on the specimen. Measurements were made, however, on a device used by G. W. Pierce as a powerful sound source, consisting of a split nickel tube with open aluminum cans attached to the ends of the tube and a driving coil about its center. A sketch of this device is shown below with the circle diagrams.

The results are given in terms of direct current and r. m. s. alternating current flowing in the coil.

These results agree in character with those for the first nickel tube. The magnitude of the motional impedances is, of course, much greater. The damped value of resistance is about 300 ohms, while the maximum motional resistance is about 1800 ohms. It is also to be noted that one particular polarizing current gives a maximum resonance motional impedance.

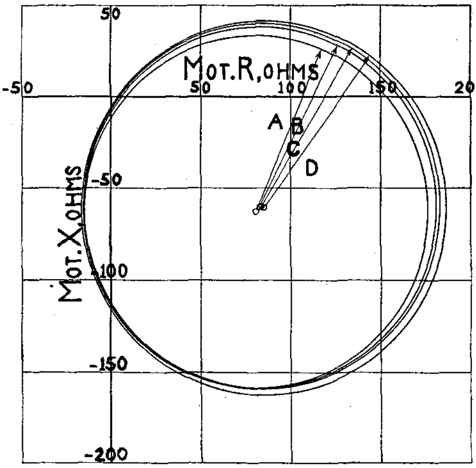

Figure 13.

Motional R plotted against motional X.
Nickel Tube # 1.
Polarizing field = 26.5 gauss.

Curve	*Alternating current in coil amperes*	*Max. motional impedance ohms*
A	.0253	191.0
B	.0178	193.6
C	.0136	196.8
D	.0079	201.0

The motional impedance of the system increases when the cans are removed, though the amount of sound radiated decreases. This is to be expected, it is analogous to the increase in motional impedance of a telephone receiver as the air pressure is reduced, an experiment described by Kennelly[3]. The actual motion of the tube is increased when the cans are removed, and therefore the electrical reaction increases.

Removal of the cans decreases the decrement.

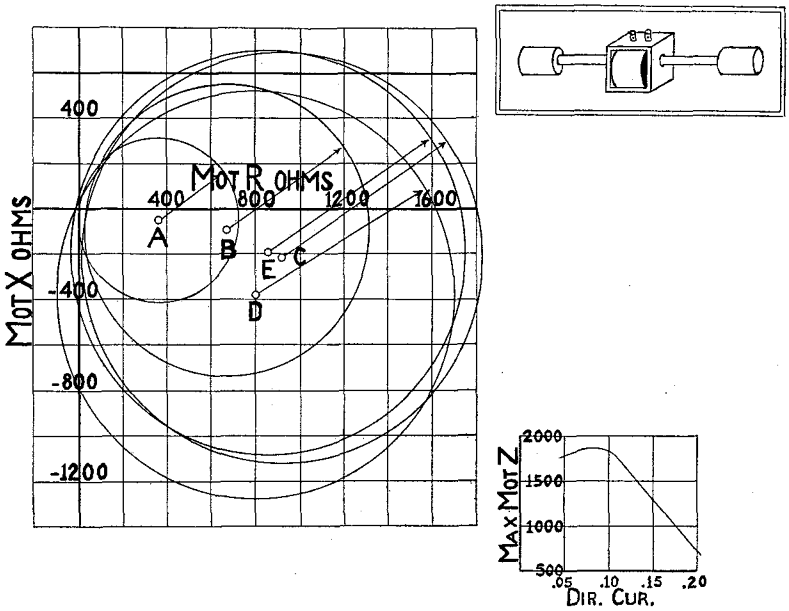

FIGURE 14.

Motional *R* plotted against motional *X*.
Maximum motional impedance plotted against direct current in the coil.
Nickel Tube # 2.
Inset shows arrangement of coil, tube, and cans.
Alternating current in the coil = .0070 amperes throughout.

Curve	*Direct current in coil amperes*	*Max. motional impedance ohms*
A	.200	730
B	.151	1274
C	.111	1836
D	.050	1790

Curve E taken with cans removed.

E	.151	1790

Damped *R* and *X* at 3300 cycles.
R 297 ohms *X* 4550 ohms

Resistance and reactance of the coil without the nickel tube.
R 200 ohms *X* 3300 ohms

Summary.—There follows an outline of the various results obtained.

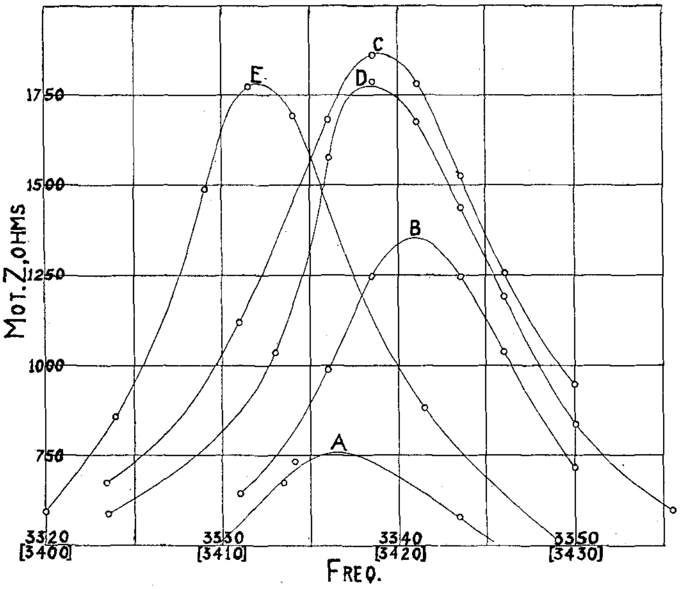

FIGURE 15.

Motional impedance plotted against frequency.
Nickel Tube # 2.
Alternating and direct current values as in Figure 14.
Frequency scale in brackets applies to curve E.

1. Alternating magnetic fields of a frequency equal to the natural mechanical frequency of a ferro-magnetic rod may give rise to displacements which are relatively several hundred times the displacements that occur with a stationary field.

2. The amplitude of vibration and all motional effects are found to be functions of the intensity of magnetization, or if one always works with a specimen initially demagnetized, the phenomena are functions of the applied polarizing field.

3. Motional resistance when plotted against motional reactance gives a circle of the same general nature as the circle diagrams that are obtained in work on telephone receivers. From these circles much information can be drawn.

4. The amplitude of vibration and the motional impedance are related to one another linearly to a reasonable approximation.

5. A study of the effect on the motional impedance of variation of

the polarizing field was made. Plotting maximum motional impedance against the polarizing field shows that in the case of stoic metal and stainless steel these curves have maxima; in the case of nickel these curves have no maxima up to a field of 30 gauss. This indicates that the electrical reaction of the system reaches a maximum for a low polarizing field in the case of the first two metals named; while for nickel the reaction increases to the highest fields used in this experiment.

6. A study of the effect on the motional impedance of variation of the alternating field shows that to a reasonable approximation the motional impedance is independent of alternating field; actually, *for small alternating fields the motional impedance shows a slight* linear decrease with increasing field.

7. The resonance amplitude of vibration is plotted against polarizing field, and it is found to reach a maximum for all cases studied at a field strength of less than 30 gauss. For small alternating fields, the amplitude of deflection is proportional to the field.

8. The angle of lag of the flux in the rod behind the current in the solenoid is found to be not far from 45 degrees for all the rods used. In most cases the lag angle appeared to be independent of polarizing or alternating field strength. The lag angle in tubes is much less.

9. From the circle diagrams the decrements of the system can be calculated. In all cases decrement increases with increasing polarizing field except in the case of stainless steel where there is a decrease.

10. In most cases the natural frequency of vibration of magnetostrictive rods increases with increasing polarizing field. In some cases the natural frequency is practically independent of the field. In no case was a decrease of frequency observed with increasing field.

11. Measurements on magnetostrictive sound sources show that a large motional power at resonance is obtainable, also that addition of sound radiating members, although increasing the amount of radiant sound energy, decreases the motional impedance and increases the decrement.

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Vol. 63. No. 3.—May, 1928.

THE PETROLOGY OF THE NORTH CONWAY QUADRANGLE IN THE WHITE MOUNTAINS OF NEW HAMPSHIRE.

By Marland Billings,
Harvard University, Cambridge, Massachusetts.

Presented by Esper S. Larsen.

Contributions from the Department of Mineralogy and Petrography, Harvard University, No. 39.

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(Continued from page 3 of cover.)

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FIGURE 1. Location of the North Conway quadrangle and the White Mountains.

FIGURE 2. Geological map of the slate belt on Mt. Pequawket.

FIGURE 3. Diagram of a disrupted siliceous stratum in mica schist.

FIGURE 4. Geological map of the White Mountains alkaline batholith.

FIGURE 5. Geological cross-section of South Moat Mountain.

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FIGURE 9. Diagrammatic cross-section illustrating the masses of Moat volcanics as isolated xenoliths immersed in the plutonic phases of the batholith.

FIGURE 10. Diagrammatic cross-section to illustrate the masses of Moat volcanics as down-faulted blocks attached to the roof along fault planes.

FIGURE 11. Diagrammatic cross-section of the batholith at Julianehaab, Greenland.

CHAPTER I.

Introduction.

Location. The North Conway quadrangle, mapped by the United States Geological Survey on a scale of 1 : 62,500, is in the east-central part of the State of New Hampshire and is approximately one hundred and fifty miles north of Boston, Massachusetts (see figure 1). The quadrangle is bounded by latitudes 44° 00′ and 44° 15′ and longitudes 71° 00′ and 71° 15′. The region may be reached from Boston by the Boston and Maine Railroad or from Portland, Maine, by the Maine Central Railroad, or by excellent highways from either city.

Topography and Drainage of the White Mountains. North Conway is in the southeastern part of the White Mountains section of the New England physiographic province (see figure 1). As Fenneman (1)[1] has pointed out, the boundaries between the White Mountains and the New England Upland are of necessity vague. The White Mountains physiographic section, as it is termed by Fenneman, trends northeasterly from central New Hampshire into western Maine, and the total length along the axis of the section is somewhat more than one hundred miles. The mountains comprise a glaciated and maturely dissected mountainous region of relatively high relief developed on crystalline rocks. The culminating summit of the whole section is Mt. Washington, which is somewhat southwest of the geometric center of the group. It attains an elevation of 6290 feet above sea level and of more than a mile above the valleys at its base. A number of peaks in the Presidential and Franconia Ranges exceed 5000 feet in elevation and many summits are higher than 4000 feet.

As shown in the accompanying map (Figure 1) the Presidential Range, in the southern part of the White Mountains section, is the center of a drainage system that radiates in all directions. The Saco River flows southeastward into the Atlantic Ocean, the Pemigewasset southward to the Merrimack, the Ammonoosuc westward to the Connecticut, and the Peabody northward to the Androscogin.

Although many of the major ranges, such as the Presidential, Baldface, Carter-Moriah, Carrigain, Dartmouth, and Randolph Ranges, have a general northeasterly trend, the fact that impresses one in the field is the apparent lack of definite plan in the distribution of the mountains. One range may trend northwesterly, as the Moat Range,

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[1] The numbers following an author's name refer to the bibliography at the end of the chapter.

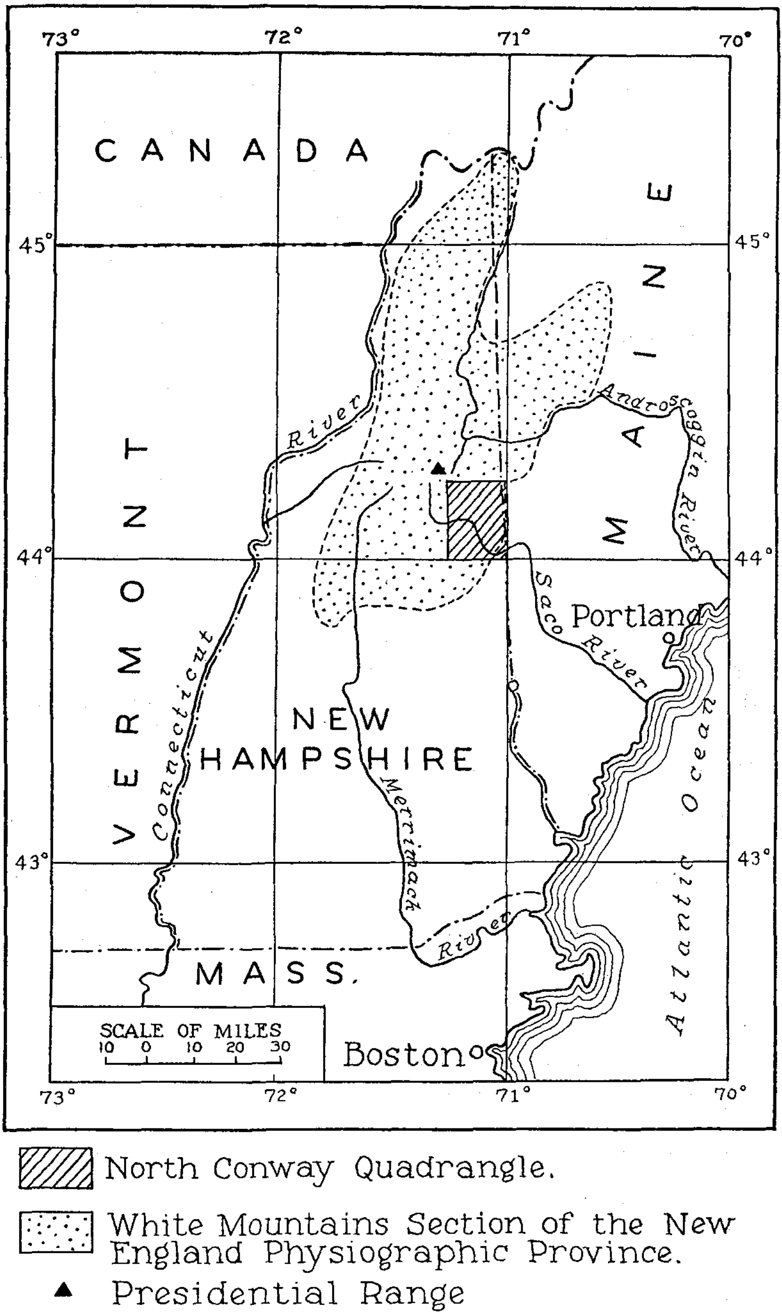
73°
72°
71°
70°
45°
44°
43°
CANADA
MAINE
VERMONT
NEW HAMPSHIRE
MASS.
Connecticut River
Androscoggin River
Saco River
Merrimack River
Portland
Boston
Atlantic Ocean
SCALE OF MILES
10 0 10 20 30
▨ North Conway Quadrangle.
White Mountains Section of the New England Physiographic Province.
▲ Presidential Range

FIGURE 1. Location of the North Conway quadrangle and the White Mountains section of the New England physiographic province.

but a closely adjacent series of peaks may trend easterly, as the Sandwich Range. Some groups show no systematic arrangement as, for example, in the southern third of the Crawford Notch quadrangle, where Table Mountain, Bear Mountain, Bartlett Haystack, Mt. Tremont, Owl Cliff, Greens Cliff, Mt. Kancamagus, and Mt. Huntington have a very erratic distribution.

Many of the summits, such as Moat and Chocorua, are bare and open due to forest fires which have swept over them in the past and destroyed the soil cap. The summits of the Presidential Range are treeless because they rise above timber-line, which is at approximately 4800 feet above sea-level but varies with several factors. The peaks are not sharp pinnacles, but many of them have relatively steep slopes; others are rounded or even flat-topped.

Topography, Drainage, and Culture of the North Conway Quadrangle. The topography and drainage of the quadrangle are adequately expressed by the United States Geological Survey's topographic map of the North Conway quadrangle. The mountain masses are irregularly distributed; the summits are subdued and in the higher ranges attain elevations from 3000 to 3600 feet. The only important stream is the Saco River, which meanders across a flood-plain, incised in the sand terraces of a valley which varies from half a mile to two miles in width (see Plate IIA).

North Conway, with 1,500 inhabitants, is the largest town in the quadrangle. Good roads and trails make most portions of the quadrangle readily accessible, but Perkins Notch, Sable Mountain, and Chandler Mountain are relatively isolated and inaccessible. The chief industry of the area is catering to summer tourists. A number of hotels are located at North Conway and Jackson, and many small boarding houses are found in the valleys. The lumber industry was once important, but in 1920 the major operations ceased. Farming along the flood-plains of the rivers is a profitable business, the chief crops being hay and corn. The corn is canned each fall at Conway, North Fryeburg, and West Fryeburg; all three of these villages are just beyond the limits of the North Conway quadrangle. A large proportion of the timber land is now owned by the Federal government and is part of the White Mountains National Forest.

Previous Geological Work. The earliest work on the geology of the whole state of New Hampshire is that by C. T. Jackson (2) in 1844, but it is of no practical value for the present work. For the North Conway quadrangle the chief geological reference is C. H. Hitchcock's "Geology of New Hampshire," in five volumes, published fifty years ago. On the whole I have followed rather closely Hitchcock's classification of the formations, although some re-definition and subdividing has seemed advisable. He made a lithological classification of the granites and syenites of the White Mountains alkaline batholith. His classification of the metamorphic rocks will probably have to undergo considerable revision, but it has not been found necessary to consider that problem in the North Conway quadrangle. As for the structural and chronological relations of the various groups, Hitchcock's work is of little value. The mapping was poor in details, but, considering the conditions under which the survey was carried on, Hitchcock did a wonderful piece of work. The following table is given to facilitate a comparison of the terminology used by Hitchcock (3) with that of the present paper.

TABLE I.

Comparison of Formational Names.

Hitchcock.	This Paper.	
Modified drift	Stratified drift	
Glacial till	Glacial till	
Conway granite	Biotite granite	
	Black Cap biotite granite	
Chocorua group	Riebeckite granite	
	Hastingsite granite	
	Nordmarkite	
Albany granite	Porphyritic nordmarkite	
	Certain contact phases of the Conway biotite granite.	
	Nordmarkite porphyry	
(Not recognized)	Diorite	
Pequawket breccia	Pequawket breccia	Moat volcanics
Quartz porphyry	South Moat flows	
Porphyritic gneiss	Meredith porphyritic granite	
(Kearsarge andalusite group)[2]	Intervale clay-slates	
Montalban group	Chatham granite	
	Montalban schists	

In determining the chronological sequence and the relations of the various formations, the writer found little help in the work of the past. Perry's (4) analyses of the comendites from Mt. Pequawket (Mt. Kearsarge) are of value, and Daly (5) published a short paper on the Pequawket breccia. Other papers that have been of great aid, although they were concerned with areas outside of the North Conway quadrangle, are Daly's (6) paper on the New Hampshire (Meredith) porphyritic granite, the paper by Pirsson and Washington (7) on the nepheline syenite of Red Hill, and the paper by Pirsson and Rice (8) on Tripyramid Mountain. Hawes' (9) paper on the Albany contact zone is classic. The recent booklet by Goldthwait (10) on the "Geology of New Hampshire" contains a bibliography and gives a popular account of the geology.

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[2] In the North Conway quadrangle Hitchcock mapped certain clay-slates as "Kearsarge andalusite group"; in the opinion of the present writer the clay-slates of the North Conway quadrangle do not belong to Hitchcock's "Kearsarge andalusite group" and for this reason they are mapped as a new formation.

Nothing has been written directly on the physiography of the North Conway quadrangle, but the papers by Goldthwait on the glacial geology (11) and physiography (12) of the Presidential Range are of great value. Lane (13), Johnson (14), Lobeck (15), and I. B. Crosby (16), have contributed to the studies on the glacial geology and physiography of the White Mountains.

A number of mineralogical papers have been published by Kunz (17), Eakle (18), Farrington and Tillotson (19), Schaller (20), and the present writer (21), on the topaz and phenacite from Baldface Mountain. J. L. Gillson (22) has recently published a paper on the paragenesis of the minerals in the Conway biotite granite.

Method of Study; Acknowledgments. The field work for the present paper was done during the summers of 1925 and 1926 under the direction of Dr. E. S. Larsen. Dr. Larsen was in the field with the writer for ten days during the first season. The Government topographic sheet, surveyed in 1892, was used as a base, and although grossly inaccurate in many details, it suffices for an area composed primarily of igneous and metamorphic rocks. All elevations were determined by aneroid barometer, and as a rule are probably accurate to within thirty feet. The petrographic studies were made under the direction of Dr. Larsen, the crystallography was studied under the guidance of Dr. Palache, and the physiographic studies were discussed with Dr. Mather. Dr. Daly suggested the North Conway area as a possible thesis subject. The thin sections for the petrographic study, nearly three hundred in all, and the chemical analyses by W. H. Herdsman and Mrs. S. Parker were financed from the Holden fund at Harvard University.

Advance Summary. The geological history of the North Conway quadrangle is summarized in the following table, which also indicates the various chapters in which the various sections are discussed.

TABLE II.

Geological History of the North Conway Quadrangle.

Pre-Cambrian (?)	*Montalban Schists. Chapter II.* Deposition of shales, limestones, and sandstones which were later to become the Montalban schists.
Pre-Cambrian (?)	*Chatham and Meredith Granites. Chapter III.* Folding and metamorphism of the sediments to form the Montalban schists, accompanied by the intrusion of granites.
Silurian (?)	*Intervale Clay-Slates. Chapter II.* Deposition of muds and sands was followed by a mountain making movement. (This phase of the history is a little uncertain, because the clay-slates may be merely the less intensely metamorphosed equivalent of the Montalban schists.)
Devonian (?)	*White Mountains Alkaline Batholith. Chapter IV.* Lava flows, with associated tuffs and breccias, were extruded upon the surface; diorite, syenites, and granites were intruded as plutonic phases.
Mesozoic and Tertiary	*Subaerial denudation.* The region was subjected to profound subaerial denudation, with repeated uplifts.
Quaternary	*Glaciation and post-glacial erosion.* The region was subjected to glaciation, alpine glaciers in the high ranges preceding the continental ice sheet, which overwhelmed the whole area. Post-glacial erosion has been relatively slight.

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CHAPTER II.

Montalban Schists and Intervale Clay-Slates.

MONTALBAN SCHISTS.

Distribution and General Relations. The oldest rocks exposed in the North Conway quadrangle form a group of mica schists, lime-silicate rocks, and micaceous quartzites, which were designated the Montalban group by Hitchcock (1). They are often intimately injected by granite intrusives which I have named the Chatham granite. The only large area of the Montalban group is found in the northwestern part of the quadrangle, but unfortunately the exposures are not good, because the outcrops are largely confined to a few of the streams. A second area about two and a half miles long is exposed west of Sloop Mountain, but the heavy covering of glacial drift makes it very difficult to determine the exact boundaries. Small patches of the Montalban schists have been noted on the east slopes of Baldface Mountain and elsewhere as xenoliths in the Chatham granite. Two patches were found west of Mountain Pond apparently as inclusions in the younger nordmarkites.

The best exposures of the Montalban schists are on the Ellis River west of Eagle Mountain and also along the river, just below where it flows out of the Crawford Notch quadrangle. Excellent exposures may be seen in the bed of Meserve Brook, on Tin Mountain, and on the south slopes of Black Mountain. In the bed of the stream to the west and south of Sloop Mountain the exposures are good but difficult to reach.

In general the Montalban schists of the North Conway quadrangle strike northeast and dip on the average about 45° to the northwest. The schistosity is parallel to the original bedding. The present writer has not been able to unravel any broad structures, but occasionally local folding may be detected, as for example along the Ellis River, northwest of Spruce Mountain, where a small anticline is exposed. If the major structure is a monocline dipping toward the northwest, as it seems to be, the Montalban schists must be many thousands of feet thick. There is no field evidence for close isoclinal folding, but the complications introduced by the two series of later intrusives, rock flowage, the heavy glacial drift, the forest cover, and probable faulting have prevented the solution of the structural problem.

The crystalline schists of the North Conway quadrangle may be divided into three major lithological types: (1) lime-silicate rocks, composed essentially of hornblende, white diopside, feldspar, and quartz; (2) mica schists, composed essentially of muscovite, biotite, and quartz; and (3) quartzites, composed essentially of quartz with accessory mica. Intermediate types are common. This classification is genetic: the lime-silicate rocks being metamorphosed limestones, dolomites, and calcareous sandstones; the mica schists, recrystallized shales; and the micaceous quartzites, metamorphosed impure sandstones.

Lime-silicate group. The lime-silicate rocks are rather widespread in the Montalban schists of the North Conway quadrangle and elsewhere in New Hampshire and Maine. Excellent exposures are seen on the south slopes of Black Mountain, between the 1340-foot and 1950-foot contours, where mica schists and sillimanite-mica schists are associated with them. The dip is 33° N. A second series of exposures are displayed in the bed of Meserve Brook, beginning at about 1120 feet and continuing intermittently to 1700 feet; the lime-silicate rocks are here associated with mica schists, injection gneisses, and a few granite dikes. The dip increases upstream from 32° NW to 80° NW. A third occurrence is found on the Ellis River about half a mile south of the uppermost highway bridge over the stream. The lime-silicate rocks are interbedded with mica schists and injection-gneisses, and all are cut by a twenty-foot dike of a muscovite-biotite granite with radiating crystals of black tourmaline. The strata here dip only 12° NE. On the eastern slopes of Spruce Mountain, half a mile north of the Spruce Mountain Camps and about one hundred and twenty-five feet above the valley floor, a lime-silcate rock is exposed beneath mica schists.

In general the lime-silicate group comprises pure white, greenish-gray, or greenish-black, thinly bedded rocks, with or without distinct schistosity and composed essentially of feldspar, quartz, hornblende, diopside, and titanite. Minor accessories include biotite, rutile, apatite, hematite, magnetite, and ilmenite. Both orthoclase and plagioclase feldspar are present, occasionally in the same hand specimen. Although the composition of the plagioclase feldspar is uniform for any given hand specimen, it is very variable in the group as a whole and may range from oligoclase to calcic labradorite (Ab_{28}). The feldspar occurs only in the groundmass and never as porphyroblasts. The diopside is pure white, is anhedral to subhedral in habit, and usually occurs as porphyroblasts set in a granoblastic ground mass of quartz and feldspar. The amphiboles vary in composition. At one end of the series the amphibole is black in the hand specimen:

the pleochroism is X = practically colorless, but with a slight tinge of yellow, Y = green with a brownish tinge, Z = green with a brownish tinge somewhat less noticeable than in Y; $X < Y = Z$. The indices are $\alpha = 1.649$, $\gamma = 1.678$. At the other end of the series the amphibole is green in the hand specimen and practically colorless under the microscope, with the following indices: $\alpha = 1.630$, $\gamma = 1.647$. The optical orientation for both types is $Y = b$, $Z \wedge c = 12° - 17°$. The variations in indices and pleochroism is probably due to differences in the iron content, the amphibole with the lower indices and slight pleochroism being a lime-magnesia amphibole. When the amphibole is very abundant in the rock, as in the amphibolites, it is compact and shows only a slight tendency toward sieve structure, but when it is not abundant the sieve structure is developed to the extreme, the hornblende at times being hardly more than a skeleton crystal. One of the most characteristic features of the metamorphic hornblende is the fact that it has a unique habit, quite unlike that of the typical igneous amphibole. In this metamorphic hornblende the front pinacoid (100) is often strongly developed but the side pinacoid (010) is usually suppressed. In the igneous amphiboles the side pinacoid is usually well-developed. Wherever the amphibole stalks are all parallel to the original bedding and to one another, a distinct nematoblastic structure results.

Obviously, with so many possible mineral combinations a great variety of rocks may result. One of the most striking types is a *white diopside rock,* composed of numerous white diopside porphyroblasts set in a granoblastic groundmass composed of quartz, orthoclase, and plagioclase. Titanite is relatively abundant in wedge-shaped crystals. A variety of this type carries occasional black hornblende porphyroblasts. A second type, a *speckled amphibolite,* is composed of black hornblende porphyroblasts showing pronounced sieve texture, set in a granular groundmass of quartz, orthoclase, plagioclase, and titanite. One variety contains a few diopside porphyroblasts, and another variety contains considerable biotite in the groundmass. In the third major group are found *amphibolites,* composed of abundant stalks of amphibole, with or without distinct parallel arrangement, set in a fine groundmass composed of orthoclase, plagioclase, and quartz. A subtype carries biotite.

Mica schists. The mica schists are platy rocks, composed essentially of mica and quartz with variable amounts of andalusite, sillimanite, garnet, feldspar, and minor accessories. The *andalusite schists* are found in only two localities in the North Conway quadrangle: one at the southern end of Black Mountain, the other in the stream west of Sloop Mountain. They are composed essentially of porphyroblasts of andalusite, often associated with sheaves of sillimanite and flakes of muscovite, set in a groundmass of quartz, muscovite, and biotite. The groundmass is generally lepidoblastic but varies in grain. Accessories include orthoclase, albite, red garnet, black tourmaline, chlorite, magnetite, apatite, and zircon. The *mica schists* are similar except that macroscopic andalusite and sillimanite are lacking. *Garnetiferous mica schists* are rare; they have been noted on North Baldface Mountain.

Hornfels. Hornfelses are relatively wide-spread. They are dense, compact, brittle, purplish-gray rocks without schistosity or cleavage. They may show well-defined bedding planes, particularly on weathered surfaces. The individual strata vary widely in composition, but the essential minerals are biotite and quartz, with important quantities of pyrite, sericite, andalusite, and plagioclase.

Quartzites. Pure white quartzites are practically lacking in the White Mountains; *micaceous quartzites* and *quartzose mica schists* are common.

Character of the original Sediments. From the preceding descriptions it is apparent that the original sediments from which the Montalban schists have been derived were impure limestones and dolomites, calcareous sandstones, shales, arenaceous shales, and sandstones.

Age of the Montalban schists. The age of the Montalban schists is not well determined. In the literature they are generally supposed to be Paleozoic (2), on the basis of a supposed correlation with the dubiously Paleozoic schists of western Massachusetts. No one has actually traced the Montalban schists southward. Recent work by E. B. Knopf (3), moreover, casts doubts on the Paleozoic age of the Massachusetts crystalline rocks. Evidence in the North Conway quadrangle demonstrates clearly that the Montalban schists are older than the Moat volcanics, which are probably Devonian. The present writer has traced the Montalban group eastward to Lewiston, Maine. The work of Perkins and Smith (4) suggests that these gneisses and schists are pre-Silurian. The evidence as a whole is admittedly inconclusive, but it favors an early Paleozoic or pre-Cambrian age for the Montalban schists.

INTERVALE CLAY-SLATES.

One small area of clay-slate has been found in the quadrangle. It may be either the less intensely metamorphosed equivalent of the Montalban schists or it may be a fragment of a westward extension of the great Silurian slate belt of central Maine. The present writer favors the latter interpretation, but more field work in the adjacent quadrangles is necessary before the problem can be considered settled. It is proposed to call this group the Intervale clay-slates because the known occurrence in the North Conway quadrangle is found on the southern slope of Mt. Pequawket. The outcrop is just east of the Kearsarge Village trail at an elevation of 1500 feet (see figure 2). Two miles NE of Intervale, in the vicinity of the trail, the slate strikes N 75° E and dips vertically. The cleavage and original bedding are essentially parallel, a fact that may be determined from the occasional quartzite layers interbedded with the slate. This slate is the basement upon which the Moat volcanics, exposed to the north on the mountain slope, rest unconformably; to the south and west the Albany porphyritic nordmarkite is intrusive into the slate and in places shows a quartz-porphyry contact-phase. Most of the fragments in the Pequawket breccia, which is exposed higher up on the mountain, are composed of this clay slate. Fragments of an andalusite-bearing phyllite, which are found in the breccia at the

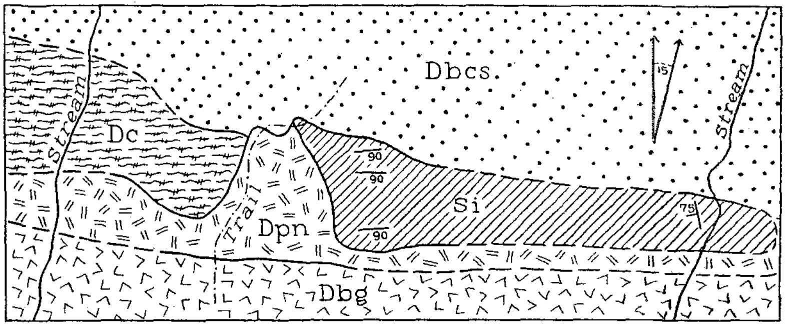

FIGURE 2. Geological map of the slate belt at an elevation of 1500 feet on the Kearsarge Village trail up Mt. Pequawket. Scale: one inch equals about 750 feet. *Si*, Intervale clay-slate; *Dc*, comendite; *Dbcs*, some comendite, but mainly breccia with many fragments of clay-slate; *Dpn*, porphyritic nordmarkite; *Dbg*, biotite granite. The comendites and breccias were deposited uncomformably on the clay-slate; this group then settled as a great block into the batholith and was intruded by the porphyritic nordmarkite and the biotite granite.

very summit of North Moat Mountain, may represent a contact-metamorphosed phase of this slate.

The clay-slates are black or very dark gray, with a more or less conspicuous silky luster. They show in addition to the regular cleavage, which is parallel to the original bedding, a cross-cutting fracture-cleavage. The slates are composed essentially of somewhat flattened quartz grains and flakes of sericite lying parallel to the slaty cleavage. Important accessories include chlorite, pyrite, and ilmenite.

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CHAPTER III.

Older Granites and Regional Metamorphism.

Introduction. In addition to the granites, quartz syenites, syenites, and diorites of the Devonian (?) alkaline batholith, the White Mountains contain large areas of earlier granites, usually characterized by muscovite and biotite. At least three major groups may be recognized. (1) The Bethlehem granite of Hitchcock (1) is a coarse, red, biotite granite; it is not exposed in the North Conway quadrangle and is therefore not discussed in this paper. (2) The Meredith granite (the porphyritic gneiss of Hitchcock) is a coarse muscovite-biotite granite with large orthoclase phenocrysts. (3) I have named a coarse two-mica granite which often intrudes the Montalban group in lit-par-lit fashion the Chatham granite. For a fine-grained and somewhat younger phase of the Chatham granite, which intrudes the injection gneisses, I propose the term Randolph granite. That this phase is actually younger than the granites forming the injection gneisses may be observed in the Ravine of the Castles, Mt. Washington quadrangle. In the areal mapping the Randolph granite has not been separated from the Chatham granite. In some places it was mapped as "Winnepesaukee Gneiss" by Hitchcock, but, inasmuch as this name covers a number of very different gneisses and granites, a new term is proposed. The Randolph granite is considered to be a late manifestation of the Chatham igneous activity.

Chatham granite. The Chatham granite, including the Randolph phase, has a wide distribution throughout the White Mountains region. Hitchcock (1) did not satisfactorily separate it, for in some places he mapped it as part of the Montalban group, but in others, as for example in the eastern part of the North Conway quadrangle and in western Maine, he mapped it separately as "granites." The name Chatham granite is proposed for the non-porphyritic, two-mica granite which is intrusive into the Montalban schists but older than the White Mountains alkaline batholith.

We may list three general types of occurrence. (1) Large areas, covering many square miles, such as the mass in Chatham township. (2) Small isolated patches in the granites of the White Mountains group which vary from blocks a few feet in diameter to others covering several acres. Typical exposures of this type may be seen in the smaller reservoir at Kearsarge Village, at Swift River Falls (Crawford Notch quadrangle), and at the upper end of Jackson Falls; in the last locality the granite is associated with the Montalban group as a xenolith in the porphyritic nordmarkite. (3) In the third type of occurrence the Chatham granite is intimately associated with the schists of the Montalban group, either intruding them in lit-par-lit fashion or containing streaky patches of schist which have been strewn about through the granite. Exposures of this type may be seen along the upper part of the Ellis River, above the uppermost highway bridge.

The Chatham granite is a group of white or yellow medium to fine-grained granites with conspicuous black specks of mica. The essential minerals are white feldspar, quartz, biotite, and often muscovite. Locally black tourmaline and red garnet are present.

Under the microscope the hypidiomorphic-granular rock is composed essentially of quartz, orthoclase, oligoclase, biotite, and often muscovite. The orthoclase is completely lacking in some varieties, but in others it is the dominant feldspar. It occasionally shows intergrown albite lamellae. Carlsbad twinning is noted in both the orthoclase and the plagioclase. The oligoclase usually shows polysynthetic twinning and varies from albite (Ab_{88}) to andesine (Ab_{70}). Undulatory extinction is common in the quartz. The biotite contains pleochroic haloes, is in places partially altered to chlorite, and is always in irregular flakes, either as individuals or as aggregates.

Zones of magnetite dust around the biotite are found in some specimens. The muscovite may occur in irregular flakes or, less commonly, in aggregates of small flakes, clearly working along cleavages in the feldspar; a fact that demonstrates a secondary origin for at least some of the mica. Among the accessory minerals the most noteworthy are andalusite and sillimanite. The former often shows a strong euhedral tendency; the latter occurs in bundles of needles. These minerals are considered to have been derived from the schists which have been shredded and strewn about through the granite. A little epidote is rarely associated with the andalusite. Among the accessories we find apatite, zircon (very rare), titanite (very rare), magnetite, ilmenite, pyrite, and leucoxene. Small needles, probably apatite, are sometimes common in the quartz.

According to the micas present, the Chatham rocks may be classified as two-mica granite, biotite granite, and muscovite granite.

One of the most striking differences between the Chatham granite and the younger White Mountains batholith is the large amount of pegmatite associated with the former. In addition to feldspar and quartz, the pegmatites contain considerable muscovite and biotite and smaller amounts of black tourmaline and red garnet. The pegmatites may be seen almost everywhere in the Chatham granite, but particularly large areas are seen on Little Deer Hill (northeast corner of the quadrangle) and on the east slopes of North Baldface, especially on the ridge north of Charles Brook.

Meredith granite. One of the most striking rocks in the whole state is the porphyritic granite, a formation studied in detail by Daly (2) thirty years ago. Hitchcock (1) termed it the porphyritic gneiss and Daly called it the porphyritic granite. Inasmuch as other porphyritic granites are present in the state, it is proposed that this particular type be designated the Meredith granite, for typical exposures may be seen in the township of Meredith. Only one large area of this formation is found in the North Conway quadrangle. It covers the whole summit of North Baldface, the hills to the west, the lower portion of the west slope of South Baldface, and a bench south of this mountain. The position of the western and southern boundaries are unknown, due to the heavy covering of drift and forest.

The Meredith granite of the North Conway quadrangle is a coarse-grained rock composed of white tabular orthoclase phenocrysts, which are frequently an inch long and show Carlsbad twinning. In places the phenocrysts have a distinctly parallel orientation which gives a trachytoid structure. The attitude of this structure is not regional, however, for the strike and dip may greatly vary within short distances. More commonly such a structure is lacking and the individual feldspar phenocrysts show a great diversity of orientation.

Under the microscope the orthoclase phenocrysts are found to enclose small poikilitic patches of oligoclase. The groundmass is composed of irregular and interlocking quartz, orthoclase, microcline, and oligoclase with numerous flakes of biotite and muscovite. In some varieties the groundmass oligoclase is distinctly dominant over the groundmass orthoclase. The muscovite is clearly secondary in many cases, for it is in flakes parallel to the feldspar cleavages. Minor accessories include apatite, andalusite, titanite, magnetite, and zircon.

Relationship of the Chatham and Meredith granites. Unfortunately the contacts of the Chatham granite and the Meredith granite are everywhere deeply buried by drift. Several lines of evidence indicate, however, that the two granites are closely related. (1) The mineralogy is strikingly similar, both for the major constituents, such as orthoclase, oligoclase, muscovite, biotite, and the accessories, apatite, zircon, and andalusite. (2) On the west slopes of North Baldface, where exposures are excellent, the number of phenocrysts in the Meredith granite in places approaches zero and the resulting rock is thus similar to the Chatham granite. (3) Both granites are shattered by the same type of pegmatite, characterized by black tourmaline and red garnet. (4) Both are intrusive into the Montalban schists, but are in turn intruded by the White Mountains alkaline batholith. These facts are clearly shown in the Baldface region, where blocks of the Montalban schist are found in the Chatham and Meredith intrusives; moreover, the Conway granite of South Baldface shows a typical porphyritic marginal facies against both of the earlier granites. These facts indicate that the Chatham and Meredith granites are closely related in chronology and mineralogy.

Regional Metamorphism. In the Montalban group there are no conglomeratic rocks to demonstrate rock flowage, but the differential effects of flowage on the tough lime-silicate rocks and the readily yielding mica schists is well illustrated by an outcrop in the Ellis River west of Eagle Mountain. The accompanying diagram (figure 3) shows a line of blocks of the highly siliceous lime-silicate type, each block being about a foot wide. The most reasonable interpretation is that a one-foot stratum of siliceous rock was interbedded with mica schists. During the metamorphism the micaceous rocks readily yielded by flowage, but the more brittle siliceous layer was broken into blocks. With the continuation of rock flowage the siliceous blocks would be strewn about through the mica schists. Numerous blocks of lime-silicate rocks in the mica schists and injection gneisses are to be explained in this same way.

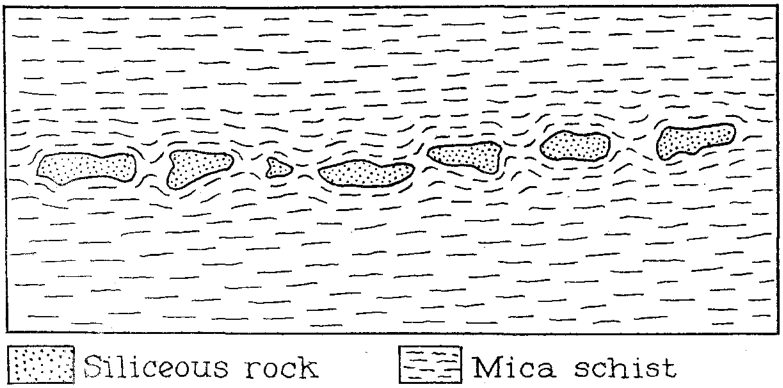

FIGURE 3. Diagram of blocks of siliceous rock in mica schist, Ellis River, west of Eagle Mountain. Scale: one inch equals about four feet. This group of siliceous blocks is considered to represent a brittle zone which was broken up during general flowage and metamorphism of the rocks.

The manner in which the Chatham granite has intruded the Montalban schists is very clearly shown in the White Mountains. Between true schist and true granite we find all intermediate stages. The first step in the injection shows narrow stringers of granite intruded parallel to the schistosity of the mica schists. With an increase in the relative amount of granite the layers of schist begin to break up into individual plates surrounded by granite. A still further stage shows a rock that is mainly granite with small shreds of schists strewn about through it. The final stage shows what is ostensibly a normal two-mica granite, but which under the microscope shows andalusite and sillimanite. The most reasonable way to explain the andalusite and sillimanite, so persistent in the Chatham granite, is to consider them as originally metamorphic minerals that have been strewn about in the granite by the shredding up of schists.

The process of lit-par-lit injection and the shredding and strewing-about of the Montalban schists through the Chatham granite may be well studied along the Ellis River above the uppermost highway bridge. On the west slope of Wildcat Mountain the granite contains shreds of sillimanite-mica schist. Beyond the limits of the North Conway quadrangle a large area of injection gneiss is exposed on the Southern Peaks of the Presidential Range and particularly good exposures may be seen between Mt. Clinton and Mt. Webster. In the bed of Meserve Brook injection gneisses are interbedded with lime-silicate rocks.

The problem of regional metamorphism is too involved to discuss in this place. The facts in the White Mountains indicate, however, that the Montalban schists are closely associated with the Chatham granite. The known evidence in the White Mountains is admittedly inadequate to prove any theory of metamorphism. But the interpretation that best fits the known facts is Barrell's (3) conception that metamorphism, orogenic movements, and granitic intrusions are essentially contemporaneous. Juices from the invading Chatham granite soaked through the folding sediments, greatly augmented the tendency to rock flowage, and caused recrystallization. As the granite moved up into a given horizon of the metamorphosed zone, it first intruded the schists in lit-par-lit fashion and then gradually ripped them to pieces, strewing the shreds throughout the granite.

In contrast with the main phases of the Chatham granite, the medium-grained Randolph phase intruded the schists with sharp contacts and shows no tendency to intrude lit-par-lit. It apparently represents the closing stages of the batholithic intrusion, after the orogenic movements had ceased (3).

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CHAPTER IV.

The White Mountains Alkaline Batholith.

General Relations.

The White Mountains alkaline batholith is located just south of the main axis of the White Mountains (see figure 4). The main batholith covers approximately 680 square miles, and if we include the two satellitic stocks of Red Hill and the Ossipee Mountains, the

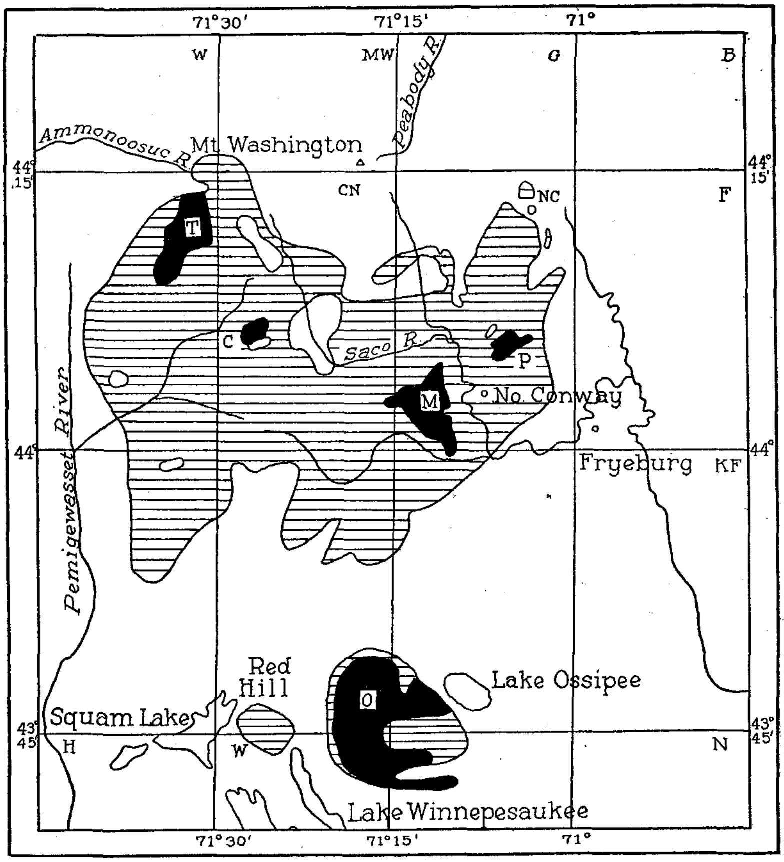
71°30'
71°15'
71°
W
MW
G
B
Ammonoosuc R.
Mt Washington
Peabody R.
44° 15'
CN
NC
F
T
C
Saco R.
P
M
No. Conway
River
Pemigewasset
44°
Fryeburg
KF
Red Hill
Lake Ossipee
O
Squam Lake
43° 45'
H
W
N
Lake Winnepesaukee

FIGURE 4. Geological map of the White Mountains alkaline batholith. Scale, about one inch to twelve miles. White areas represent mainly the older gneisses, schists, and granites. Horizontal lining indicates the plutonic phases of the batholith (diorite, nordmarkites, hastingsite granite, riebeckite granite, biotite granite, and nepheline syenite). Solid black represents extrusive phases (tuffs, breccias, trachyte, and comendites): O = Ossipee Mountains, M = Moat Mountain, P = Mt. Pequawket, C = Mt. Carrigain, T = Twin Mountain. U.S.G.S. topographic maps indicated thus: B = Bethel, G = Gorham, MW = Mt. Washington, W = Whitefield, F = Fryeburg, NC = North Conway, CN = Crawford Notch, KF = Kezar Falls, N = Newfield, W = Winnepesaukee, H = Holderness. The other sheets have not yet been published. The data for the lithology of the North Conway quadrangle are original; the rest of the map has been compiled from other authors, mainly C. H. Hitchcock. A number of stocks, possibly satellitic to the alkaline batholith, have been omitted; these include Pleasant Mountain, Burnt Meadow Mountains, and Tripyramid Mountain.

area is augmented to 740 square miles. The Burnt Meadow Mountains and Pleasant Mountain in Maine are also probably satellitic stocks. The main batholith extends from near the Maine boundary on the east to Franconia Notch on the west, a distance of thirty-six miles. The maximum north-south dimension is twenty-six miles; the northern limits are Fabyans on the west and Baldface Mountain on the east; Mt. Chocorua is somewhat north of the southern boundary. Within the confines of the batholith is a number of areas of the older formations. The Ossipee stock, covering approximately fifty-eight square miles, is four miles south of the main batholith; and the Red Hill satellitic stock, which covers nearly five square miles, is ten miles south of the main batholith and four miles west of the Ossipee Mountains.

Both extrusive and intrusive phases are well exposed, the former being preserved as great blocks, from a few feet to eight miles in diameter, that have settled into the batholith. The various phases may be classified chronologically as follows, but it should be noted that within the plutonic group many transitional types are found, and the named types can be considered only as members of a more or less continuous series of rocks. The groups are arranged in the order of increasing age.

TABLE III.

Chronology of the White Mountains Alkaline Batholith.

Intrusive phases	[Nepheline syenite] Conway biotite granite (Conway granite) Riebeckite granite Hastingsite granite (Chocorua granite) Nordmarkite (Chocorua syenite) Porphyritic nordmarkite (Albany granite) Nordmarkite porphyry Diorite
Extrusive phases Moat volcanics	Comendites, riebeckite comendite, trachyte, microgranite, feldspathic tuffs, clay-slate breccia, polygenous breccia, and quartz porphyry breccia. Total thickness at least ten thousand feet.

The following qualifications of the table should be made:

1. The position of the nepheline syenite is not known from field evidence; it is considered to be the youngest intrusive, from evidence in other similar batholiths.
2. The diorite may be older than the extrusive phases.

3. The existence of a biotite granite older than the Albany porphyritic nordmarkite is discussed on a latter page.

The names in parentheses are those used by C. H. Hitchcock (1).

Geological Age.

No fossiliferous horizons are associated with the White Mountains batholith, and its geological age is therefore difficult to determine. All that we definitely know is that the alkaline rocks are younger than the Pre-Cambrian (?) Montalban schists and Chatham granites and antedate the long period of subaerial erosion which has carved the White Mountains. The separation of some of the allanite from the Conway biotite granite to determine the lead-uranium ratio might solve the problem. At present the only method available is to correlate the White Mountains group with similar alkaline rocks of eastern North America and western Europe. As far as they can be dated, all the alkaline rocks of the Appalachian belt of eastern North America are Devonian, with two possible exceptions. One exception is in southeastern Maine, where the alkaline rocks are intrusive into metamorphic rocks which are considered to be Carboniferous by the United States Geological Survey (2). They are correlated with the Carboniferous phyllites at Worcester, Massachusetts, but Wandke (3) states that this conclusion is by no means certain. An occurrence of alkaline rocks that are clearly not of Devonian age is found at Litchfield, Maine, where the litchfieldite and associated soda-syenites are probably Pre-Cambrian (4).

Moat Volcanics.

General Relations.

In the North Conway quadrangle two large areas of volcanics are exposed, the type locality on Moat Mountain and a second large area on Mt. Kearsarge (Mt. Pequawket). These volcanics are composed of siliceous flow rocks and interbedded tuffs and breccias, I propose the name Moat volcanics to include the Pequawket breccia and "quartz porphyry" of Hitchcock (1). Most of the comendites (quartz porphyries) are considered to be lava flows, but some occurrences may represent shallow intrusives.

The type locality for the Moat volcanics is on South Moat Mountain where an excellent section is exposed from an elevation of 1100 feet upwards on the South Moat trail to the Red Ridge (see figure 5). The lowest exposures are composed of Pequawket breccia, which continues up to an elevation of 1250 feet. From this point to 1410

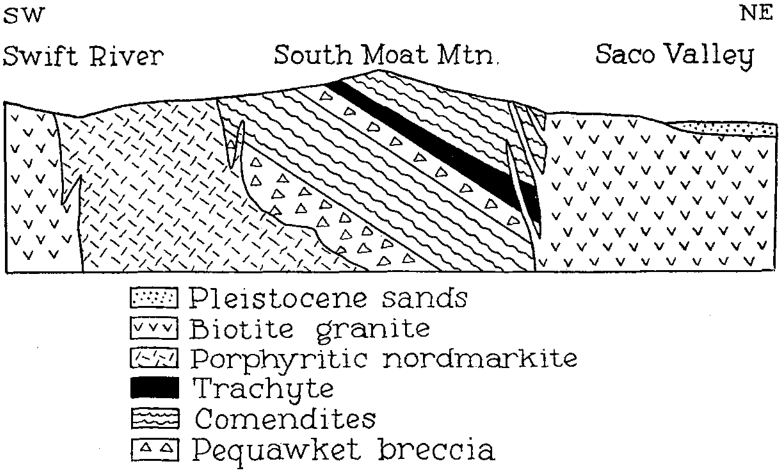

FIGURE 5. Structure section across South Moat Mountain. Scale about one inch to one mile and a half.

feet is a quartz porphyry (comendite) flow, above which comes a breccia that gradually passes upwards into a feldspathic tuff. The stratification becomes more pronounced and may be readily viewed in the cliffs east of the trail between 1670 and 1760 feet. The strata strike northwest and dip about 32° NE. Immediately above a dense tuff is a red trachyte which extends up the trail to 2400 feet. The upper fifty feet of this flow is strongly brecciated. Above the breccia zone comes the great series of gray quartz-porphyry flows which comprise the South Moat Ridge. The structure of these flows may be readily seen on a large scale as one looks north from South Moat Mountain; the dip of thirty two degrees to the northeast is clear. Looking south toward South Moat, the same structure is observed, and in the winter the alignment of the spruces, as seen from Conway station, shows that this structure is characteristic of South Moat. In details the same attitude is found, not only in the tuffs and breccias already mentioned, but also in the flow structure and the contacts of individual flows seen on South Moat Mountain and the ridge to the north. The whole aspect of the group indicates clearly that the Moat volcanics form a series of interbedded flows and breccias. The same structure is shown on the spur leading from the South Moat Ridge to the Albany Haystack, where tuffs, breccias, a trachyte flow, and comendite flows are conspicuously interbedded. The accompanying structure section (figure 5) shows the relations of the various groups. The area of breccia west of the Moat Range, exposed along Little Deer Brook, shows frequent bedding and a structure similar to that of the South Moat area. Summing up, all the evidence indicates that the Moat Mountain block of volcanics forms a monocline striking N 45° W and dipping 32° NE. The attitude of the volcanics of North Moat and Big Attitash is not clearly shown, but the general distribution of rock types indicates an attitude similar to that on South Moat. The structure of Mt. Pequawket is very poorly shown, but the detailed mapping and an occasional stratified layer indicate in general an east-west strike and a vertical dip (see figure 6).

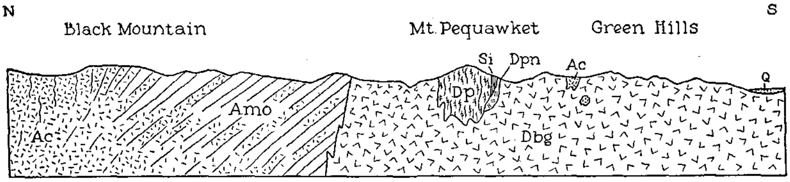

FIGURE 6. Semi-diagrammatic cross-section through the North Conway quadrangle along a north-south line. Scale one inch equals about four and a half miles. *Amo* = Montalban schists, *Ac* = Chatham granite, *Si* = Intervale clay slates, *Dp* = comendites, tuffs, and breccias, *Dpn* = porphyritic nordmarkite, *Dbg* = biotite granite, *Q* = Quaternary deposits. This section illustrates the difference in the mechanics of intrusion of the Chatham granite and the Devonian (?) granites and syenites. The former intruded the schists lit-par-lit, and was essentially contemporaneous with a period of folding; it belongs to a *synchronous* batholith. The Devonian plutonics were intruded with sharp contacts after a period of orogeny; they belong to a *subsequent* batholith. Mt. Pequawket is an isolated block of volcanics immersed in the granites and syenites of the Devonian batholith; a small part of the floor upon which these volcanics were deposited (*Si*) is preserved.

The relative age of the Moat volcanics and the other formations in the White Mountains is readily determined. The volcanics are younger than the Montalban schists, Intervale clay-slates, Chatham granites, and Meredith porphyritic granite, for fragments of these formations are found in the breccias. On the other hand, the plutonic phases of the White Mountains batholith, with the possible exception of the diorite, are younger; not only are fragments of them completely lacking in the tuffs and breccias, but the Albany porphyritic nordmarkite and Conway biotite granites are clearly intrusive into the Moat group. On the east slopes of Moat, where the biotite granite and volcanics are in contact for five miles, the biotite granite is distinctly finer-grained, and apophyses from it cut the volcanics. The striking manner in which the biotite granite cuts off the structures southeast of South Moat Mountain is seen on the geological map (Plate I). That the porphyritic nordmarkite (Albany granite of Hitchcock) is intrusive is shown by its general distribution, by a detailed map on the south slopes of Mt. Pequawket (figure 2), by the manner in which the Albany type cuts off the structures of the volcanics on the southeast slopes of Mt. Pequawket, and by the fact that the porphyritic nordmarkite shows contact modifications against the volcanics. On the south slopes of Mt. Pequawket a hastingsite sölvsbergite (nordmarkite) dike cuts the Pequawket breccia.

The exact contact of the Moat volcanics with the basement upon which they were extruded is everywhere lacking in the North Conway quadrangle, except at the small slate patch on the south slopes of Mt. Pequawket, which is exposed at an elevation of 1500 feet on the Kearsarge Village trail (Figure 2). That a mountain-making movement followed the deposition of the slates and preceded the formation of the Moat volcanics is demonstrated by several facts: (1) slate fragments which show fracture cleavage and slaty cleavage, the latter being essentially parallel to the original bedding, are abundant in the volcanics; (2) the detailed map (figure 2) suggests that a distinct angular unconformity exists between the slate and the volcanics at the east end of the slate belt.

Since the Moat volcanics on Moat Mountain form a simple monocline striking northwest and dipping 32° NE, the thickness of the formation may be readily determined, assuming no faulting parallel to the strike. A section in the vicinity of the Red Ridge gives a thickness of 8,300 feet; a section from the northwestern slopes of Big Attitash toward Humphrey Ledge gives a value of 9,700 feet; and a measurement on the Moat mass as a whole gives a value of 11,800 feet. These are minimum figures for the thickness of the formation, because both the top and the bottom have been stoped away. Assuming Mt. Pequawket to be a vertically-dipping monocline, the volcanics are found to be 9,200 feet thick. Again this is a minimum figure, for although the basement is exposed in the south, the top of the series has been carried away by intrusions along the northern border.

South Moat Flows.

Comendites. (*Quartz porphyries*). The comendites are typically exposed on South Moat Mountain, but they also occur in large areas on the north slopes of Big Attitash Mountain, on the north and east slopes of North Moat Mountain, on the Red Ridge (the Red Ridge is a mile southeast of North Moat), and also on the spur extending southwest from the Red Ridge. On Mt. Pequawket the comendites are found well exposed on the southwest slopes of Bartlett Mountain; the exposures on the west and northwest slopes of Bartlett Mountain and on the north and east slopes of Mt. Pequawket are poor.

The comendite outcrops vary in color. On the South Moat trail they are typically gray, but on the Red Ridge a red color is pronounced. On the east slopes of South Moat a yellow tinge is present; certain phases with a coarse groundmass are white; in varieties with a dense groundmass a black color is prominent. On the west slopes of South Moat the upper ten feet of one of the porphyry flows is much redder than the main portion of the flow. Flow structure is not common. On South Moat Mountain and the ridge to the north, broad flow bands, one or two inches thick, conform with the monoclinal structure, and on the west slopes of peak 2750 (South Moat Ridge) a zone fifteen feet thick shows typical rhyolitic flow texture.

The comendites greatly vary in detail, but they are all similar in showing quartz and feldspar phenocrysts, set in a relatively dense groundmass. The quartz phenocrysts are smoky or less commonly milky white, vary in size from one to three millimeters, and in many cases show the bipyramidal termination characteristic of high-temperature quartz. In any one hand specimen the feldspar phenocrysts are uniform, but throughout the area the color may be milky white, pink, or occasionally a light greenish-gray. They are usually from one to three millimeters in length. The groundmass is dense or fine-grained and may be bluish-gray, greenish-gray, yellowish-gray, or white.

Under the microscope we note in addition to the characteristic phenocrysts of quartz and microperthite an occasional phenocryst of biotite or hastingsite. The feldspar phenocrysts are microperthite, antiperthite, or, much less commonly, a homogeneous feldspar, probably soda orthoclase. Although the quartz phenocrysts are occasionally euhedral in outline, they are more commonly irregular and in many cases are corroded and embayed. The ferrous amphibole, hastingsite, is not abundant and is smaller than the quartz and feldspar phenocrysts but larger than the groundmass. It is very irregular in outline and frequently shows sieve structure; its detailed description would merely duplicate that of the hastingsite of the porphyritic nordmarkite (see below). Fayalite was observed in one of the thin sections, the largest grain being more than a millimeter in its greatest dimension. Fluorite is frequently associated with the fayalite and hastingsite. Biotite flakes locally constitute as much as four per cent of the rock. The hornblende and biotite may occur together in the same hand specimen, or only one may be present, or both may be lacking.

The mineralogy of the groundmass seems to be fairly constant, but the textural pattern and grain-size are very variable. Within a given specimen, however, the groundmass is usually uniform. It is composed of quartz and alkali feldspar, and is generally panallotriomorphic or microfelsitic (cryptocrystalline), in the former case showing frequent micropoikilitic or micrographic tendencies. In general the feldspar is homogeneous, with indices considerably lower than those of quartz and balsam, and is therefore considered to be orthoclase, probably with more or less albite in solid solution. Some twinned albite and oligoclase, however, are present in small grains. Minor accessories include zircon, magnetite, and fluorite. The latter is relatively widespread and occurs in irregular isolated patches in the groundmass, but occasionally it is clearly associated with the corroded hastingsite. Small needles of apatite occur. Secondary minerals include kaolin, sericite, titanite, clinozoisite, biotite, magnetite, chlorite, and calcite.

A variety which contains very few phenocrysts may be termed a *microgranite*. It has been noted on the northwest slopes of Bartlett Mountain and on the southeast slopes of Mt. Pequawket. In the latter area we find flow structure. A *riebeckite comendite* was found on Dry Brook (Albany) at an elevation of 1480 feet. The riebeckite occurs as microlites in both the groundmass and the phenocrysts. An *aegerine-augite comendite* found on the ridge between Big and Little Attitash Mountains contains abundant grains of what Washington and Merwin (5) would term acmitic diopsidic hedenbergite. The pyroxene occurs as small irregular blebs, a habit that suggests a deuteric origin. The optical data follows: Biaxial positive, $\alpha = 1.718$, $\beta = 1.728$, $\gamma = 1.742$; $2V = 70° \pm 5°$; $Y = b$; $X \wedge c =$ high, about 35°. $X =$ yellowish green, $Y =$ yellowish green, and $Z =$ light yellowish green; $X = Y > Z$. These optical data correspond rather closely to the data given by Washington and Merwin (5) for the aegerine-augite from Salem Neck, Massachusetts.

Perry (6) has made four analyses of the Moat volcanics, but, inasmuch as he did not recognize the true origin of the group, one of his analyses seems to have been made from the tuffs and is therefore omitted. An analysis of the riebeckite comendite has been made for this paper by W. H. Herdsman of Glasgow, Scotland. A paisanite dike from Mt. Ascutney is very similar and is therefore quoted.

The similarity between the Mt. Ascutney rocks and those from the White Mountains batholith is very striking and analyses of rocks from the former locality will be frequently quoted.

	1	2	3	a	b	4	c
SiO_2	75.38	73.33	72.25	73.65	74.44	72.05	73.03
Al_2O_3	11.85	12.95	13.40	12.73	11.27	14.72	13.43
Fe_2O_3	1.78	0.98	1.10	1.29	2.78	1.02	0.40
FeO	0.88	1.66	1.53	1.36	0.94	1.46	1.49
MgO	0.00	0.00	0.00	0.00	0.35	0.15	0.14
CaO	0.33	0.98	0.74	0.68	0.21	0.79	0.79
Na_2O	3.68	3.46	4.27	3.80	4.18	4.42	4.91
K_2O	5.37	5.61	5.56	5.51	4.95	4.00	4.54
H_2O+	0.50	0.30	0.31	0.37	0.59	0.55	0.35
H_2O-	0.15	0.11	0.10	0.12		0.55	0.18
MnO	0.10	0.13	0.11	0.11	0.08	tr	0.15
TiO_2	nd	nd	nd	nd	0.19	0.23	0.30
P_2O_5	nd	nd	nd	nd	0.02	nd	0.06
Total	100.02	99.51	99.37	99.62	100.00	99.94	99.77

1. Bluish-gray comendite with fine groundmass. From 1550 feet elevation, south slope of Mt. Pequawket, just above the clay-slate. Sp. Gr., 2.62. J. H. Perry, analyst.

2. Dark gray comendite. East slope of Mt. Pequawket, about 2000 feet elevation. Sp. Gr., 2.64. J. H. Perry, analyst.

3. Dark gray comendite, summit of Mt. Pequawket. Sp. Gr., 2.643. J. H. Perry, analyst.

a. Average of the three analyses above. (Average sp. gr., 2.63.)

b. Average comendite. (Daly, Igneous Rocks and Their Origin.)

4. Riebeckite comendite, Dry Brook, Albany, 1480 feet elevation. W. H. Herdsman analyst. Sp. Gr., 2.61.

c. Paisanite dike, Mt. Ascutney, Vermont. (R. A. Daly, Bull. 209, U. S. G. S., p. 75, 1903.) Sp. Gr., 2.63. Additional data on analysis: $ZrO_2 = 0.06$, $Cl = 0.03$, $F = 0.08$, $FeS_2 = 0.09$; total = 100.03.

Norms. In the Norm classification these specimens are liparose and give the following normative composition:

Specimen	1.	2.	3.	4.
Symbol	I.″4.1.3	I.4.1″.3	I.4.1.3	I.4.1″.3
Quartz	32.76	28.62	23.94	28.44
Orthoclase	31.69	33.34	33.34	23.91
Albite	31.44	29.34	36.15	37.20

Specimen	1.	2.	3.	4.
Anorthite		3.06	0.56	3.89
Corundum				1.63
Diopside	0.49	1.73	2.97	
Hypersthene	0.53	1.72	0.40	1.85
Magnetite	2.15	1.39	1.62	1.39
Ilmenite				0.46

Trachyte. The trachyte is found as a distinct flow only on the south slopes of South Moat Mountain, but fragments of it are persistent, though rare, throughout the tuffs and breccias of Moat Mountain and Mt. Pequawket. On the South Moat trail and even better in the small cliffs to the east of the trail, at an elevation of about 1760 feet, one may see the base of the trachyte overlying a dense, water-laid tuff. The top of the flow is found at 2400 feet; it is brecciated and overlain by a well stratified tuff. The dark color of the rock differentiates it from the other volcanics, and, looking west from the Saco Valley, the dark cliffs of trachyte stand out in sharp contrast with the white, red, and gray comendite ledges. The rapid thickening of the flow to the southeast is well shown on the geological map. The base of the flow, usually underlain by a water-laid tuff, is conformable to the general structure, but the top is not. The writer is not prepared to state whether the rapid thinning is due to the fact that the flow originally came from the southeast, thinning toward the northwest, or is the result of erosion after the trachyte was extruded but before the quartz-porphyry flows of South Moat Mountain were extruded. A northeast section taken across the South Moat trail at 1750 feet elevation indicates that this flow is about 1100 feet thick at maximum. The manner in which the trachyte is cut off by the younger biotite granite southeast of South Moat Mountain is striking.

The trachyte of South Moat Mountain is composed of pinkish-white phenocrysts of feldspar, set in a dense, red or purplish groundmass. The phenocrysts are somewhat larger than the feldspar phenocrysts of the quartz porphyries and often reach half a centimeter in their maximum dimension. A few smoky quartz phenocrysts are present.

The microscope shows two generations of feldspar phenocrysts—a larger group averaging about 2.5 millimeters in length, and a smaller group about 0.2 millimeters long. The feldspar crystals of the groundmass are approximately 0.04 millimeters in length. The trachytes have suffered considerable alteration and for this reason the feldspars are clouded by clinozoisite, sericite, and kaolin. The feldspar phenocrysts, irregular in outline, are composed of a patchy mixture of twinned albite and orthoclase. The quantity of albite greatly varies, comprising from 25 to 95 percent of the crystal. Quartz phenocrysts are very rare and have suffered considerable corrosion.

The groundmass is strikingly different from that of the quartz porphyries and microgranites, as it is composed chiefly of tabular feldspar crystals, usually parallel to one another. It is homogeneous and probably a soda-orthoclase; microperthite and albite are also present. Much dust of hematite clouds the slide. Minor accessories are zircon and rather abundant, ragged grains of magnetite. Epidote and clinozoisite in large irregular masses are not uncommon, and chlorite is also present.

The following analysis of the South Moat trachyte has been made by W. H. Herdsman.

	5.	6.	m.	d.
SiO_0	65.05	64.40	62.63	60.68
Al_2O_3	16.80	16.75	17.06	17.74
Fe_2O_3	4.97	1.31	3.01	2.64
FeO	1.12	3.44	1.98	2.62
MgO	0.20	0.62	0.63	1.12
CaO	1.68	1.85	1.51	3.09
Na_2O	3.94	4.44	6.26	4.43
K_2O	5.22	4.96	5.37	5.74
H_2O+	0.30	1.05	0.71	1.26
H_2O-	0.45	0.22		
TiO_2	0.25	0.88	0.62	0.38
P_2O_5	Tr	0.11	0.09	0.24
MnO	Tr	0.05	0.13	0.06
Total	99.98	100.08	100.00	100.00

5. Trachyte from the South Moat trail, 2040 feet elevation. W. H. Herdsman, analyst. Sp. Gr. 2.65.

6. Albany porphyritic nordmarkite, Jackson Falls. W. H. Herdsman analyst. Sp. Gr. 2.68.

m. Average alkaline trachyte (Daly, Igneous Rocks and Their Origin).

d. Average trachyte. (Daly, Igneous Rocks and Their Origin.)

Comparing our trachyte with average trachyte, the most striking features are the high silica and ferric-iron content, and the low lime and magnesia. The high silica is due to the presence of more quartz than is usual in the average trachyte; the high ferric iron is due to the unusual amount of hematite. The low lime and magnesia indicate a trachyte with alkaline affinities. The close chemical similarity of the trachyte to the Albany porphyritic nordmarkite, hereafter described, is noteworthy. The chief difference is in the state of oxidation of the iron.

From analysis No. 5 the following norm has been calculated.

Name	Toscanose
Symbol	I''.4.(1)2.''3.
Quartz	18.30
Orthoclase	30.58
Albite	33.54
Anorthite	8.34
Corundum	1.63
Hypersthene	0.50
Ilmenite	0.61
Magnetite	2.55
Hematite	3.20

The percentage of quartz is much higher than one would expect from a microscopical examination of the rock, but the high hematite content checks well.

Pequawket breccia.

Interbedded with the quartz porphyry and trachyte flows of the Moat volcanics is a group of clastic rocks varying greatly in composition and appearance. On Moat Mountain three relatively large areas are found in addition to a number of smaller lenses. The structural relations are best shown on the south and southwest slopes of South Moat Mountain, where a great group of tuffs, breccias, and quartz porphyry flows underlie the trachyte. A second large area of breccias is found on North Moat Mountain, and, although the exposures are good, the structural relations are by no means clear. A third area is well exposed along that branch of Little Deer Brook that drains the southwestern slopes of Big Attitash Mountain. Smaller areas have been found on the ridge between Little Attitash Mountain and Humphrey Ledge, on the east slope of South Moat, and in that branch of Little Deer Brook flowing west from the Red Ridge. On Mt. Pequawket the whole southern slope and the two main peaks are zones of clastic rocks. A second area is found on the northern slopes, but it is very poorly exposed. A third area is found at the extreme eastern end of the Pequawket mass; this area is likewise poorly exposed.

In describing the various tuffs and breccias I will first discuss in some detail the fragments occuring in the clastic rocks and then describe the matrix; finally, I will group them into lithologic types. A fragment may be described rather arbitrarily as a rock or mineral of macroscopic size; anything smaller is classed as matrix. In maximum the fragments may reach three, four, or even five feet in diameter. Blocks reaching these dimensions are rare; but bowlders of two-mica granite of this size occur on the southwest slopes of South Moat, and on the summit of Mt. Pequawket a diabase bowlder ten feet across is found in the feldspathic tuffs. Blocks a foot or two in diameter are very common in all the areas of breccia. As a general rule these clastic fragments are angular to subangular, the feldspar, quartz, slate, and schist being particularly angular; the quartz-porphyry and granite bowlders as a rule show considerably more rounding than the other rocks.

In certain respects the lithology of the fragments occurring in the clastics of Moat Mountain and Mt. Pequawket are similar, but there are certain striking differences. On Mt. Pequawket the most common fragment is clay-slate with important quantities of quartzite and quartz porphyry; on the upper slopes quartz and feldspar become very abundant. The clearness of the quartz and the frequent magmatic corrosion imply that most of it is of volcanic origin; some of it may be detrital, the product of the decay of older granites. The feldspar, usually orthoclase with irregular albitic patches, is apparently all of volcanic origin. Pebbles of trachyte have also been found, but diabase bowlders are rare. Fragments composed of fine-grained tuffs are not uncommon. Relatively rare minerals that are found embedded in the groundmass include rounded zircon (probably originally from the quartzites which occur so commonly as pebbles) and sericite (probably chipped off from the clay-slate fragments).

On Moat Mountain these types are found, and in addition, andalusite clay-slate (on the summit of North Moat), quartz-mica schist, quartz-chlorite-muscovite schist, two-mica granites, muscovite granites, Meredith porphyritic granite, rhyolite showing flow-structure, and detrital feldspar. That some of the feldspar is of detrital origin rather than volcanic, is implied by the fact that it is a cross-hatched microcline, similar in every respect to the microcline found in the Chatham and Meredith granites and quite unlike any of the feldspar of the quartz-porphyry group. Most of the quartz and feldspar, however, is apparently volcanic. Rounded apatite stubs, titanite, and muscovite are found.

From this survey of the fragments it is obvious that those from Mt. Pequawket, being essentially clay-slates and quartzites, represent less deep-seated rocks than the andalusite clay-slate, two-mica granite, muscovite granites, Meredith porphyritic granite, and schists of Moat Mountain.

The matrix is macroscopically black, greenish-black, or light gray; it is very dense, occasionally sprinkled with a few recognizable muscovite flakes. The mosaic is often impossible to resolve under the microscope; where sufficiently coarse, it is seen to be composed essentially of quartz and orthoclase, often with important quantities of chlorite, sericite, and biotite in small flakes. Although some of the biotite and chlorite is probably detrital, much of it occurs in aggregates of many flakes and is apparently secondary. Irregular patches of magnetite are frequently present, and hematite dust sometimes clouds the slide.

The proportion of matrix to fragments greatly varies. In some of the slate-bearing feldspathic tuffs of Mt. Pequawket, there is practically no matrix; any original cavities have been filled by squeezing and crushing of the clay-slate fragments. In many of the feldspathic tuffs of South Moat the matrix makes up half the rock. In still other specimens the fragments may be completely absent, resulting in dense red or black tuffs, the groundmass of which can not be resolved under the microscope.

Because of the variation in the lithology and in the relative proportion of matrix and fragments, numerous rock-types may be differentiated. On the southern slopes of Mt. Pequawket there is exposed a breccia, composed essentially of clay-slate, with small amounts of quartzite, quartz porphyry, quartz, and feldspar; it may be classed as a *clay-slate breccia.* Lithologically similar rocks are found on the South Moat trail between 1100 and 1250 feet. On the summit of North Moat, covering an area of ninety feet by forty-five feet, is a breccia composed essentially of andalusite clay-slate. On the west slope of South Moat, at an elevation of about 2300 feet, are exposures of a *coarse polygenous breccia* which shows a great variety of bowlders, including muscovite granite, two-mica granite, pegmatite, quartz porphyry, quartzites, quartz-mica schist, and sillimanite-mica schist. One of the granite blocks is five feet long. Similar types are found in two small patches east of Little Attitash Mountain. In these occurrences many of the bowlders are composed of the Meredith porphyritic granite, the orthoclase phenocrysts show the typical Carlsbad twinning, but they are seldom over half an inch long. In the bed of Dry Brook, at an elevation of 950 feet, on the southern border of the North Conway quadrangle, is a similar rock, composed largely of a white muscovite graphic-granite. The widespread breccia on the northern slopes of North Moat may be assigned to the polygenous type, but in this case the most abundant fragments are red and black tuffs; other fragments are quartz-biotite schist, micaceous quartzites, quartz porphyries, porphyritic granite, two-mica granite, and pegmatite.

Still a third type of clastic rock, a *feldspathic tuff*, is exposed on the southern slopes of Mt. Pequawket between 2100 feet and the summit. The most common fragments are composed of microperthite or orthoclase with irregular albite patches, and clear, unstrained quartz, sometimes showing evidence of magmatic corrosion. Clay-slate fragments are relatively abundant, and occasional pieces of quartzite, sericitic quartzite, volcanic tuff, trachyte, diabase, and zircon are observed. The feldspathic tuff of Mt. Pequawket grades downwards into the clay-slate breccia on the lower slopes of the mountain. Feldspathic tuffs have also been observed below the trachyte of South Moat Mountain, on the north slopes of North Moat at about 2300 to 2400 feet, and on the Rocky Branch of the Saco River, about a mile below Jericho, where a xenolith of Pequawket tuff is found in the biotite granite. A *quartz-porphyry breccia*, composed essentially of fragments of quartz porphyry, is found on the summit of North Moat, extending down to about 2750 feet on the northern slope and to 3160 feet on the southern slope. A *dense, black tuff* is found typically developed on the west slopes of South Moat at 2330 feet, immediately underlying the trachyte. Somewhat similar, but *gray tuffs* are found east of the South Moat trail at 1760 feet. In the Little Deer Brook locality black, dense tuffs and *dense, red tuffs* are interbedded with polygenous breccias. Dense, red tuffs are also found in Dry Brook (southwest of South Moat) at about 2050 feet; this lithological type is rather commonly present as fragments throughout the clastic phases of the Pequawket volcanics.

PLUTONIC PHASES.

The plutonic phases of the White Mountains batholith form a nearly continuous series of closely allied rocks. Although the end members of the series are very different chemically, mineralogically, and texturally, almost complete gradation may be seen between many of the phases. For example, the nordmarkite porphyry, although locally in sharp contact with the porphyritic nordmarkite, elsewhere seems to grade into it. In the nordmarkite group there are all gradations from pulaskites through nordmarkites to hastingsite granites. The hastingsite granite of Doublehead grades southward into the biotite granite; some phases of the biotite granite are richer in hastingsite than biotite and are thus closely allied to the hastingsite granites. Yet, in spite of the close association and frequent gradations, the following chronological sequence may be determined: diorite, nordmarkite porphyry, porphyritic nordmarkite (Albany granite of Hitchcock), nordmarkites (Chocorua syenites), hastingsite granite (Chocorua granite), riebeckite granite, and biotite granite (Conway granite).

Diorite. A diorite, which is an early phase of the White Mountains batholith, is found in numerous small dikes, two to five feet wide, in a stream bed east of Hurricane Mountain, and also in a small stock-like mass east of Mt. Pequawket along that branch of Weeks Brook north of Mirror Lake. In the latter occurrence actual outcrops are found along the stream-bed between an elevation of 1560 feet and 1450 feet, and the distribution of stream bowlders indicates that the formation extends up to at least 1740 feet. The total area underlain by this stock-like mass of diorite is thus about half a mile long, measured parallel to the stream; the other dimensions are unknown due to the heavy covering of glacial drift.

The normal diorite, as exposed in the dikes and parts of the stock, is a medium to fine grained black to gray granular rock with the typical "salt and pepper" appearance of diorite. With the hand lens the rock is resolved into white plagioclase of tabular habit, hornblende, and small amounts of biotite. An occasional striated plagioclase phenocryst dots the specimen and irregular patches of pyrite are observed in the dike variety.

In the stock-like mass the diorite has been remarkably shattered and impregnated by the biotite granite. Small dikes of medium-textured biotite granite, from a fraction of an inch to half a foot in diameter, cut the diorite. Large crystals of pink microperthite, some half an inch across, are irregularly distributed through the diorite. Somewhat smaller quartz crystals are present. In other parts of the diorite irregular hornblende crystals, as much as a quarter of an inch long, are abundantly distributed; and honey-yellow patches of titanite are visible to the naked eye. The microperthite, the quartz, the hornblende, and the titanite have clearly soaked in from the biotite granite.

Under the microscope the diorite is seen to be composed of frequently zoned, tabular plagioclase, abundant anhedral hornblende, and ragged biotite flakes. Minor accessories include quartz, apatite, magnetite, and pyrite. Alteration minerals are sericite and chlorite. The unzoned plagioclase is andesine (about Ab_{55}) and shows both polysynthetic and Carlsbad twinning. The zoned feldspar is similarly twinned; the core is labradorite (about Ab_{40}) and the periphery is usually oligoclase (about Ab_{80}). The amphibole is in irregular grains and has the following optical properties. Biaxial, negative; dispersion medium, $\rho > \nu$; optic angle medium; $Y = b$, $Z \wedge c = 17°$. Pleochroism: $Y > Z > X$; X = light yellow, Y = olive green, and Z = deep green. Indices $\alpha = 1.670$, $\gamma = 1.698$. These indices are about 0.025 lower than those for hastingsite, and indicate that the amphibole in the diorite is a normal hornblende, relatively rich in magnesia compared to hastingsite.

In the less metamorphosed types the chief change has been the introduction of microperthite, either interstitial to the plagioclase or in aggregates of half a dozen grains; quartz, allanite, and titanite are also introduced. In the more completely soaked varieties the number and size of the introduced amphibole crystals greatly increases, the microperthite is much more abundant, large, irregular patches of titanite are noted (one is seven millimeters long), and apatite appears. A little epidote was found in the titanite. The significance of these observations and their bearing on the origin of essexite is discussed on a later page.

The diorite is obviously older than the biotite granite, as the former is intensely shattered by the latter. On the other hand, it cuts the Montalban schists and the Chatham granites. Since no igneous cycles other than the Chatham and White Mountains periods of activity have been recognized in this quadrangle, and because the law of decreasing basicity seems to be fairly general, we may consider the diorite as an early phase of the White Mountains batholith.

It is obvious that much of the diorite mass was stoped away by the magma of the younger Conway granite, for we have seen this very process "caught in the act" in the shatter-zone along Weeks Brook. We may reconstruct the geological history as follows. A diorite stock, with associated satellitic dikes, was intruded into the Chatham granite occupying the area east of Mirror Lake; the stock may have extended as far south as Hurricane Mountain. The diorite crystallized, and, after a long period of magmatic differentiation, the great mass of Conway granite rose by stoping away most of the diorite stock and the Chatham granite. In the vicinity of Weeks Brook a small part of the stock, intensely shattered, was still preserved, and at South Chatham the Chatham granite and the satellitic diorite dikes were untouched.

Nordmarkite porphyry. The nordmarkite porphyry has been observed in five small areas. Two represent small stocks satellitic to the main batholith. Two others are border phases of the nordmarkite of the main batholith. The fifth body is completely surrounded by the Albany porphyritic nordmarkite. The satellitic stock on Mt. Eastman is about half a mile in diameter, although the exact boundary on the northeast slope could not be established. The Sloop Mountain stock is a long, dike-like body striking north; it is a mile and a half long and a quarter of a mile wide. On the northeast slope of Sable Mountain a belt at least half a mile long and perhaps a quarter of a mile wide is found between the nordmarkites and the older Meredith granite. Still a fourth area is found on the west slopes of Chandler Mountain. This region, although densely wooded and characterized by poor exposures, is important because the nordmarkite porphyry is here intensely shattered by the nordmarkites of Chandler Mountain. The fifth area is well exposed on the northeast slopes of Little Attitash Mountain, between the elevations of 2300 feet and 1750 feet; a few isolated exposures between 1000 and 1200 feet along Stony Brook indicate that the area extends at least that far west. As a rule the contact between the nordmarkite porphyry and the Albany porphyritic nordmarkite is sharp, but locally the two grade into each other.

Dikes of the nordmarkite porphyry cut the Montalban schists at an elevation of 1450 feet, in the brook west of Sloop Mountain; and the Chatham granite, at an elevation of 1040 feet in the "flume" of Chandler Brook. It is thus clearly younger than the Pre-Cambrian (?) formations. On the other hand, it is older than the nordmarkites of Chandler Mountain, as already stated. Thus the nordmarkite porphyry is to be considered an early phase of the White Mountains batholith. The age relative to the diorite is not known from field evidence, but presumably the diorite is the older.

In the hand specimen the nordmarkite porphyry is composed of rather scattered greenish-gray or white, tabular feldspars, set in a greasy greenish-gray or dense, black groundmass. The feldspar phenocrysts vary in length from several millimeters to over a centimeter.

Under the microscope the nordmarkite porphyry is seen to consist essentially of ragged feldspar phenocrysts set in a micropegmatitic or panallotriomorphic groundmass. The feldspar phenocrysts vary from orthoclase with small areas of twinned albite to crystals of untwinned albite containing small shreds of orthoclase. The groundmass consists essentially of orthoclase (which may contain albitic patches), quartz, hastingsite, and hedenbergite. The hastingsite is exceedingly ragged and is frequently peppered by round grains of quartz and orthoclase. The hedenbergite is colorless, has a high extinction angle, and possesses closely-spaced cleavage, along which some magnetite dust is observed. It is often enclosed in the hastingsite. That the pyroxene actually is hedenbergite is demonstrated below, in the description of the nordmarkites. In one slide a number of rather ragged grains of fayalite, some phenocrystic, are present. Another slide showed euhedral phenocrysts of titaniferous augite. Another rare mineral is yellow in thin section and has a low birefringence. Apparently a chlorite, pseudomorphic after fayalite, it is enclosed either in hedenbergite or hastingsite, the former being in turn enclosed by the latter. Minor accessories include large grains of zircon and apatite, biotite, and small irregular magnetites. Secondary minerals include matted sericite, leucoxene, and a little zoisite.

Porphyritic nordmarkite. A porphyritic hastingsite granite or nordmarkite was termed the Albany granite by Hitchcock from its good exposures in the town of Albany. Two areas are found in the North Conway quadrangle, one of which lies to the north and west of the volcanics of Moat Mountain. The second fringes the volcanics of Mt. Pequawket on the southeast, south, west, and northwest, whence it extends to Spruce Mountain as a dike-like mass, varying from a quarter of a mile to a mile wide; at Spruce Mountain the outcrop belt takes a right angle turn and strikes southwest, finally ending in the Crawford Notch quadrangle.

The porphyritic nordmarkite is very close to the boundary between granite and nordmarkite. In the particular specimen analysed the percentage of silica is 64.40; Brögger (7) considered 66 per cent of silica as marking the boundary-line between nordmarkite and granite. It is very possible that some of the porphyritic nordmarkite has more than the maximum allowed; the contact phases, described below, certainly have more than 66 per cent of silica. Hawes's analyses (8) of the so-called Albany granite of Mt. Willard show 72 per cent of silica, thus placing the rock definitely in the granite group; but the rock classified by Hawes is not the same as the original rock from Albany. The present writer would classify the rock described by Hawes as a hornblendic, porphyritic contact-phase of the Conway biotite granite and similar in many respects to the green type described on a later page.

One of the most striking features of the porphyritic nordmarkite is the existence of a chilled contact-phase, which has the chemical and mineralogical composition of a quartz porphyry. Usually this contact zone is twenty feet or less in width; it is often entirely absent. Progressing from the typical porphyritic nordmarkite to the contact with an older rock the groundmass becomes fine-grained and lighter in color, due to the decrease in the amount of hastingsite. The feldspar phenocrysts are of the same size as those in the typical porphyritic nordmarkite. Euhedral quartz crystals bounded by the hexagonal pyramid become very abundant, and, although as a rule smaller than the feldspar phenocrysts, they are more numerous; quartz phenocrysts a quarter of an inch across have been noted. This contact phase has usually been seen against the Moat volcanics, but it has also been seen where the porphyritic nordmarkite is intrusive into the Intervale clay-slates. Unfortunately the relations are best exposed in rather inaccessible localities: one example is a large ledge three quarters of a mile north of Mirror Lake and a mile east of the summit of Mt. Pequawket, where the porphyritic nordmarkite intrudes a tuffaceous phase of the Moat volcanics (2220 feet elevation). The same phenomena were noted north of the Albany Haystack and at an elevation of 2650 feet on the southeast slopes of Big Attitash, peak 2884. In both places the intruded rock is quartz porphyry. The same relation was noted southeast of the slate belt on the south slopes of Mt. Pequawket and on the southwest side of the clay-slate exposed in the East Branch of the Saco River.

The occurrence of such a chilled phase rich in quartz violates the common rule that the marginal phase of a plutonic mass is less siliceous than the main intrusive. Other exceptions have, however, been noted in a number of the alkaline batholiths. Daly (9) records a quartz-porphyry, marginal phase of a nordmarkite at Mt. Ascutney. Ussing (10) found the same relation at Julianehaab, and Brögger (7) observed in the Christiania (Oslo) region quartz-porphyry, marginal phases for the äkerites and nordmarkites. At Red Hill, one of the satellitic stocks of the White Mountains alkaline batholith, Pirsson and Washington (11) noted that the main nepheline syenite intrusion has a nordmarkite marginal facies. I have no explanation of the phenomenon. Ussing's suggestion that it is the result of the assimilation of sandstone is unsatisfactory. It is true that the example cited by Ussing might be explained in this manner, for the intruded rock was sandstone; likewise in the North Conway quadrangle where the country-rock is quartz porphyry a similar explanation might hold. But it does not account for the cases where clay-slate is the intruded rock.

The relative age of the porphyritic nordmarkite is fairly well established. It is younger than the Montalban schists and Chatham granites, for a large xenolith of these two formations has been observed in the porphyritic nordmarkite at Jackson Falls. Dikes of the latter cut the Montalban group at Thorn Mountain. The quartz-porphyry, marginal facies against the Intervale clay-slates has already been noted. The general distribution of the porphyritic nordmarkite as a shell around the Moat volcanics implies that the former is the younger rock; the chilled, marginal phase of the Albany against the volcanics confirms this impression; and on the southeast slopes of Mt. Pequawket blocks of tuff are found in the Albany formation..

The porphyritic nordmarkite is very probably older than the hastingsite granites, because on the south slopes of South Doublehead, between 1980 and 2220 feet, a porphyritic nordmarkite is intensely shattered by a hastingsite granite. Southwest of South Moat Mountain a medium-textured hastingsite granite cuts the porphyritic nordmarkite. Not only does the general distribution of the porphyritic nordmarkite imply that it is older than the biotite granite, but also two small dikes of biotite granite cutting the porphyritic nordmarkite were noted at an elevation of 1730 feet on the northwest slopes of Attitash Mountain, Crawford Notch quadrangle. A number of dikes of biotite granite, described on a later page, cut the porphyritic nordmarkite near Jackson. The mineralogical and chemical relations indicate that the porphyritic nordmarkite is to be correlated as essentially the same as the nordmarkites. The position of the porphyritic nordmarkite is thus determined younger than the Montalban schists, Chatham granites, Intervale clay-slates, and the Moat volcanics; it is older than the hastingsite and biotite granites, but nearly contemporaneous with the nordmarkites.

The porphyritic nordmarkite often contains many inclusions, such as hornfels and schist. This feature may be noted either on the south slopes of Mt. Pequawket or on the ledges in the Swift River at the mouth of Little Deer Brook. A large amount of biotite granite has been found associated with the porphyritic nordmarkite southwest of the volcanics of Moat Mountain. In the hand specimen it is a fine-grained, pink granite speckled with black flakes of biotite. In the field it frequently shows horizontal sheeting, a feature which is excellently displayed near the mouth of Little Deer Brook on the south side of the new road up the Swift River Valley. Under the microscope the textural pattern is conspicuously hypidiomorphic-granular, the albite and microperthite crystals forming a striking network which contains the interstitial, anhedral grains of quartz. The biotite contains abundant poikilitic quartz and feldspar; it is often altered to chlorite, and a muscovite-like mica has occasionally resulted from the "bleaching" of biotite. Minor accessories include fluorite, zircon, and iron ore. Hematite is noted as a secondary mineral.

This biotite granite is assigned to the White Mountains batholith rather than to the earlier granites, because of the nature of its feldspars, the presence of fluorite, and the absence of true muscovite. On the other hand, it is definitely older than the Conway biotite granite, which encloses blocks of this granite. I have not succeeded in conclusively demonstrating that the fine biotite granite is present as inclusions in the porphyritic nordmarkite, but the manner in which they are associated in the field makes the conclusion almost inevitable. The existence of inclusions of biotite granite in the porphyritic nordmarkite is important for a theory of the magmatic differentiation, for it demonstrates that a more siliceous rock (granite) may be older than the less siliceous (nordmarkite), contrary to the general rule in composite batholiths.

In the hand specimen the porphyritic nordmarkite consists of irregular or tabular feldspar phenocrysts, averaging about five millimeters in length, set in a relatively fine groundmass, speckled with black hastingsite. Both the groundmass and the phenocrysts may be pink, white, bluish-gray, or greenish-gray. Twinning in accordance with the Carlsbad law is common. The percentage of phenocrysts is variable, in places forming more than half the specimen, in others only twenty-five per cent of the rock. Quartz phenocrysts, irregularly spheroidal in habit, are as a rule rather scarce; in fact, the typical porphyritic nordmarkite shows on the average only three or four quartz phenocrysts in a standard hand specimen. The groundmass varies in grain-size, but micrometer measurements show that in the typical type the feldspar and quartz of the groundmass are from 0.1 to 0.2 millimeters in diameter; in the contact phase, described on a previous page, the groundmass grains are from

0.01 to 0.02 millimeters across, being actually smaller than the groundmass of some of the comendites.

Under the microscope the porphyritic nordmarkite is a holocrystalline rock with phenocrysts of microperthite and quartz, the former greatly predominating. The feldspar phenocrysts are as a rule irregular, tending to be tabular in shape. Aggregates of feldspar crystals are sometimes found. The quartz, on the other hand, is rounded and invariably shows more or less corrosion. The groundmass is composed essentially of a hypidiomorphic intergrowth of orthoclase, microperthite, albite, quartz, and hastingsite. Micropegmatitic intergrowths are present. The hastingsite occurs in the typical subhedral crystals elongated parallel to the *c*-axis and is somewhat larger than the typical groundmass quartz and feldspar. Minor accessories include zircon, allanite, apatite, fluorite, magnetite, and biotite (both primary and secondary). Secondary minerals include sericite (poikilitic in the albitic feldspars), chlorite, and very rarely clinozoisite.

Of widespread distribution throughout the White Mountains batholith is an amphibole belonging to the soda-iron series. It forms the essential dark mineral in the nordmarkite porphyry, porphyritic nordmarkite, nordmarkite, and hastingsite granite, and it is present in small quantity in the biotite granite and the comendites. The habits of this amphibole are very variable. In the comendites it is very spongy, enclosing poikilitically quartz and feldspar. In the porphyritic nordmarkite it is more compact with euhedral tendencies; in the nordmarkites it is frequently zoned about hedenbergite.

The optical properties are as follows. Biaxial negative. Indices (determined to within three in the third decimal place): $\alpha = 1.698$, $\beta = 1.719$, $\gamma = 1.722$. $2V = 47° \pm 3°$. Dispersion is medium-strong. Optical orientation, $Y = b$, $Z \wedge c = 20°$. Pleochroic formula $Y > Z > X$, but Y and Z are almost equal: X = light yellowish brown, Y = olive green, Z = deep green. The specific gravity, determined by pycnometer, is 3.375 at 4° C.

The high indices and high specific gravity imply richness in iron and inasmuch as an analysis of the widespread amphibole in the alkaline syenites and granites of the White Mountains has not been made, it seemed advisable to separate some for chemical analysis. The material was taken from the porphyritic nordmarkite at Jackson Falls, for an analysis of this rock had already been made. The rock was ground up so as to pass through a screen with 60 mesh to the inch; all the material passing through a 200-mesh screen was discarded. A magnetic concentration was first made; this concentrate consisted essentially of hastingsite and some magnetite, but some quartz, feldspar, and biotite were present as impurities. The powder was then run through methylene iodide (specific gravity about 3.3); the feldspar, quartz, and biotite floated but the hastingsite and magnetite (of which there was very little) sank in the liquid. After drying the powder, it was run under a hand magnet and the electromagnet to eliminate any magnetite that might be present. A careful check was kept by microscopic methods during the course of the separation. The final product was more than 99 per cent pure. The analysis of this amphibole and the porphyritic nordmarkite from which it comes are given below, along with the analysis of the original hastingsite (12) and similar analyses by Quensel (13), Shannon (14), and Nelson (15).

	6.	C.	g.	h.	i.	j.
SiO_2	64.40	37.40	34.18	37.49	38.50	36.86
Al_2O_3	16.75	12.34	11.52	10.81	10.88	12.10
Fe_2O_3	1.31	4.16	12.62	7.52	6.70	7.41
FeO	3.44	25.84	21.98	25.14	27.28	23.35
MgO	0.62	2.20	1.35	1.34	1.40	1.90
CaO	1.85	9.72	9.87	9.77	11.30	10.59
Na_2O	4.44	1.80	3.29	2.06	1.22	3.20
K_2O	4.96	1.36	2.29	1.91	1.66	1.20
H_2O+	1.05	Tr	0.35	2.01	1.27	0.60
H_2O-	0.22	0.60			0.00	0.70
TiO_2	0.88	3.20	1.53	0.86	tr	1.04
P_2O_5	0.11	nd	nd	nd	nd	nd
MnO	0.05	1.24	0.63	0.95	tr	0.77
BaO	nd	0.00	nd	nd	nd	nd
Cr_2O_3	nd	0.00	nd	nd	nd	nd
Totals	100.08	99.86	99.61	99.86	100.16	99.72

6. Porphyritic nordmarkite, Jackson Falls. W. H. Herdsman, analyst.

C. Hastingsite from porphyritic nordmarkite, Jackson Falls. W. H. Herdsman, analyst.

g. Hastingsite, Dungannon, Ontario.

h. Hastingsite, Seglinge, Almunge.

i. Metamorphic amphibole from Custer County, Idaho.

j. Hudsonite, Cornwall, Orange County, New York.

The Rosiwal method has been employed to determine the proportions of the various minerals in the analyzed porphyritic nordmarkite.

The result is here stated, along with the norm. The percentage of allanite is probably a little high for the average porphyritic nordmarkite.

Mode		Norm	
Quartz	12.6	Quartz	13.08
Feldspar	74.1	Orthoclase	29.47
Hastingsite	12.3	Albite	37.73
Allanite	0.4	Anorthite	8.34
Biotite	0.5	Corundum	0.92
Apatite	tr	Hypersthene	5.43
Magnetite	0.1	Apatite	0.34
Zircon	tr		
Total	100.0		

The following chemical analysis has been made by W. H. Herdsman. Along with it are given the analyses of average nordmarkite and also a nordmarkite from Mt. Ascutney.

	6.	e.	f.
SiO_2	64.40	64.36	64.88
Al_2O_3	16.75	16.81	16.24
Fe_2O_3	1.31	1.08	1.37
FeO	3.44	2.71	2.70
MgO	0.62	0.72	0.89
CaO	1.85	1.55	1.92
Na_2O	4.44	5.76	5.00
K_2O	4.96	5.62	5.61
H_2O+	1.05	0.70	0.46
H_2O-	0.22		0.19
TiO_2	0.88	0.45	0.69
P_2O_5	0.11	0.09	0.13
MnO	0.05	0.15	0.14
Total	100.08	100.00	100.22

6. Porphyritic, nordmarkite, Jackson Falls. W. H. Herdsman, analyst. Sp. Gr. 2.68.

e. Average nordmarkite (Daly, Igneous Rocks and Their Origin).

f. Nordmarkite from Mt. Ascutney. Sp. Gr. 2.68. Contains in addition $ZrO_2 = 0.13$, $Cl = 0.04$, $F = 0.08$, $BaO = 0.06$; total 100.53.

Comparing this analysis with the average nordmarkite, we note that the ferrous iron is somewhat higher and the alkalies lower than the average. The alkalies are lower than in the average, because the analyses used in Daly's computations come mainly from the unique Oslo region. Similarity in the proportions of the other oxides and with the Mt. Ascutney analysis as a whole is noteworthy.

According to the Norm classification, analysis No. 6 is a toscanose with the symbol I(II).4(5).″2.3.

Nordmarkites. The nordmarkites are a group of medium-textured quartz-hastingsite syenites. Hitchcock applied the name Chocorua Group to the hastingsite-quartz syenites and granites of the White Mountains group; these granites are well exposed on Mt. Chocorua, just south of the North Conway quadrangle. The proportion of quartz is very variable; in some places it is almost lacking, giving a pulaskite, but in others it is so high that the rock must be classed as a hastingsite granite. In fact, any division between nordmarkites and hastingsite granites is purely arbitrary and an exact separation has not been possible in the areal mapping. In the Christiania (Oslo) region Brögger (6) similarly observed all gradations between quartz-poor and quartz-rich rocks. He wrote: "The less siliceous rocks of the nordmarkite series appear to be transitional into the äkerite series; the more acid are transitional into the later soda granites."

The only large area of nordmarkites and associated hastingsite granites is found in the north-central part of the quadrangle, on Sable and Chandler Mountains, and on the hills around Mountain Pond. The nordmarkite types are much the more numerous, but the granitic phases are common. Good exposures may be seen on the northeast, south, and southwest slopes of Sable Mountain and on the north slope of Chandler Mountain. Although heavily wooded, the south slope of Chandler Mountain and the hills north and south of Mountain Pond show outcrops of similar rocks. Unfortunately the northern part of the quadrangle is at present very inaccessible, for the mountains are far back from the roads, trails are lacking, and a heavy underbrush makes traveling extremely difficult. South of Mountain Pond the heavy forest-cover and the paucity of outcrops makes mapping difficult. A small patch of syenite has been noted on Tin Mountain near the Tin Mines, apparently as an inclusion in the Albany porphyritic nordmarkite. On the west slopes of Mt. Bartlett a small syenite area is associated with the Albany type.

The isolation of the nordmarkites from the other phases of the batholith makes it difficult to determine the exact age relations. They are slightly younger than the nordmarkite porphyry, for they shatter it on the west slopes of Chandler Mountain, and also the marginal distribution of the nordmarkite porphyry implies its greater age. The pulaskite phase of the nordmarkites exposed on the southwest slopes of Chandler Mountain is intruded by dikes of hastingsite granite. These facts indicate that the nordmarkites are younger than the nordmarkite porphyry but older than the hastingsite granites. Moreover, the similarity in mineralogical and chemical composition of the nordmarkite and porphyritic nordmarkite implies that they are essentially contemporaneous; since the latter is older than the biotite granites, the nordmarkites are likewise considered to be pre-Conway.

In the hand specimen the hastingsite-quartz syenite is a medium-textured rock composed essentially of greenish feldspar and black hastingsite in the proportion of about seven to one. The feldspar crystals tend to be tabular, average three to five millimeters in length, and occasionally show Carlsbad twinning. In some varieties the feldspar is pink, and in the more weathered specimens white. The hastingsite is usually irregular in habit, but tends to be either circular or oval in cross-section. The amount of quartz varies from five to fifteen per cent.

Under the microscope the nordmarkites are found to be holocrystalline hypidiomorphic-granular rocks composed essentially of microperthite and hastingsite. Quartz is sparingly present as small grains interstitial among the feldspars. Hedenbergite, constituting as much as ten per cent of the rock in the pulaskitic phases, is commonly zoned in an extremely patchy manner by the hastingsite, but it has also been noted without any associated amphibole. Minor accessories include albite, apatite, magnetite, zircon, biotite, allanite, fluorite, and chlorite.

Pyroxene is very rare in the granitic phases of the White Mountains batholith, but it may be relatively abundant in the syenitic phases; in the pulaskite it is the dominant ferromagnesian mineral. It usually occurs as a core surrounded by hastingsite. From the analysis, No. 7, stated below, the core pyroxene is inferred to be hedenbergite rather than augite. The low magnesia implies only a small content of that oxide in the pyroxene, which makes up 3.6 per cent of the rock. Knowing the relative proportions of all the minerals in the rock analysed and the chemical composition of all the minerals except the pyroxene, it has been possible to determine that the pyroxene is composed essentially of lime, ferrous iron, and silica; rather abundant ferric iron and titanium accompany small quantities of soda, potash, and magnesia.

The modes of two specimens, one from a type without pyroxene (A), a second with considerable pyroxene (7), are given below. These two specimens are lower in quartz than the average nordmarkite of the quadrangle.

	7.	A.
Quartz	6.3	4.0
Feldspar	75.9	86.0
Hastingsite	12.6	9.5
Hedenbergite	3.6	
Biotite	0.4	
Apatite	0.1	0.2
Magnetite	1.0	0.3
Zircon	0.1	
Totals	100.00	100.00

7. Nordmarkite from an elevation of 2410 feet on the ridge extending south from Chandler Mountain.

A. Nordmarkite from the northwest slope of Bartlett Haystack, elevation of 1980 feet. (Crawford Notch Quadrangle.)

The following chemical analysis has been made for this paper by Mrs. S. Parker of Zürich, Switzerland. The specimen is the same as that for which the mode is given above.

	7.	e.	k.
SiO_2	62.24	64.36	62.99
Al_2O_3	15.82	16.81	14.25
Fe_2O_3	1.94	1.08	2.78
FeO	4.69	2.71	5.15
MgO	0.07	0.72	1.30
CaO	2.65	1.55	2.72
Na_2O	4.80	5.76	4.86
K_2O	6.26	5.62	6.35
H_2O+	0.56	0.70	0.18
H_2O-	0.07		n.d.
TiO_2	0.87	0.45	0.16
P_2O_5	0.14	0.09	n.d.
MnO	0.24	0.15	0.18
Total	100.35	100.00	100.92

7. Nordmarkite from an elevation of 2410 feet on the ridge extending south from Chandler Mountain. Sp. Gr. 2.67.

e. Average nordmarkite. (Daly, Igneous Rocks and Their Origin.)

k. Umptekite, Beverly, Massachusetts (16). Sp. Gr. 2.73.

The White Mountains nordmarkite is richer in iron and lime than the average nordmarkite, due to the greater percentage of dark minerals in the former. Some specimens, such as *A* in the modal determinations, would have a proportion of dark minerals more like that of the average nordmarkite. The umptekite from Beverly, Massachusetts, is similar in many respects, but is richer in dark minerals.

From analysis No. 7 the following norm has been calculated:

Name	Ilmenose
Symbol	(I)II.5.1.″3
Quartz	4.44
Orthoclase	37.25
Albite	40.35
Anorthite	3.06
Magnetite	2.78
Diopside	8.15
Hypersthene	1.82
Ilmenite	1.67
Apatite	0.34
Total	99.86

Hastingsite Granite. A few miles southwest of the North Conway quadrangle, the typical hastingsite granite is exposed on Mt. Chocorua. The upper seven hundred feet of Mt. Chocorua has very little vegetation and the bare ledges give superb exposures. The granite weathers white, but it is often stained a dirty yellow by limonite. The rock is relatively uniform in composition, but on the Champney Falls trail a number of other rock types are exposed. In the North Conway quadrangle the area of hastingsite granite centers about South Doublehead, although a few patches of riebeckite granite have also been observed. The exposures on the great white ledges on the southern slopes of the mountain are excellent. Working toward the south we find that the hastingsite granite is transitional into the biotite granite south of Doublehead; transitional types containing equal amounts of biotite and hastingsite may be observed in the field. The Robbins Ridge region shows some areas of a hastingsite granite, which is finer-textured than usual.

The hand specimens of the hastingsite granites are somewhat variable in color and texture. The most common variety is a medium-textured, granular rock composed essentially of white feldspar, colorless to smoky quartz, and black amphibole. The individual crystals vary from two to five millimeters in diameter. It is very probable that the white color of the feldspar is not primary, but rather the result of profound chemical alteration of this rock. Carlsbad twins are readily recognized. Apparent transitions into the nordmarkites have already been mentioned.

These granites are holocrystalline hypidiomorphic-granular rocks composed of microperthite, albite, quartz, and hastingsite. Minor accessories are zircon, allanite, apatite, fluorite, magnetite, biotite, and hedenbergite. Much of the biotite is in veins composed of small flakes, a habit that indicates a secondary origin for part of the mica. Occasionally the potash feldspar shows microcline cross-hatching. The pyroxene, noted in specimens low in quartz, is free or else zoned by hastingsite. Occasionally the hastingsite shows small blue patches, suggesting a riebeckitic tendency. Micropegmatite is rare. Alteration products include kaolin from the potash feldspar, chlorite associated with the hastingsite, and some sericite in the albite.

A modal determination on a specimen from Mt. Chocorua, Champney Falls trail, 2940 feet elevation, shows that the percentage of quartz is too high to place the rock among the nordmarkites.

Quartz	17.4
Feldspar	72.6
Hastingsite	8.3
Magnetite	0.7
Chlorite	0.3
Biotite	0.6
Zircon	0.1
Total	100.00

In other specimens the quartz runs higher, reaching thirty per cent as a maximum.

Riebeckite Granite. The riebeckite granites occur only in small, widely separated patches, closely associated with nordmarkites and hastingsite granites. What may be considered the type locality is found on the summit of North Doublehead. On the southwest slopes of South Doublehead, at 1680 feet elevation, a biotite-bearing riebeckite granite has been observed. A third area was noted on the eastern end of the range of hills southeast of Mountain Pond, at an elevation of 1970 feet. The exact size of each area is not well known, partly due to the scarcity of outcrops and partly due to the failure to separate the riebeckite granites and hastingsite granites during the field work. The riebeckite granite has also been noted on Stony Brook, Hart Location, in the Crawford Notch Quadrangle.

The age relations of the riebeckite granites are not known from any field observations. They are closely allied, both in distribution and in composition, to the nordmarkites and hastingsite granites. They are therefore considered as essentially contemporaneous with the hastingsite granites.

In the hand specimen the riebeckite granites are snowy white, black riebeckite being set in a granular or micropegmatitic somewhat drusy intergrowth of quartz and feldspar. The grain-size averages about two millimeters and is thus distinctly smaller than that of the normal hastingsite granite. The quartz, in contrast to the smoky quartz of the hastingsite granites, is clear and vitreous. The riebeckite usually has a distinct stub-like habit and is somewhat less abundant than the amphibole of the hastingsite granites. Prominent accessories, which are readily seen with the hand lens, are astrophyllite, zircon, and titanite. The astrophyllite occurs as elongated, light brown, micaceous sheaves, frequently with a radiating habit. A single crystal may be two millimeters long and in exceptional cases five millimeters. Zircon is reddish-brown in color; the largest crystal observed was about a millimeter across. One showed its habit so well that it was set up on the two-circle goniometer. The unit pyramid (111) was the only form present, the prisms being completely suppressed. Titanite, in dirty yellow patches, is interstitial among the feldspars.

Under the microscope the riebeckite granites are seen to be hypidiomorphic-granular to micropegmatitic, and composed essentially of microperthite, quartz, and riebeckite. Accessories include astrophyllite, often in radiating groups, zircon, magnetite, ilmenite, titanite, apatite, fluorite, and biotite.

The complex titanium silicate, astrophyllite, is very persistent and may constitute one per cent of the rock. Microscopically it is readily recognizable by its radiating habit and distinctly yellow pleochroism. Optical data obtained from the astrophyllite in the granite of North Doublehead are as follows: Biaxial positive, $\beta = 1.711$, and $\gamma = 1.738$. $2V$ = large; dispersion strong; $\rho < \nu$. The plane of the optic axes is parallel to the base, the acute bisectrix Z emerges on the front pinacoid, and the obtuse bisectrix X on the side pinacoid; that is, $X = b$, $Y = c$, and $Z = a$. The pleochroic formula is X = deep orange, Y = yellowish orange, Z = lemon yellow; $X > Y > Z$.

A following modal determination on a specimen from North Doublehead shows the abundance of quartz and the importance of astrophyllite. This specimen is richer in quartz than the average riebeckite granite of the White Mountains batholith.

Quartz	39.4
Feldspar	54.0
Riebeckite	5.5
Astrophyllite	1.1
Total	100.0

Conway Biotite Granite. The Conway group is primarily composed of a biotite granite with occasional phases of hastingsite granite. It is widespread in the central part of the quadrangle, and a satellitic stock is found on Baldface Mountain. At least six distinct phases may be recognized, some of which are segregation phases and others distinct intrusives: (1) the Red phase, a coarse biotite granite; (2) the Green phase, a relatively coarse, hastingsite granite; (3) the Baldface phase, a medium-textured biotite granite low in dark minerals; (4) the Diana phase, a porphyritic biotite granite; (5) the Black Cap phase, a fine-grained biotite granite; and (6) contact phases.

The Red phase shatters the diorite in Weeks Brook, as has been described on a previous page. The general distribution of the rock types indicates that the biotite granite is younger than the porphyritic nordmarkite; also two small dikes of the former cut the latter on the northwest slopes of Big Attitash (Crawford Notch Sheet). On the east slopes of Moat Mountain and on the north slopes of Mt. Pequawket the biotite granite clearly dikes the Moat volcanics, against which the biotite granite shows contact modifications, more fully described below. South of South Doublehead, however, the biotite granite grades into the hastingsite granite. These facts indicate that the biotite granite is the youngest of all the phases of the alkaline batholith, though perhaps contemporaneous with the hastingsite granites; the relation with the nepheline syenites of Red Hill is, however, unknown.

That the biotite granite is younger than the Montalban schists and the Meredith granite is eloquently demonstrated on South Baldface Mountain. The fine exposures on the west slope show that toward the contact with the Meredith granite the biotite granite becomes distinctly porphyritic, quartz and microperthite phenocrysts being set in a relatively fine-grained groundmass. Similar phenomena have been observed near Charles Brook, where the country rock is Montalban schist.

The exact relations of the various phases of the Conway itself are not clear. That the Red and Baldface phases actually represent two distinct intrusions is shown near the Hurricane gold mine and also on

Redstone Ledge. In both of these localities a sharp contact exists between the two phases, but it has not been possible to determine the age relations. The Diana phase is merely a variety of the Red phase that takes on a porphyritic habit, due to the assimilation of inclusions. The Green hastingsite phase is considered a segregation from the main biotite granite.

The *Red phase* of the biotite granite is a coarse-grained holocrystalline rock composed of pink feldspar, gray smoky quartz, and biotite. Occasionally the quartz is milky white. The minerals occur very distinctly in aggregates. The average grain-size of the feldspar is seven millimeters, quartz five millimeters, and biotite two millimeters; some of the feldspars, however, attain a length of two centimeters. They tend to be tabular, frequently show Carlsbad twinning, and sometimes show patches of white, striated albite. In the more weathered specimens the feldspar may be entirely white and the rock spattered by specks of limonite. In a medium-grained type from Thompson's Falls on Moat Brook, white albite is as abundant in macroscopic grains as the pink microperthite.

Under the microscope the Red phase is found to be coarse and hypidiomorphic-granular, the essential minerals being microperthite, quartz, and biotite. The accessories include albite, allanite, fluorite, apatite, zircon, magnetite, and hastingsite. The tendency for the three dominant minerals to gather together into aggregates has been noted above. The potassic member of the microperthite shows microcline cross-hatching in some cases. Some slides show the development of a thin shell of fine-grained quartz and albite along the contacts of microperthite crystals. The biotite occurs as large, ragged, primary grains, and as small, secondary flakes, showing simultaneous extinction. Secondary minerals include sericite poikilitic in the albite, chlorite developed from the biotite, and kaolin in the potash feldspar.

A striking feature is the intense, post-glacial weathering undergone by the coarse Red phase. This may be observed in the glacial drift, where bowlders a foot in diameter have completely disintegrated; bowlders larger than this usually show a relatively solid core. Large glacial erratics, some of them five feet across, that have been exposed to weathering since the close of the Wisconsin stage have completely broken down. The weathering may also be observed in the bed-rock. At Goodrich Falls, for example, weathering has affected horizons thirty feet below the surface. (See Plate II B.) Exactly how much of this weathering has progressed from above and how much from the sides one can not say. Certain layers, controlled by horizontal sheeting, have yielded much more readily than others, producing a distinctly stratified appearance in the rock. The unweathered portions of certain horizons remain as ellipsoids. This weathering may also be seen on the brook leading up to the Hurricane gold mine, in exposures on the east slopes of Moat Mountain, on Peaked Mountain, and on the west slopes of Mt. Stanton.

That the weathering is post-glacial is demonstrated by a number of facts. (1) The decay of the boulders of biotite granite in the glacial till has been mentioned. The only logical explanation is that the granite was carried as blocks of fresh rock by the ice, was then deposited in the till, and was finally disintegrated. It might be thought that after pre-glacial weathering the resulting arkosic sand was frozen to form a compact gravel that was ripped up and carried as frozen blocks to be deposited in the till. The textural appearance of the weathered boulders and the fact that the larger boulders may show unweathered cores disprove this suggestion. (2) Heaps of arkosic sand, obviously of the Conway type, have been found on the east slopes of Black Cap at an elevation of 2100 feet. The only explanation is that glacial erratics have there disintegrated. (3) Slopes, such as that on the south side of Peaked Mountain, whose general outline indicates glacial plucking, are covered by a large amount of arkosic sand.

In the hand specimen the *Green phase* is a coarse hastingsite-biotite granite, composed essentially of green feldspar, smoky quartz, hastingsite, and biotite. The average sizes of the grains are respectively seven, four, three, and two millimeters. The feldspars tend to be tabular, but as a rule they are irregular in habit; Carlsbad twinning is seen occasionally.

Under the microscope the Green variety does not differ from the red phase except that hastingsite is more abundant than biotite. This phase is considered to be a segregation from the coarse biotite granite. A problem that arises is how to separate the type under consideration from the normal hastingsite granite. The green phase is coarser, is more deficient in hastingsite, has a less granular texture, and is closely associated areally with the coarse biotite granite.

The *Diana phase* is a porphyritic biotite granite composed essentially of large pink feldspar phenocrysts and smaller light, gray quartz phenocrysts set in a fine-textured granular groundmass made up essentially of feldspar, quartz, and biotite. The respective sizes of the various components are: feldspar phenocrysts, ten to twenty millimeters; quartz phenocrysts, three to five millimeters; groundmass material, one to two millimeters. Due to weathering the feldspars are often pure white. In the field the Diana phase is associated with large, slightly darker patches, which vary in diameter from one to ten feet. These patches are ghosts of xenoliths and the Diana phase has resulted from the wholesale replacement of inclusions by the Conway granite. Various stages in the replacement of inclusions have been noted in different parts of the quadrangle.

The best exposed *contact-phase* of the Conway biotite granite is to be seen on the west slopes of South Baldface Mountain, where the country rock is the Meredith granite. The contact as a whole has a relatively regular trend, somewhat west of north (N 25° W), but in detail it is extremely jagged. No blocks of the older granite in the younger were observed, but numerous dikes of the biotite granite cut the Meredith granite. The contact zone is rather variable in width, varying from one to ten feet. As is typical in the contact facies of the Conway, the porphyritic contact type does not form a definite zone, but rather occurs as irregular patches in the normal granite along the contact belt. The type developed is a white porphyry in which smoky quartz and white feldspar phenocrysts, about two millimeters in diameter, are set in a white granular groundmass, speckled with black biotites. An occasional small pegmatite druse is present, similar to those described below on the east slope of Moat Mountain.

Although the lower eastern slopes of Moat Mountain are heavily wooded, a detailed study of the chilling effects is readily made. The porphyritic tendency is not so pronounced as on South Baldface; a fine-grained granite is the common contact type. Small druses of pegmatite, with crystals of clear quartz and white feldspar, are abundant in this fine contact phase. As on South Baldface, the contact facies does not occur as a definite zone, but is very erratic in its distribution, as irregular patches in the more normal granite. At the immediate contact the granite is so fine-grained that in places it is at first difficult to separate it from the Moat porphyries. For a distance of half a mile west of the contact the comendites are injected by dikes of a fine biotite granite, biotite granite porphyry, and biotite pegmatites.

Contact phases of the biotite granite have also been noted (1) on Cedar Brook about half a mile east of the point where the North Moat trail leaves the valley to climb the mountain; (2) on the northeast slopes of Mt. Pequawket; (3) in the bed of the East Branch of the Saco River just north of the xenolith of Montalban schist; and (4) between North and South Twin Mountains. Where the Conway is in contact with the Albany porphyritic nordmarkite no contact modifications are observable, implying that the Albany was still warm at the time of the Conway intrusion.

Black Cap Biotite Granite. The Black Cap granite is a fine-grained, pink biotite granite, typically exposed on the southeast slopes of Black Cap. A second area is on Thorn Mountain. Occasionally the rock is white instead of pink and thus superficially resembles certain phases of the Chatham granite. With the microscope the Black Cap granite is seen to be very similar to the biotite granite found as inclusions in the Albany porphyritic nordmarkite southwest of Moat Mountain. It is a fine-grained, hypidiomorphic-granular rock, composed essentially of albite, microperthite, quartz, and chloritized biotite. Accessories include fluorite, zircon, magnetite, and apatite.

This granite belongs to the White Mountains alkaline batholith, a fact that is demonstrated by its mineral composition and its chemical analysis (see below). Is it to be correlated with the pre-nordmarkite biotite granite of the Swift River Valley or with the Conway biotite granite? The evidence seems to be somewhat contradictory; but at present the writer is inclined to consider the Black Cap granite as an early phase of the Conway biotite granite.

Two facts seem to show that the Black Cap granite is post-nordmarkite, but pre-Conway (or early Conway):

(1) The Black Cap granite is shattered by the Conway granite on the southeast slopes of Black Cap.

(2) On the west slopes of Middle Mountain a dike of fine-grained biotite granite, apparently an apophysis from the mass of Black Cap granite on Thorn Mountain, cuts the porphyritic nordmarkite.

Apparently contradicting evidences are:

(1) The general distribution of outcrops around Thorn Mountain suggests that the Black Cap granite is older than the porphyritic nordmarkite.

(2) Lithologically the Black Cap granite is similar to the biotite granite found as inclusions in the porphyritic nordmarkite southwest of Moat Mountain.

From these two facts the Black Cap granite appears to be pre-nordmarkite. However, the first two evidences seem the stronger and the writer considers the Black Cap granite as an early phase of the Conway biotite granite, though shown on the areal map as a distinct lithological unit.

The following modal determinations have been made of the various phases of the Conway biotite granite.

	8.	9.	10.
Quartz	26.7	18.5	30.0
Feldspar	69.4	77.8	68.4
Biotite	3.0	0.2	1.6
Hastingsite	0.2	3.5	
Allanite	0.6		
Magnetite	0.1		
Apatite	tr		
Total	100.00	100.00	100.00

8. Conway granite, Red phase, Redstone granite quarries.

9. Conway granite, Green phase, Redstone granite quarries.

10. Conway granite, Baldface phase, east slopes of South Baldface Mountain.

These modes emphasize the paucity of dark minerals in the Conway granite.

Five analyses of the Conway granite are available and are listed below. One is quoted from Perry (6). The other four are new analyses made for this paper. An analysis of the biotite granite from Mt. Ascutney is entered for comparison.

	8.	9.	10.	11.	12.	1.
SiO_2	70.45	69.85	73.65	73.01	72.35	71.90
Al_2O_3	14.87	14.44	13.73	13.73	12.48	14.12
Fe_2O_3	1.07	0.94	0.23	0.44	1.22	1.20
FeO	1.54	2.37	0.95	1.48	1.25	0.86
MgO	0.35	0.24	0.11	0.01	0.42	0.33
CaO	1.28	1.25	0.90	0.94	1.19	1.13
Na_2O	4.28	3.93	4.47	3.50	3.76	4.52
K_2O	4.16	5.04	4.27	5.62	5.98	4.81
H_2O+	0.65	0.66	0.77	0.18	0.60	0.42
H_2O-	0.50	0.35	0.56	0.05	0.17	0.18
TiO_2	0.69	0.74	0.22	n.d.	0.39	0.35
P_2O_5	0.04	0.03	tr	n.d.	0.27	0.11
MnO	0.00	tr	tr	0.09	0.03	0.05
Total	99.88	99.84	99.86	99.05	100.11	99.98

8. Conway granite, Red phase, Redstone quarries. W. H. Herdsman, analyst. Sp. Gr. 2.63.

9. Conway granite, Green phase, Redstone quarries, W. H. Herdsman, analyst. Sp. Gr. 2.65.

10. Conway granite, Baldface phase, east slopes of South Baldface Mountain. W. H. Herdsman, analyst. Sp. Gr. 2.60.

11. Conway granite, Red phase, south slope of Mt. Pequawket. J. H. Perry, analyst. Sp. Gr. 2.62.

12. Black Cap granite, east slope of Black Cap. Mrs. S. Parker, analyst.

1. Biotite granite from Mt. Ascutney, Vermont. Sp. Gr. 2.62. Contains in addition CO_2 = 0.21, ZrO_2 = 0.04, Cl = 0.02, F = 0.06, FeS_2 = tr, BaO = 0.04, SrO = tr, LiO_2 = tr; total 100.35.

Norms. The norms of these rocks have been determined as follows:

Specimen	8	9	10	11	12
Name	Toscanose	Liparose	Liparose	Toscanose	Liparose
Symbol	I.4.(1)2.3	I.4.1(2).3	I.4.1''.3	I.4.(1)2.3	I''.4.1.3
Quartz	25.80	23.64	28.50	28.08	25.80
Orthoclase	25.02	29.47	25.58	33.36	35.58
Albite	36.15	33.54	37.73	29.34	30.92
Anorthite	6.39	6.12	4.45	4.73	.....
Corundum	1.63	0.20	.....	0.20	.....
Acmite	.....	.....	.....	.....	0.92
Hypersthene	1.56	2.98	1.49	2.51	0.23
Diopside	.....	.....	.....	.....	3.19
Magnetite	1.62	1.39	0.23	0.70	1.39
Ilmenite	1.37	1.37	0.46	.....	0.76
Apatite	.....	.....	.....	.....	0.67

Pegmatites associated with the Conway granite are rare. The pegmatites in the Red phase at Redstone have been described by J. L. Gillson (17). My crystallographic study on topaz and phenacite from the pegmatites associated with the Baldface phase on South Baldface has been presented in another paper (18).

DIKE ROCKS.

A great variety of dike rocks occur in the North Conway quadrangle and to describe them in detail would demand far more space than is here available. All of the dikes described below are phases of the alkaline batholith. The granite and pegmatite dikes associated with the Chatham granite are not considered. For convenience the dikes may be divided into three major groups: (1) granitic, (2) syenitic, and (3) lamprophyric.

Granitic Types. The granitic types are composed essentially of quartz, alkali feldspar, and such ferromagnesian minerals as biotite, riebeckite, and hastingsite. Minor accessories include fluorite, allanite, zircon, magnetite, and chlorite. Dikes of *biotite granite,* often porphyritic, were observed cutting the Albany porphyritic nordmarkite near Jackson. *Paisanite* types carrying riebeckite are associated with the hastingsite and riebeckite granites. Some varieties are porphyries, the phenocrysts being microperthite. Wide *quartz porphyry* dikes, up to eighteen feet in width, were noted on Black Mountain, cutting the amphibolites of the Montalban schists. A medium-textured *hastingsite granite* dike cuts the Albany porphyritic nordmarkite southwest of South Moat Mountain. *Aplite* dikes are often associated with the Conway granite. Apophyses of the Conway granite include *biotite pegmatites*, and *biotite granites;* they intrude the Moat volcanics, Meredith granite, and diorite.

Syenitic Types. The syenitic types are composed essentially of alkali feldspar with such major accessories as augite and hastingsite. The typical *augite syenite porphyry* shows three generations of minerals, an earliest stage of microperthite phenocrysts, an intermediate stage of alkali feldspar, augite, and biotite phenocrysts, and a groundmass stage of quartz and alkali feldspar. The type specimen occurs near the Tin Mines in Jackson. At Champney Falls (Mt. Chocorua) a similar dike has been eroded to a depth of twenty feet to form the basin into which the falls plunge. A medium-textured *hastingsite sölvsbergite* cutting the Moat volcanics of Mt. Pequawket is composed of a net of euhedral alkali feldspar with interstitial hastingsite. The *bostonite* is a dense, red rock, which under the microscope is found to be composed essentially of tabular crystals of microperthite showing a typical trachytic arrangement. Dikes of the *nordmarkite porphyry* which cut the Montalban schists and Chatham granites have been described on a previous page. Dikes of the Albany *porphyritic nordmarkite* have also been mentioned.

Lamprophyric Types. The lamprophyric dikes are composed essentially of soda-lime feldspar with such major accessories as hornblende, augite, biotite, and barkevikite. The *diorite* dikes of Weeks Brook have already been described. The *kersantites* are composed of tabular labradorite and biotite or its alteration products, particularly chlorite. The phenocrysts, which are very variable in amount, are andesine or labradorite or augite. The rocks are usually badly altered, a fact indicated by the large amount of chlorite, calcite, and epidote. These kersantite dikes intrude the Conway biotite granite, the Moat volcanics, and the Albany porphyritic nordmarkite. The *camptonites* are composed essentially of labradorite and an amphibole, which under the microscope is pleochroic brown. The optical properties of this amphibole suggest either barkevikite, or basaltic hornblende. Some of the camptonites are porphyritic with phenocrysts of either augite or barkevikite. Such dikes cut the schists of the Montalban group.

Economic Products of the White Mountains Batholith.

The only product of the Conway granite that is of economic importance at present comes from the Redstone Quarries, about three miles south of North Conway. Two of the three pits are located in the coarse Red phase, but the third is in the Green phase. Operations began in the fall of 1886 and the quarries are still being actively worked. When they were first opened, paving stones for New York City were among the chief products. Many fine buildings have been constructed in part of Conway granite. The masonic temple at Alexandria, Virginia, is being built of the Red and Green phases of the Conway granite. In the past as many as three hundred men have been employed; during the summer of 1926 about one hundred and fifty men were working in the quarries and sheds.

Two miles southwest of North Conway is the defunct White Mountains Quarry. The drydock at the Charlestown Navy Yard, Boston, is reputed to have been built of granite from this quarry.

On the south slopes of Iron Mountain, between an elevation of 2000 and 2150 feet, there are three openings that have been excavated for hematite ore. Two of the openings are open pits and one is a tunnel thirty-five feet long. The ore occurs as irregular bodies in the Conway granite and is composed of hematite and quartz with small amounts of galena and other sulphides. Field and microscopic evidence indicate that the ores are a replacement of the Conway biotite granite: (1) the ore bodies are very irregular in shape and difficult to delimit; (2) although the contact of the granite and ore is often relatively sharp, a small transition zone may be found showing blebs of hematite in the granite; (3) specimens in the dump show granite in all stages of replacement. Under the microscope radiating needles of hematite may be seen cutting across the hypidiomorphic granular texture of the granite. The writer has no data concerning the operation of the mines.

The "gold mine" at 1620 feet elevation on the west slopes of Hurricane Mountain is apparently more famous for its hopes than for its products. The shaft is located in the bed of a brook and is reported to have been originally eight feet deep and five feet wide. The bait is a siliceous rock zoned into white, red, and brown wavy belts a few inches wide. The country rock in Conway granite, Baldface phase. The siliceous rock is apparently an irregular replacement of the granite, for it is possible in the field and under the microscope to observe siliceous rocks which still preserve a granitic texture.

A number of small pits and tunnels on the west slopes of Tin Mountain, near Jackson, mark the site of the Tin Mines, which are no longer operated. The present writer has been unable to find any ore, but there is no doubt that it exists, for in 1843 C. T. Jackson (19) gave a complete description of the tin veins of this locality.

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CHAPTER V.

Conclusions.

A. ROOF-COLLAPSE IN THE LATE-PALEOZOIC ALKALINE BATHOLITHS.

Evidence from the White Mountains. The area of the White Mountains batholith incloses five major volcanic masses, which vary from two to eight miles in diameter—Mt. Pequawket, Moat Mountain, the Ossipee Mountains, Mt. Carrigain, and Twin Mountain (Figure 4). Are the volcanics merely roof-pendants projecting down into the batholith or are they blocks that have settled from above? I believe that they are great blocks that have settled from above. The basis of the argument is the fact that the volcanics occur only within the limits of the batholith and its satellitic stocks; nowhere are they found beyond the limits of the batholith.

The volcanics were extruded upon a surface composed of slates, schists, and muscovite-biotite granites, for fragments of these rocks are found in the breccias. These relations are illustrated in the accompanying diagram (Figure 7A). If we assume the batholith to have stoped up into its roof, any roof-pendants that are left, as for example "R" in the diagram, would be composed of the same formation as the walls of the batholith, Montalban schists in the case illustrated. If stoping continued into the volcanics the same rule would hold; the walls and any roof-pendants would be composed of volcanics (figure 7B). But in the case of the White Mountains neither of these relations holds. We have great blocks of Moat volcanics within the limits of the batholith and its satellitic stocks, but nowhere outside of these areas do we find these volcanics; we find only the rocks of the basement on which these volcanics were deposited. This observation in itself might be explained as due to the fact that the volcanics were extruded only directly above the batholith. Although this is possible, some observations remain unexplained. If a batholith were to stop into a volcanic pile which existed only directly above the intrusion, we might now have volcanic rocks exposed only within the area of the batholith. But if this were so, the exposures of volcanics would occupy only the highlands and the older basement rocks would be in the lowlands (see figure 8). However, in the White Mountains the relations are just the reverse, for the schists and gneissic mountains are higher than the volcanic mountains. The most logical explanation is that the volcanics have settled into the batholith from above. The down-sunken blocks may represent isolated xenoliths which were

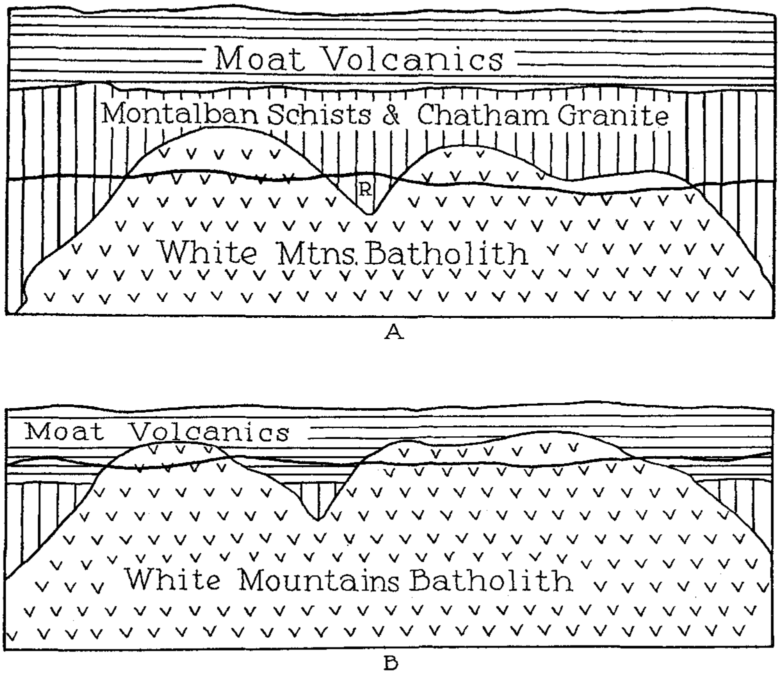
Moat Volcanics
Montalban Schists & Chatham Granite
R
White Mtns. Batholith
A
Moat Volcanics
White Mountains Batholith
B

FIGURE 7. Diagrammatic cross-section of a subsequent batholith, showing the lithological similarity of the walls and roof-pendants. In 7 A the batholith is assumed to have stoped into the schists; the roof-pendant (R) and the walls are both composed of schists. In 7 B the batholith is assumed to have stoped into the volcanics; in this case both the walls and the roof-pendant would be composed of volcanics. For simplicity the diagrams indicate (1) that the lavas were extruded before roof-collapse began, and (2) that the lavas covered an area extending beyond the limits of the batholith.

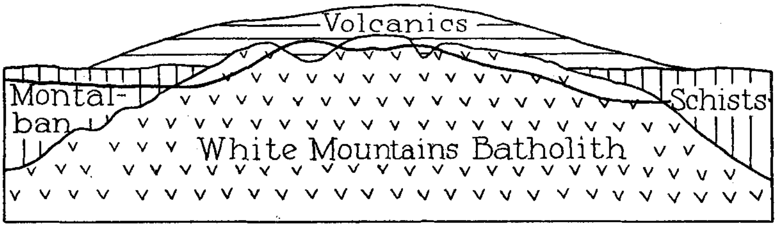
Volcanics
Montal-ban
Schists
White Mountains Batholith

FIGURE 8. Diagrammatic cross-section of a batholith stoping into a volcanic pile, which is confined to an area immediately above the batholith.

completely immersed in the magma (figure 9), or they may be blocks that were attached to the roof along fault-planes (figure 10). The difference in strike and the steep dips of the Moat and Pequawket masses suggest that they are isolated xenoliths.

We may determine in a general way a minimum value for the amount of settling. Outside the batholithic area, in those places where there was no collapse, the base of the volcanics must have been higher than the present summits of the highest schist mountains fringing the batholith. That is, the base of the volcanics was higher than a level which now stands at 6,000 feet. In Moat Mountain the lowest exposed horizon of the volcanic group is at 900 feet. In other words, the elevation of the base of the volcanics is less than 900 feet within the batholith; how much lower, we have no way of telling. The western part of the Moat mass has settled therefore at least 5,000 feet below its original level. By simple trigonometry it may be demonstrated that in the northeastern portion of Moat Mountain the lowest known horizons are at least 17,000 feet below their original level, if we assume that piecemeal stoping has not shattered the large block at depth (see figure 5). All these figures are clearly only a minimum, for the actual basement on which the volcanics of Moat Mountain were extruded is lower within the batholith, and higher beyond its limits, than the horizons from which we have made our measurements. Thus the volcanics of Moat Mountain seem to have settled at least 5,000 feet into the batholith, and probably at least 17,000 feet on the northeastern end of the block. That this down-sinking took place during the intrusion of the batholith and not later is shown by the intrusive nature of the contacts between the Moat volcanics and the biotite granites.

It is frankly admitted that the exact value of the minimum amount of collapse can not be determined for a number of reasons. Late-Paleozoic or post-Paleozoic crustal warping has undoubtedly introduced complications. But the most difficult problem is to find a plane of reference from which to make measurements. In the discussion we have constantly referred to the floor upon which the volcanics were extruded as the reference plane. That floor may have had considerable relief. What we have actually used as a plane of reference is the *plane formed by the lowest points in that floor*. We find no volcanics beyond the limits of the batholith, so that we assume even the lowest points in the floor to have been above what are now the schist mountains surrounding the batholith. I wish to emphasize this point, lest it be thought that the surface upon which the volcanics

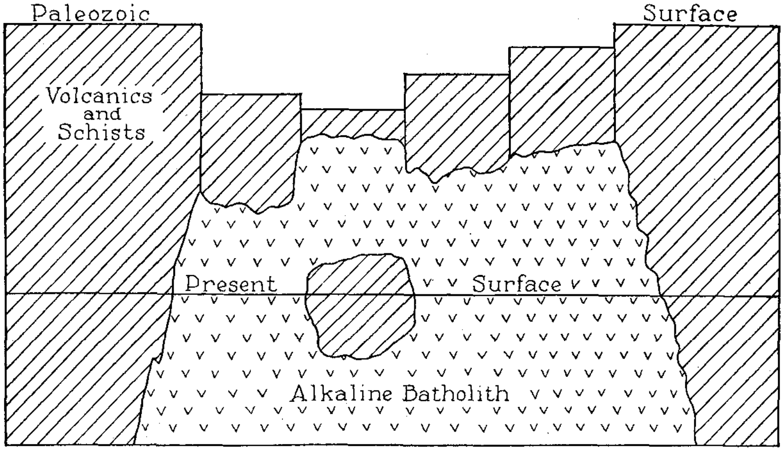
Paleozoic
Surface
Volcanics
and
Schists
Present
Surface
Alkaline Batholith

FIGURE 9. Diagrammatic cross-section showing the masses of Moat volcanics as isolated xenoliths in the plutonic phases of the batholith.

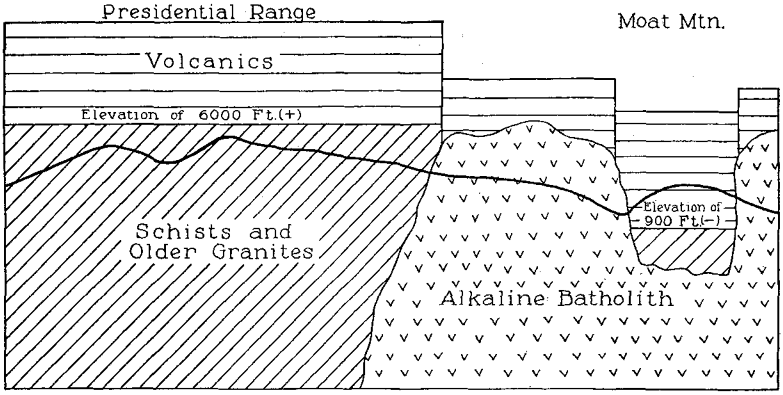
Presidential Range
Moat Mtn.
Volcanics
Elevation of 6000 Ft.(+)
Elevation of
900 Ft.(−)
Schists and
Older Granites
Alkaline Batholith

FIGURE 10. Diagrammatic cross-section showing the masses of Moat volcanics as down-faulted blocks attached to the roof of the batholith along fault-planes. The heavy black line represents the present surface. In the Presidential Range the floor upon which the volcanics were deposited would lie more than 6000 feet above sea level if it had not been eroded away. Within the confines of the batholith this same floor lies less than 900 feet above sea level. The Moat Mountain block has therefore settled at least 5000 feet below its original level.

were extruded is assumed to have been either a peneplane or a region of moderate relief. On the contrary, I have taken the plane formed by the lowest points in the floor as the reference plane and have made no assumptions as to the topography.

Having established the facts, we may briefly speculate as to the details of the process. We may assume that after a long period during which the magma was working upward, presumably by stoping, a stage was reached in which the roof was too thin for full support and began to break up into large blocks that would sink into the magma below. By this collapse of the roof over the batholith great differences in relief would result and great quantities of coarse sediment would pour into the collapsing area. Presumably the whole roof would settle, but it would undoubtedly break up into subsidiary blocks sinking at different rates. Accompanying collapse, great quantities of magma would well up along the fracture-planes to form surface flows, accompanied by considerable explosive activity. As collapse continued, great flows would flood the region, and vast quantities of detrital matter, mingled with explosive ejecta, would be swept in from the highlands to the lowlands. Probably at no time would the uncongealed magma in the upper portions of the batholith be exposed directly to the air; instead it would well up around the collapsing blocks to pour forth as lava flows. That all of the flows were extruded before roof collapse began does not seem probable. The high relief essential for the production of the coarse breccias is best explained by the contemporaneous collapse; moreover, collapse without accompanying volcanic activity seems improbable.

Evidence from other Late-Paleozoic Alkaline Batholiths. If the White Mountains are compared with other of the Late-Paleozoic alkaline batholiths of eastern North America, Greenland, and Norway, the conception of roof-collapse is clearly substantiated. The extrusion of flows and the contemporaneous collapse of the roof is illustrated by the following batholiths: Oslo (Christiania) batholith in Norway (1); Julianehaab batholiths in Greenland (2); East Greenwich batholith in Rhode Island (3); and the Quincy batholith in Massachusetts (4, 5). In the first two cases the field workers, Brögger (1) and Ussing (2) respectively, fully recognized the phenomenon of roof-collapse.

At Julianehaab in Greenland a series of Devonian (?) red sandstones, 1200 meters thick, underlie diabases, porphyrites, quartz porphyries, and subordinate volcanic tuffs that are 1000 meters thick (2*a*) (see figure 11). Just as in the White Mountains the sandstone and volcanics are found only within the limits of the alkaline plutonics (2*b*). Ussing has conclusively shown that their preservation in this region is due to the down-sinking of the roof of the batholith (2*c*); moreover, he shows that the portion of the volcanic series now preserved is older than the plutonic phases of the batholith and also that *subsidence accompanied batholithic intrusion* (2*d*). The subsidence amounts to at least 2000 meters and probably at least three kilometers (2*e*).

In the Oslo (Christiania) region of Norway, the classical works of Brögger have adequately demonstrated the collapse of the crust over the batholith (1*a*). In this down-faulted tract we find preserved Cambrian, Ordovician, Silurian, and Devonian (?) shales, limestones, and sandstones, overlain by a great series of extrusive augite porphyry, melaphyr, labradorite porphyry, and rhombenporphyries. To quote Brögger (1*b*): "The whole Christiania region forms, as I have already demonstrated, a long down-sunken graben about 230 kilometers long and within this great graben the collapsed portion of the earth's crust has broken up into numerous blocks shoved against one another. The blocks have subsided varying amounts, and some blocks are twisted or even pushed up." Brögger estimates that the collapse amounts to at least 1300 meters (1*a*). The fact that the graben has broken up into many subsidiary blocks should be emphasized.

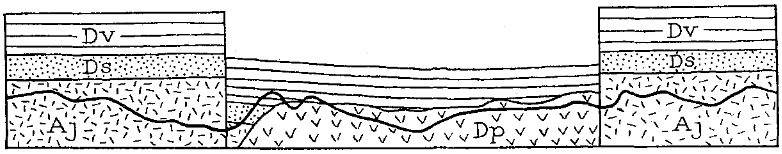

FIGURE 11. Diagrammatic cross-section of the batholith at Julianehaab, Greenland, to show roof-collapse. (After Ussing.) *Aj*, Julianehaab granite (Pre-Cambrian); *Ds*, red sandstone (Devonian ?); *Dv*, volcanic phases of the alkaline batholith; *Dp*, plutonic phases of the alkaline batholith. The heavy line represents the present surface. The collapse amounts to at least three kilometers.

There are certain great contrasts between the White Mountains batholith on the one hand and the Julianehaab and Oslo batholiths on the other. In the White Mountains the down-faulted volcanics form only a very small proportion of the whole igneous area; in Oslo and Julianehaab the volcanics and associated sediments form more than half the batholithic belt. Moreover, the down-sunken blocks of New Hampshire have suffered much more rotation than in the Greenland example.

In the Shetland Islands volcanics and plutonic phases of the same alkaline batholith are known (6, 7). The Old Red Sandstone, with associated sandstone, conglomerate, tuff, agglomerate, porphyrite, diabase, and rhyolite, has been dropped down into a late-Devonian batholith composed essentially of a biotite granite with associated riebeckite-bearing dike rocks.

In the East Greenwich batholith of Rhode Island (3) approximately a square mile is represented by a microgranite breccia, which locally shows a rude stratification and contains fragments of rhyolite, quartzite, and granite. The strike is east-west, at right angles to the normal trend-lines of the region, and the dip is about twenty-five degrees to the north. I suggest that the microgranite and associated breccias are to be interpreted as a series of interbedded flows and fanglomerates which have settled into the biotite granites due to roof-collapse.

In the Quincy-Blue Hill batholith of eastern Massachusetts there are associated with the riebeckite granites a series of rhyolites, quartz porphyries, tuffs, and breccias, which Crosby (4) and Warren (5) have interpreted as a large, chilled, marginal phase of the batholith. The physiography, lithology, and other field evidence indicate that the rhyolite, porphyries, tuffs, and breccias represent a volcanic pile which has collapsed into the alkaline batholith.

One of the best examples of roof-collapse is found in the cauldron-subsidence of Glen Coe, so ably described by Clough, Maufe, and Bailey (8). A great ellipsoidal tract of volcanic rocks, nine miles by five miles, has settled into a tonalite magma of Late-Devonian age. This is a subalkaline batholith and is therefore not described further.

In summary, roof-collapse seems to have taken place in a number of the Late-Paleozoic alkaline batholiths, not only in the White Mountains of New Hampshire, but also in Norway, Greenland, Rhode Island, and eastern Massachusetts.

B. SYNCHRONOUS VERSUS SUBSEQUENT BATHOLITHS.

The Chatham batholith and the White Mountains batholith were intruded under very different conditions and by very different mechanisms (see figure 6). The Chatham batholith was intruded during an orogenic period under such conditions of pressure and temperature that it regionally metamorphosed the surrounding sediments, then intruded them in lit-par-lit fashion, and finally ripped them up and strewed them about through the granite. Folding was going on at the same time and the batholithic roof was always intact, greatly hindering the escape of magmatic gases; as a result great quantities of pegmatite were formed. Some large-scale stoping took place, but much of the space occupied by the granite originated through the folding, squeezing, and flowage of the schists. It is a *synchronous* batholith, for it accompanied a period of folding.

The White Mountains batholith on the other hand was not intruded during a period of orogeny and produced no regional metamorphism. It formed its chamber, at least in the closing stages, by stoping off small and large blocks of the older rocks, blocks which in some cases reached miles in diameter. The roof of the batholith was broken, and as a result the gases were not confined. Their escape caused extensive vulcanism at the surface and their loss prevented the formation of pegmatites in any great quantities. This batholith was intruded shortly after the compressive forces of the Acadian disturbance (9) had ceased, and the intrusion is therefore *subsequent* to a period of orogeny.

C. Assimilation of Inclusions in the White Mountains Batholith.

Small inclusions immersed in the granites and nordmarkites of the White Mountains batholith were not melted or corroded, but they were converted into granite by a process of impregnation. Each inclusion preserved its form and solutions soaking through the granite gradually caused microperthite, hastingsite, quartz, biotite, fluorite, allanite, and other minerals to replace the original minerals of the inclusion. At the same time the elements displaced must have diffused outward into the granite. The process has been described in detail for the impregnation of the diorite of Weeks Brook and in a general way for the re-making of inclusions to form the Diana phase of the Conway granite. The writer has observed the process elsewhere in the North Conway quadrangle.

The same phenomenon has been described by Warren (5) in the Quincy batholith, where the rhombenporphyry has been made over after being immersed in the riebeckite granite. Clapp (10) has maintained that the original essexite of Salem Neck is a hybrid rock resulting from the impregnation of diorites by exhalations from a nepheline-syenite magma. With this conclusion the present writer is in full accord. The Salem Neck and Weeks Brook occurrences are very similar, except that in the former case the diorite was stewing in a nepheline-syenite magma and in the latter case a biotite-granite magma. In the Salem occurrence alkali feldspar, nepheline, and ferromagnesian minerals have soaked into the older rock; in the White Mountains locality alkali feldspar, quartz, and ferromagnesian minerals have been added to the diorite.

D. Summary of the Geological History.

During a long period of time, presumably in the Pre-Cambrian, the site of the present White Mountains was inundated by an epicontinental sea in which thousands of feet of shale, sandstone, limestone, and dolomite were deposited. During a latter period, also probably Pre-Cambrian, the rocks were folded. With the folding, highly aqueous granitic magmas were intruded, to aid the mountain-building forces in metamorphosing the shales, sandstones, limestones, and dolomites into mica schists, micaceous quartzites, and lime-silicate rocks. Following the metamorphism, the granite intruded the schists, lit-par-lit, and gradually strewed them about through the granite.

The later Pre-Cambrian and early Paleozoic history is shrouded in mystery. It is probable that during the Silurian the region was submerged by an epicontinental sea, in which muds and sands were deposited; a mountain-making movement followed this deposition.

In the Devonian a great batholith, ranging in composition from diorite to granite, was intruded into the older rocks. The magma stoped upwards, ripping off great blocks of the older rocks, which sank to abyssal depths, to be assimilated. Eventually the roof of the batholith became so thin that collapse began. Along the great fractures molten magma moved upwards and issued at the surface as great flows and pyroclastic deposits. Coarse breccias accumulated in the lower basins. Eventually the magma spent its energy, and the great molten mass congealed to form syenites and granites.

The bed-rock structure of the White Mountains was now essentially complete; all further processes were confined to carving the present mountains from the schists, granites, syenites, and volcanics. For a long period the rivers have been striving to reduce the surface to a plain, but without complete success. Large areas were frequently reduced to the base-level of erosion, but each time, long before the process was complete, a continental uplift of a few hundred feet forced the rivers to begin anew their work of destruction. The repetition of this process formed many terraces. There were complications; at times the uplift was slow and gradual rather than swift and periodic; at other times the region subsided. The net effect was to produce many benches rising above one another, like a series of steps.

Then came the Great Ice Age. In the first stages of refrigeration mountain glaciers formed on the higher summits, carving out cirques. Eventually the ice-sheet from the north overwhelmed the mountains, scouring, striating, and fracturing the rocks. Projecting spurs and summits were rounded off, and hill sides were planed away. Glacial till was dumped to block the valleys. In the waning stages of the Ice Age a glacial lake filled the Saco Valley around North Conway. Since that time the meandering Saco River has been engaged in cutting away the sand plain, but little has been accomplished.

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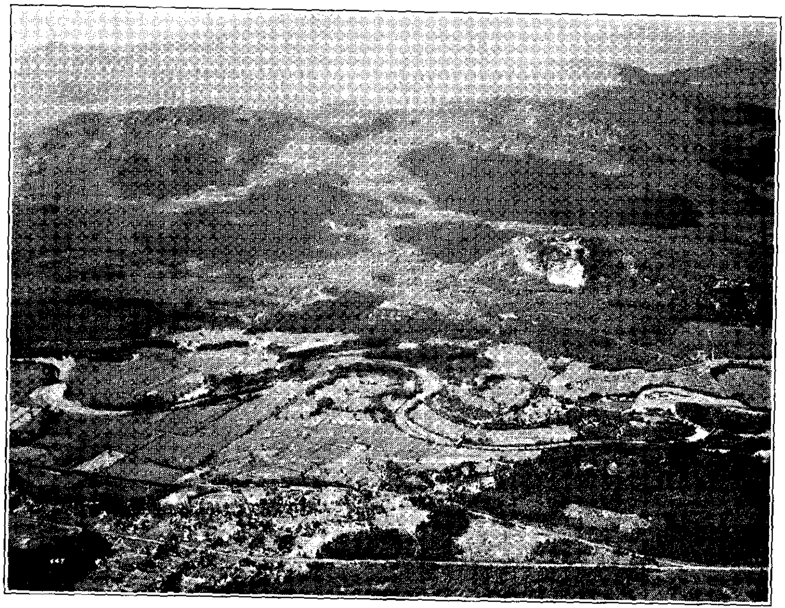
PLATE II A.

Aerial photograph of the Saco Valley at North Conway. In the foreground may be seen the meandering Saco River; in the middle distance, White Horse and Cathedral Ledges; and in the background, Moat Mountain.

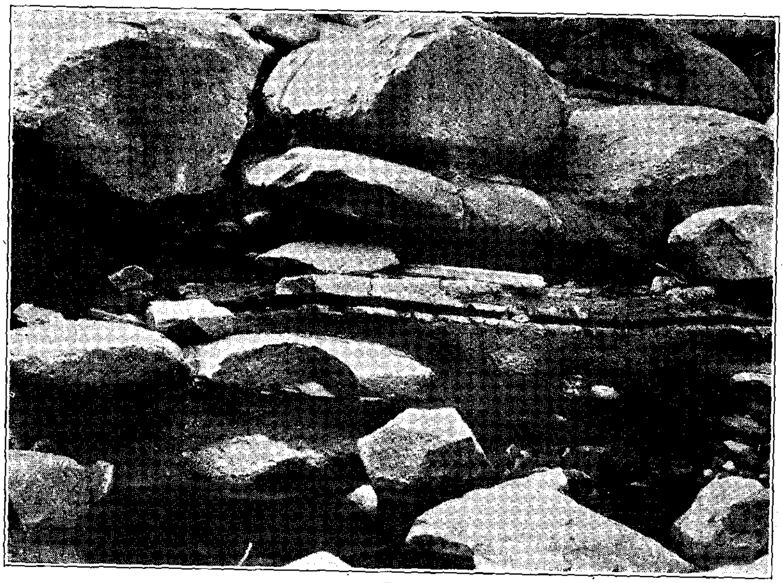
PLATE II B.

Weathering of the Conway biotite granite at Goodrich Falls.



Photograph from Hunting's Studio, North Conway.

A



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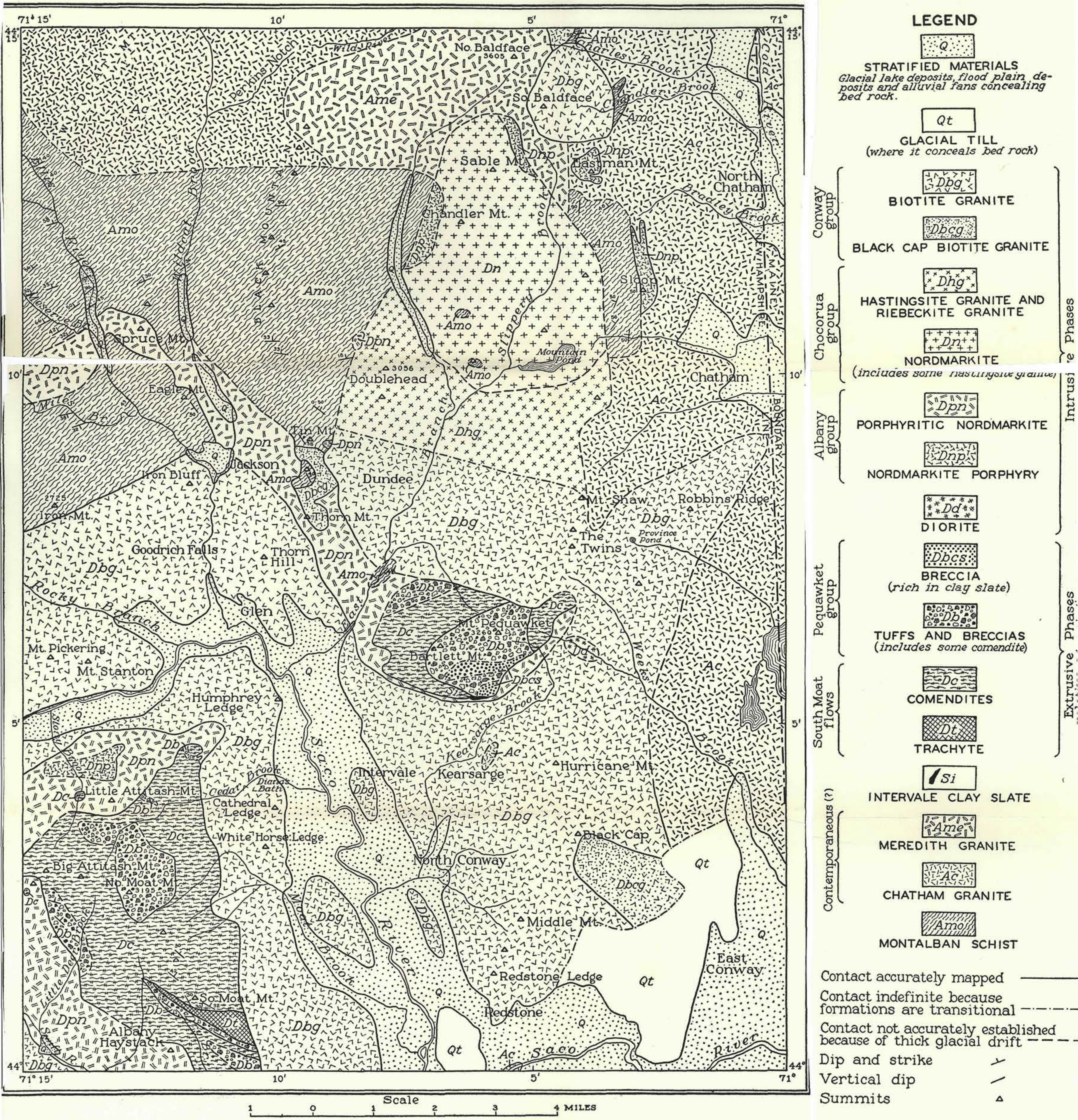
LEGEND
Q
STRATIFIED MATERIALS
Glacial lake deposits, flood plain deposits and alluvial fans concealing bed rock.
Qt
GLACIAL TILL
(where it conceals bed rock)
Conway group
Dbg
BIOTITE GRANITE
Dbcg
BLACK CAP BIOTITE GRANITE
Chocorua group
Dhg
HASTINGSITE GRANITE AND RIEBECKITE GRANITE
Dn
NORDMARKITE
(includes some hastingsite granite)
Albany group
Dpn
PORPHYRITIC NORDMARKITE
Dnp
NORDMARKITE PORPHYRY
Dd
DIORITE
Intrusive Phases
Pequawket group
Dbcs
BRECCIA
(rich in clay slate)
Db
TUFFS AND BRECCIAS
(includes some comendite)
South Moat flows
Dc
COMENDITES
Dt
TRACHYTE
Extrusive Phases
Si
INTERVALE CLAY SLATE
Contemporaneous (?)
Ame
MEREDITH GRANITE
Ac
CHATHAM GRANITE
Amo
MONTALBAN SCHIST
Contact accurately mapped
Contact indefinite because formations are transitional
Contact not accurately established because of thick glacial drift
Dip and strike
Vertical dip
Summits
71° 15'
10'
5'
71°
44° 15'
10'
5'
44°
No. Baldface
3605
So. Baldface
Wild River
Charles Brook
Chandler Brook
Perkins Notch
Sable Mt.
Eastman Mt.
North Chatham
Bradley Brook
Chandler Mt.
Slippery Brook
Sloop Mt.
NEW HAMPSHIRE
MAINE
BOUNDARY
Cold River
BLACK MOUNTAIN
Wildcat Brook
Spruce Mt.
Eagle Mt.
Doublehead
3056
Mountain Pond
Chatham
Tin Mt.
Jackson
Iron Bluff
Iron Mt.
2725
Dundee
East Branch
Thorn Mt.
Mt. Shaw
Robbins Ridge
The Twins
Province Pond
Goodrich Falls
Thorn Hill
Rocky Branch
Glen
Mt. Pequawket
Bartlett Mt.
Mt. Pickering
Mt. Stanton
Humphrey Ledge
Weeks Brook
Kearsarge Brook
Hurricane Mt.
Intervale
Kearsarge
Saco
Little Attitash Mt.
Cedar Brook
Dianas Bath
Cathedral Ledge
White Horse Ledge
Black Cap
Big Attitash Mt.
No. Moat Mt.
North Conway
Moat Brook
Middle Mt.
East Conway
Redstone Ledge
So. Moat Mt.
Redstone
Albany Haystack
Saco River
Scale
1 0 1 2 3 4 MILES

PLATE I. GEOLOGICAL MAP OF THE NORTH CONWAY QUADRANGLE, NEW HAMPSHIRE.

The following additional notes may be made to the legend: (1) the diorite of the White Mountains alkaline batholith may be older than the extrusive phases; (2) in the vicinity of Robbins Ridge small areas of hastingsite granite have not been separated from the biotite granite.

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STUDIES ON THE FAUNA OF HOT SPRINGS IN THE WESTERN UNITED STATES AND THE BIOLOGY OF THERMOPHILOUS ANIMALS.

By Charles T. Brues.

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STUDIES ON THE FAUNA OF HOT SPRINGS IN THE WESTERN UNITED STATES AND THE BIOLOGY OF THERMOPHILOUS ANIMALS.[1]

By Charles T. Brues.

Received May 15, 1928. Presented May 9. 1928.

During a sojourn of several weeks in the Yellowstone National Park in the summer of 1923 the writer became interested in the fauna of the abundant thermal springs in that region. A considerable amount of material was obtained at that time and this has already been discussed in a paper published in 1924, together with a brief resumé of what had been previously recorded concerning the fauna of hot springs in other parts of the world. At that time he was surprised at the dearth of knowledge relating to the animal life of the numerous hot springs which are scattered through several of our other western states, and determined to investigate some of these when an opportunity might present itself. Plans were made to visit a number of hot springs in New Mexico, Nevada, California and Utah during the summer of 1927 and an itinerary was arranged to include as extensive and varied a series of springs as appeared possible in the limited time at our command. To further the purposes of the expedition he was given a grant from the Milton Fund for Research at Harvard University, which he wishes gratefully to acknowledge as this made it possible to extend the survey considerably.

In order to indicate more clearly the location of the hot springs to be mentioned, the accompanying outline map has been prepared. On it each spring is indicated by a dot together with a number by which the spring will be designated in the discussion which follows.

Some of these springs were first mentioned in publications of the United States Geological Survey; particularly in the reports of the early explorations of the west or of surveys made to locate routes for transcontinental railways. Very few references to them appear in any more recent publication, with the exception of those contained in certain water supply papers issued by the U. S. Geological Survey. Aside from their scientific interest these springs were naturally of considerable importance to these early explorers as the patches of green vegetation commonly associated with them in this desert region form conspicuous landmarks in an otherwise monotonous country and the grass furnished much needed forage for their horses. Also the water when cool is usually fairly palatable. In some cases springs which we visited in 1927 were carefully described about 75 years previously and it has been possible to indicate some changes which have taken place during this quite considerable interval of time.

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[1] Contribution from the Entomological Laboratory of the Bussey Institution, Harvard University, No. 292.

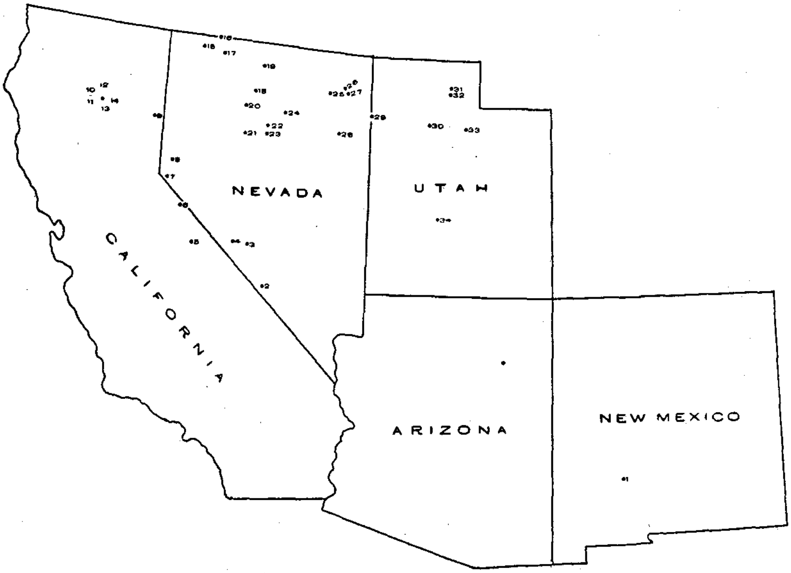

FIGURE 1. Map showing location of hot springs visited. The numbers are those cited in the descriptions and used elsewhere in the text.

From the naturalist's standpoint and from that of the nature-lover as well it is distressing to record that all except the most inaccessible springs, or those included within the boundaries of National Parks, have been either converted into natatoria, sanatoria for arthritics, radium baths and the like, or have been diverted into irrigation ditches, sometimes with the aid of dynamite, to supply a few desolate ranches with water for cattle and alfalfa. For this reason the fauna and the flora of these thermal springs is destined soon to be wiped out over considerable areas by the advance of commercialism.

As it is possible to approach most of the localities to be described over roads passable under ordinary weather conditions to automobiles we traveled by motor, carrying the necessary supplies and camp equipment, journeying somewhat over 11,000 miles in this way.

The writer was accompanied by his wife who made collections of the grasses growing in and adjacent to the various hot springs, and observations on the temperatures and other conditions determining the growth of these plants. The results of these studies will be published separately. Determinations of the temperatures, specific gravity and hydrogen ion concentration of the waters where collections were obtained were made by my son, Austin M. Brues, and these data are included in the detailed account which follows. To both companions I am deeply indebted for their unfailing cheerfulness and resourcefulness under many trying circumstances.

For temperature, a thermometer was used which read to tenths of a degree centigrade.

For determining specific gravity, a hydrometer was used which read to the fourth decimal place. This determination had then to be corrected to a uniform temperature of 15° C., which introduced a certain error. On a basis of determination of the same water at different temperatures, we should consider this error to be not over 0.0004.

The hydrogen-ion concentration was determined by the colorimetric method. The following indicators were used, with their ranges expressed in pH; Bromphenol blue, 3.0–4.6; methyl red, 4.4–6.0; bromcresol purple, 5.2–6.8; phenol red, 6.8–8.4; cresol red, 7.2–8.8; thymol blue, 8.0–9.6. It would have been impracticable to carry liquid standards of comparison, so a color chart was used. This chart was issued by a German seedsman, and contained about 450 lithographed blocks of color. This included all the noticeably different pure colors from yellow through red to blue, about 24 in number, with a series of different intensities and brightnesses corresponding to each color, so that the pure color qualities of a given indicator would be read as equal if greater or less amounts of indicator were used, and in water of white, gray or black turbidity. The color chart was calibrated by means of chemical solutions, and differential readings of the same water using different amounts of indicator showed that the error could nowhere be over 0.4, and that where the ranges of two indicators overlapped the error could not be more than 0.1. The charts retained their original colors throughout the summer.

In practically all of the areas where active hot springs occur, there are evidences of the vast extent and enormous size of earlier springs now extinct. By reason of the chalky white or characteristically colored deposits produced by calcareous hot springs and the equally distinctive sinter cones and formations resulting from the flow of siliceous springs it is possible readily to recognize the remains of extinct hot springs as features of the landscape, especially in connection with the non-conformity of the thin layers composing them which is always to be noted as a conspicuous feature.

The first of these which we encountered was a few miles southeast of the town of Beatty, Nevada, near the California state line and almost directly east of Death Valley, and it is typical of a quite recently extinct hot spring. Remains of a central elevation some thirty feet high slope off into the surrounding flat sage brush plain and there are indications of terraced slopes greatly eroded and washed away. As the pure white calcareous material becomes disintegrated it separates into very minute particles which are washed out by the spasmodic, torrential showers and form a covering of powdery, white soil. This extends for half a mile or more toward the lower part of the valley, and probably overlies similar deposits laid down when the spring was actively flowing. In this neighborhood are a number of others, each visible from a considerable distance as a white scar on the steeply sloping side of the low mountain ridge which borders the northern side of the valley. These are much more extensively eroded and evidently antedate the one first mentioned. Such groups are common throughout the hot springs area to be described, but they are frequently seen at places where no active nor very recently extinct springs occur within many miles, almost always they set well up on the slopes of hills or mountains. How many may have been present at lower levels, which have been smoothed down and covered with deposits derived from the higher slopes it is impossible to judge without more careful examination, but it is probable that those at the lower levels would rapidly disappear from view.

In one locality, particularly, an extensive group of large extinct hot springs is a conspicuous feature of the dull drab gray landscape, in an open valley well removed from any mountain slopes. These are about 35 miles south of the town of Denio, Oregon which is on the Nevada state line, and they lie therefore near the northern boundary of Nevada. A couple of miles eastward is a small active hot spring (No. 15 to be described) and within 30 miles to the north are several rather large active ones.

As is commonly the case, the deposits here are strongly tinged with ochre yellow with occasional pure white masses interspersed. They are mound-like with gradually or steeply sloping sides practically devoid of the sage brush vegetation which covers the surrounding area. Similarly colored, dome shaped or conical mounds are also numerous in Lassen National Park, California, but here they are in a very mountainous region and are sometimes built up on steep slopes. Such deposits are almost always uniformly of a soft calcareous nature and must weather very rapidly.

The harder deposits of siliceous sinter laid down by alkaline hot springs weather more slowly, but are less conspicuous as they assume a dull gray tinge. A most remarkable series of these is to be seen west of Pyramid Lake, Nevada, representing the remains of a group of hot springs and geysers many miles in extent. Pyramid Lake derives its name from an enormous sinter cone, which rises out of the bosom of the lake and along the western shore are others of great size, entirely extinct, but still retaining the characteristic structure of geysers or boiling springs (Plate 2, fig. 1) especially where the interior cavity may still be seen. To the westward are many other groups and the road passes for several miles over the rough weathered surface of a shelf which formed part of a vast hot spring terrace. The springs in this region are entirely extinct, but some miles beyond them to the west are some active springs at Ameede, California (Hot Spring No. 9).

References to other groups or series of extinct springs might be added at great length for the present active ones are but scattered remnants of a vast and gigantic series which once dotted this entire region. Most stupendous of all is the well known Bryce Canyon in southern Utah, apparently eroded from the deposits of an extinct hot spring many miles in extent. The writer has given elsewhere his reasons for believing that Bryce Canyon originated in this way.

The following list which includes 34 hot springs or groups of springs is arranged in the order in which we visited them. The arrangement is therefore one of convenience only as it does not coincide with any grouping which might be made on the basis of composition, temperature or otherwise. Also it does not include some springs visited where we failed to obtain collections, due usually to the springs having been walled in, piped for drinking water or otherwise despoiled.

List of Hot Springs

No. 1. Hot Springs, New Mexico. The springs are near the town at the foot of a small hill. Some have been turned into baths, but there is a small amount of seepage forming small pools in a large marshy meadow. Some of these pools are barely tepid, but a flowing stream which forms the overflow from several small springs near the upper edge of the marsh, traverses the marsh for a distance of several hundred yards. The temperature of this stream, which is several feet wide, is not very high, in the neighborhood of 40° along most of its course. The specific gravity, corrected for temperature is 1.0039 and pH 8.1. It supports a fauna of snails, chironomid larvæ, and fish. About 60 miles south of this locality is another hot spring known as Radium Hot Spring emitting water at 85°, but this is piped to a bath house from the rocky hill side and affords no opportunity for biological collecting.

No. 2. Hot Springs Station, Nevada. This is 5.8 miles north of Beatty, on the railroad and about a quarter of a mile off the highway leading to Goldfield. The springs, which have a temperature of 40.4°, are led into a bath house and into a cement pool containing green algæ. They flow from the lower edge of a low ridge and there is in a marsh below a small pond in which the water is cool. The spring water has a specific gravity of 1.0016 and pH of 7.2. In the neighborhood there are other places where water oozes through the desert floor and supports patches of vegetation but no other warm springs were noted.

No. 3. Goldfield, Nevada. This is about 12 miles from Goldfield on the road which leads to the mining town of Weepah. It is led into a large swimming pool, but there is a considerable overflow of hot water from the spring which forms a stream several feet in width that flows out into the semi-desert plain. The water has a temperature about 40° at the head of the stream which drops to 35° within about 50 yards. This stream flows in a ditch near the spring, but spreads out below where it is lined with bullrushes. The water contains a considerable amount of algal growth and insect life.

No. 4. Silver Peak, Nevada. This is about 50 miles west of Goldfield and about the same distance east of the California state line. The springs are at the western edge of a large alkali flat several miles in length which is dry during most of the year, but was then covered with water from rains that had preceded our visit. The country to the north is mountainous and the hot springs are at the lower edge of the steep rocky slope, only a few feet above the flat or marsh which is really a salt desert when dry, or a vast shallow lake when covered with water, from the center of which rises a conspicuous dark rocky hill. There are a number of small springs and below them the soil is wet, spongy and covered with a coarse growth of grass (*Distichlis spicata*). The individual springs vary greatly in temperature; some are extremely hot (67°; 53.5°) while others are quite cool and in one case two very hot ones only six feet apart had between them a cool one. The specific gravity of the water is high, 1.0212 with a pH of 8.3. The animal life in the springs includes larvæ of mosquitoes, Ephydra and Stratiomyia, as well as some water-bugs and the annelid, Tubifex. Nearby is a large cold spring, impounded for use in an ore-crushing mill.

No. 5. Whitmore springs, near Convict Lake, California. These are reached by a road which leaves the highway between Bishop and Mono Lake about 38 miles north of Bishop. The side road leaves the north and south highway at a right angle and the springs lie about three miles to the east over a very winding and little used road. Before reaching the springs one passes along the southern slope of a small open grassy valley through which a stream flows eastward. Waring ('15) describes two large springs of low temperature (maximum 37.7°) arising in a meadow near this point. We did not visit these, but noticed them from the road lying below and to the north where they water the pasture land. The springs to be described are in the Inyo National forest although the immediate vicinity is practically treeless. They consist of a number of boiling hot and warm pools which line the southern bank of the stream and lie quite near to the edge of a steep slope which rises about fifty feet above the stream. This slope is composed of hot springs deposits, showing that the extent of the springs has been much greater in the past. In addition to the pools there is one very small geyser which erupts a small amount of water to a height of several feet. The creek is decidedly warm but only a very small amount of water flows into it from the springs along the bank. The specific gravity of the water in the springs is about 1.0012 and the hydrogen-ion concentration 8.5 where most of our material was collected but mosquito larvæ were taken from one more alkaline pool, having a pH of 8.8. These springs are in a very attractive setting between the steep slopes on each side of the stream and, looking up the valley to the westward, the high Sierras are visible in the distance.

A few miles further north next to the main highway are the Casa Diablo Hot Springs. These are not extensive and are apparently rapidly disappearing. They are located on the slopes of a hillside just above the road and the only one which seems to be actually flowing has been boxed in to form a tiny bath house. The rapid decrease in flow of these springs is said to date from the earthquake of 1915. Waring ('15) found in 1908 several springs discharging water, but only one is now actually flowing. He also states that some fifty years previously another spring nearby, now dry, was active.

No. 6. Fales Hot Springs. These are located about 14 miles northwest of Bridgeport, California, close to the main highway between that town and Carson City. One which has a temperature of 67.8° is confined in a large concrete pool, and over the other which is of much lower temperature has been built a bath house. The overflow from both springs is combined and flows as a small stream to the northwest through a gully. Considerable quantities of gas probably carbon dioxide, bubble through the pools and the pH of the overflow is rather low, 7.5, while the specific gravity of the water is rather high, 1.0030. The stream supports a considerable growth of filamentous algæ and flows over a number of small terraces or cascades which it has deposited. Waring ('15) gives Fahr. 60.5° C. as the highest temperature, but it is now much higher, due perhaps to the cementing of the pool, although the difference is considerable. There are extensive deposits, above the present springs indicating that they were much more extensive in the past.

No. 7. Walley Hot Springs. These are several miles west of Minden, Nevada and about 15 miles south of Carson City. They are not on the main highway between these two places, but are reached by traveling westward from Minden to the old Pony Post road and proceeding northward on this road for about a mile toward Carson City. At this point the road follows along the edge of some foothills of the Sierras which rise to the westward. The springs are directly east of the road while beyond them toward Minden stretches out a vast marsh which supports a heavy growth of tall rushes. The springs are all small, scattered along the marsh which is separated from them by open water at least several feet in depth and twenty or thirty feet in width. Into the marsh there also extends an arm of open water with a slight but noticeable flow, indicating a much greater flow of water than that which comes from the springs above the water's edge. In the water which is warm (36°–37°) a number of large fish and ducks ventured out on the open water several times during our visit. The water appears quite clear and is practically fresh as we could detect no salts in solution by means of our hydrometer. The pH, is however, comparatively high, 9.6 or over. The temperature of the pools varies greatly and some are far too hot to support animal life. This one of the most extensive springs which we visited and undoubtedly all the water in the marsh comes from these and associated springs, some of the larger ones doubtless below the level of the water in the swamp. These springs (Plate 2, below) are in their natural condition, but about a quarter of a mile further north is another group similarly located which form the basis of water cure and mud bath for arthritics.

No. 8. Steamboat Springs, Nevada. These lie directly to the west and above the main highway between Carson City, Nevada and Reno, about 20 miles north of the former, and one mile north of Steamboat Springs Station. They form the remnants of what have been much larger springs in the past and emerge from an extensive sinter deposit about five hundred yards in length extending north and south, with a width of perhaps about three hundred yards. The highest part of the ridge which is parallel to the road is about sixty or seventy feet in height and composed of porous siliceous sinter. There are several narrow fissures within which the boiling of water may be heard, but in only one place is there an overflow that forms a small stream a foot or two in width which flows over the sinter for a hundred yards or more. Along its course there is a good deal of vegetation and at spots the water spreads out to form flat marshy areas a few feet in diameter. Toward the northern end of the sinter deposit the ground is hot in places and sounds hollow to the tread. These springs were visited in 1870 and are described by Hague and Emmons ('77, p. 825). It would appear that the springs were more active at that time, giving forth a much greater volume of water, but this may not be true as some hot water is now piped to a settlement below and one bored well at the northern end was spouting boiling water to a height of about 25 or 30 feet at the time of our visit. Near this are a few small pools and sinter cones which appear to be drying up as a result of the new well.

No. 9. Amedee, California. This is a group of several very hot springs, one of which is boiling at a temperature of 95.6°. They are near the southeastern border of Honey Lake, close to the highway between Pyramid Lake, Nevada and Susanville, California. One group of springs supplies some bathhouses and furnishes hot water for a hotel, but two are still in more or less natural condition and the water from these is led off in ditches for irrigation purposes. One of these is boiling and the other forms a small series ranging from 40°–50° in temperature. The specific gravity of the water in the run-off from these springs where collecting was done is 1.0012 and the pH 8.5. According to Waring the water contains a considerable amount of sodium sulphate and some silica. The latter is sufficient in amount to cause the silicification of wood which has fallen into the spring. There is but little tufa in the neighborhood of these springs, but further northward and west above the northern border of the lake at Shaffer there is a considerable amount which forms one series of crags fully 50 feet high. These are similar to the very large and extensive tufa formations which characterize the country further east near Pyramid Lake. South of these crags is a series of hot springs, said to be rapidly decreasing in size and next to them is an overgrown swampy pool of cool water overgrown by coarse vegetation.

Lassen National Park. Springs Nos. 10–14 are located in Lassen National Park which contains many thermal springs of varied types, including a hot lake, hot and boiling pools, steam vents, boiling mud-pots and some small actively boiling, almost geyser-like vents which emit steam and water. There are, however, no true geysers. The hot springs of the Park, are confined mainly to three areas. One of these is in the southeastern part of the park, about two miles south by southwest of Lassen Peak in a straight line and southwest of Lake Helen. It includes only small springs which are located mainly along the slopes of ravines, but biologically these are very interesting. Springs No. 10 and 12 are located here. A second group, much larger, more spectacular and extremely varied in character, lies almost directly south of Lassen Peak at a distance of two and one half miles. This group, known as Bumpass Hell, lies on the ancient crater rim of Lassen Peak at an elevation of 8,000 feet where it occupies a sloping area of some 80 acres bordered to the north and west, and partly on the other sides, by a ridge which rises above it. This locality is designated as spring no. 11. The third area lies near the southern boundary of the park. It extends over a large area, from five to eight

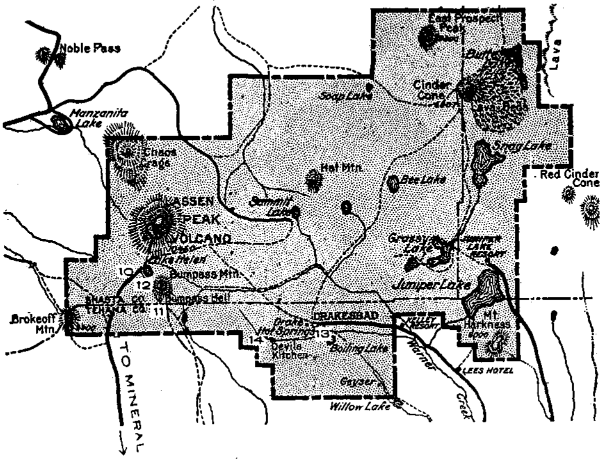
Noble Pass
Manzanita Lake
Chaos Crags
LASSEN PEAK VOLCANO
Lake Helen
10
12
Bumpass Mtn.
Bumpass Hell
11
SHASTA CO.
TEHAMA CO.
Brokeoff Mtn.
TO MINERAL
Soap Lake
Hot Mtn.
Summit Lake
Bee Lake
East Prospect Peak
Butte
Cinder Cone
Lava Beds
Lava
Snag Lake
Red Cinder Cone
Grassy Lake
Juniper Lake
Mt. Harkness
DRAKESBAD
Drake's Hot Springs
13
14
Devils Kitchen
Boiling Lake
Geyser
Willow Lake
Warner Creek
LEES HOTEL

Figure 2. Map showing location of the hot springs in Lassen National Park, California which are referred to in the text. (Slightly modified from one issued by the National Parks Service.)

miles southeast of Lassen Peak in a straight line. Here the elevation is much lower, varying from 5600 to 5900 feet.

No. 10. This is along the trail which leads to Lake Helen about two miles (by trail) below the lake. This trail leaves the road which enters the park from Mineral, about eleven miles from the latter place. At this point there are a number of small pools that water a grassy slope. The temperature of the water varies from 38°–47° where collecting was done and the pH varies from 6.7–7.3, with a specific gravity of 1.0014. These springs are well above the road and before reaching them one passes many mounds and slopes formed by deposits from springs not now active. These are in part pure white, but are mainly ochre yellow, with very little admixture of red.

No. 11. Bumpass Hell. Although this remarkable area exhibits a great variety of thermal phenomena, the springs containing animal life are not numerous. The ground about the springs is in considerable part bare, but a few of the springs contain algæ and there are grasses surrounding others. The water has a specific gravity of about 1.0016 and a rather low pH of 5.7–6.5, varying apparently with the content of hydrogen sulphide. Many of the pools are turbid with clay or sulphur in suspension while others are far too hot to support animal life. Bumpass Hell is difficult to reach as the approach from the west road involves a stiff climb, descent into a valley and another steep ascent to the springs.

No. 12. This is quite near to no. 10, in a small ravine next to a rushing brook not far above the road which leads into the park from Mineral. It is one of a considerable number, but the only one in which material was collected. The specific gravity was not determined, but the pH is unusually low, 5.1.

No. 13. Drakesbad, Lassen National Park. This is a small cemented pool about half a mile west of the hotel. The water is scarcely warm, 28.2°, specific gravity 1.0021, pH 6.7, but is interesting as the only water striders collected were obtained here.

No. 14. Devil's Kitchen, Drakesbad, Lassen National Park (See Plate 4). This is an area, several miles west of Drakesbad, quite similar to Bumpass Hell, but of lesser magnitude and at a lower elevation. In spite of the varied character it was biologically unproductive. Several miles southeast of Drakesbad is Lake Tartarus, a large "boiling" lake about four hundred feet in width. The water is very turbid with clay in suspension and the shores are so muddy that there appears to be no animal life present.

No. 15. 25 miles southwest of Denio, Oregon. This spring lies in the northern, almost uninhabited section of Nevada close to the Oregon border. It is six miles off the road from Cedarville, California, about 76 miles from the latter place. The spring is small but emits a considerable volume of clear water with a specific gravity of 1.0000 and pH of 8.4. The temperature is 32.5° in the overflow where our collecting was done. Although small, this spring is biologically very interesting. It is in an area which has been at one time thermally very active. There is much silicified wood a few miles to the southeast in a hill where opals are mined and to the west, near the main road there are the remains of other hot springs present as large, much weathered mounds of calcareous deposits formed by the action of large springs now entirely extinct.

No. 16. Six and one-half miles south of Denio, Oregon. At this place there are several small springs in a group near the highway to Cedarville, California. They are in flat grassy meadow about half a mile east of the mountains which rise abruptly from the desert. The green area extends eastward and a large pool with water near the boiling point (spring no. 16^a) is located about a quarter of a mile east of the first group. There is practically no deposit except a small amount of clayey mud formed at the margin of these springs and even the large one, which produces a considerable amount of water that is led off for irrigating the meadow, is scarcely raised above the surrounding plain. The water has a specific gravity of 1.0015 and pH 8.5. There is plenty of ground water in the meadow, probably originating from springs, and to the southward at a distance of about two miles is a large pond or lake fed by the Thousand Creeks. These springs are a very interesting group.

No. 17. Twenty-one miles southeast of Denio, Oregon. Like the last, this locality is in northern Nevada, on the valley road to Winnemucca, Nevada which lies 83 miles further on to the south. The main spring arises in a mass of pink chalcedony a few feet in height and below in a depression there is some slowly flowing hot water which forms a sinuous elongate pool bordered by sedges and grasses. Here in water at a temperature of 32°–40° a number of beetles were obtained. The water is practically free from dissolved salts with a specific gravity of 1.0001 and a high pH of 9.4. The country in the neighborhood of the three springs (nos. 15, 16 and 17) near Denio has been very interestingly described by Reeds ('27) who refers to many geological features of the region. About the rocks at this spring we noted a considerable number of cast snake skins. Evidently the warmth from the water is attractive to snakes (See Plate 4).

No. 18. Golconda, Nevada. These springs are close to the small town which is some 18 miles east of Winnemucca on the transcontinental highway. There is a hotel built at some of these, but a short distance away are several good sized pools at the head of a large open meadow. They set on low mounds of sinter which rise only a few feet above the surrounding surface and support a good growth of grass. Some are too hot for animal life, but one contains water at 50° and below while in the overflow the water is cooler. The water is nearly fresh, specific gravity 1.0008 and pH 8.1.

No. 19. Paradise, Nevada. This may be reached either from Golconda or Winnemucca; about 60 miles from the former and 40 from the latter over none too good roads. It is visible from the road about a quarter of a mile to the south, close to the Humboldt River and north of Hot Springs Peak. The spring arises from an old sinter cone or conical mound about 20 feet high. The water no longer flows over the top as the spring has been tapped for irrigation and led through a ditch toward a ranch house. The sides of the ditch are muddy, with abundant grass, and a great variety of water beetles were obtained here where the water cooled somewhat. The specific gravity is 1.0012 and pH 7.6 (See Plate 5).

No. 20. Twenty-nine miles south of Winnemucca, Nevada. These are in the broad valley between the Payute range and the Havalla mountains, reached by a passable but indescribably rough road from Winnemucca. The springs are on the gentle slope of a low hill beyond the foothills on the eastern edge of the valley. There are about 25 pools, the largest 10 or 12 feet in diameter set in white travertine sinter which forms a gradually sloping face. There are many marshy spots and there is a good carpet of grasses over these. Some pools are bubbling and near the boiling point, and furnish a considerable flow of water that is carried out into the valley for irrigation. The entire area occupies several acres, and is several times longer (in a north and south direction) than wide. These springs were described in Hague and Emmons' report ('77, p. 687) as they existed in 1870 and their description indicates that there has been but little change in the springs during the intervening period of nearly 60 years. The specific gravity of the water is 1.0014 and the pH 8.4–8.7. An analysis given in the publication cited above indicate that the main mineral content is calcium carbonate (See Plate 6).

No. 21. Sou Hot Springs. These were described in the report just cited. They lie to the south of those just mentioned, 72 miles from Winnemucca and are reached over a continuation of the same road. They lie toward the western edge of the desert valley on a low mound of travertine which slopes gently from the flat plain to a height of

60 or 70 feet. This is surmounted by a series of several warm pools scattered over the more or less level top about an acre in extent. The largest of these pools is about 25 feet in diameter and the larger ones are surrounded by tall sedges, grasses and other vegetation; some much smaller ones are hot or boiling. In addition there are a number of pits which are no longer filled with water. Much of the water from this group of springs has been diverted by blasting in an attempt to irrigate a small ranch on the desert about a mile further south and this is no doubt the reason that the springs appear to be much less active than they were in 1870. At that time as described and beautifully illustrated in Hague's account (*loc. cit.*) the water level reached the rim of the pools. The specific gravity of the water is 1.0020 and the pH 8.0. According to Hague the principal material in solution is sodium sulphate, with very little sodium chloride, although the Osobb Valley which lies directly south is a salt desert and there are strong brine springs near by, which according to Hague are also warm (125° Fahr.), but we did not have opportunity to search for these. Evidently the source of the water in the Sou springs is not derived from the seepage of meteoric water through the salt fields. Although this valley has been known as the Osobb Valley it is known to ranchmen locally only as the Dixie Valley. One of the pools in this group is shown on plate 3.

No. 22. Twenty-six miles south of the town of Battle Mountain, Nevada. This is on the eastern side of the Reese river valley on the main highway from that place to Austin, Nevada. The springs arise as two very shallow pools only a few feet in diameter with muddy bottoms and surrounded by level, soft, damp soil. They are on the slope of a low hill which encroaches on the valley as an extension of the foot-hills of the mountain range that lies to the eastward. There is a very scant overflow, but the vicinity of the springs is indicated by a noticeable amount of green grass which contrasts with the drab coloring of the surrounding desert flora. The water is nowhere very hot, but ranges in the neighborhood of 40° in the pools. The specific gravity of the water is 1.0014 and pH 7.8.

No. 23. Watt's Ranch, 37 miles south of Battle Mountain, Nevada. This group of springs is near the same road as the last, about eleven miles further south. These springs have been described by Hague ('77, p. 633) and his description of the locality leaves no doubt of their identity. However, the position and temperatures of the springs do not agree and I hesitate to make comparisons of their condition at the two dates.

It seems probable however that there have been quite considerable changes in position and an increase in the temperature of the water in some springs since 1870. One spring which is nearest the road formerly arose from a sinter deposit about eight feet high which has been cut through to lead water into the ranch house. As a result this spring is drained to the level of the opening and is not flowing perceptibly; it is not distinctly warm.

Over an area of several acres to the westward of this are a number of hot pools which flow slightly over a flat meadow covered with grass. The pools are lined with soft gray mud, but the water is quite clear unless disturbed. Various beetles, crustaceans and dragon-fly nymphs were obtained here. The water is but slightly impregnated with salts, and has a specific gravity of 1.0008 and pH of 8.0. According to an analysis given by Hague the pasty gray material deposited in these springs contains a considerable amount of silica as well as calcium carbonate.

No. 23^a. About six-tenths of a mile north of the group just mentioned is a large spring which gives off a strong flow of hot water that is led off through a ditch for irrigation. This spring is on the side of a low hill that rises above the plain on all sides. After flowing for about 100 feet the water is sufficiently cool to support animal life and here a succession of small cascades and pools afforded an opportunity to collect a number of insects and a species of planarian. The water is more saline than that of the nearby springs to the south, with a specific gravity of 1.0021 and also slightly more alkaline with a pH of 8.5 (See Plates 3 and 6).

No. 24. Beowawe, Nevada. These springs are about seven miles south of the town of Beowawe which is reached by a road that leaves the main transcontinental highway (Victory highway or U. S. 40) at the small railway station of Dunphy, 56 miles west of Elko, Nevada. The springs, together with a number of geysers, some of which erupt at frequent intervals to a height of 15 or 20 feet, occupy a shelf of siliceous sinter about half a mile in length on the western slope of the mountain that borders the eastern edge of the valley southwest of Beowawe. The sinter deposit rises at a steep slope to a height of about 200 feet and its upper nearly level face extends in places to a width of over forty feet. The overflow from the geysers and from a number of hot springs flows down the side of the sinter at several places and below in a meadow close to the edge of the sinter escarpment is a large very hot pool which is said to erupt at times. The specific gravity of the water is 1.0063 and the pH 9.6 or over. A number of beetles and chironomid larvæ were taken in the overflow from the upper springs and geysers. These springs are most remarkable for their large extent, very large sinter deposits and the presence of large, true geysers. This group was well described by Peale ('83, p. 322), as Volcano Springs but has apparently entirely escaped attention since that time. Outside of Yellowstone Park, they appear to be the only true geysers of any considerable size in the United States (See Plate 5).

No. 25. Near Deeth, Nevada. These springs are 20 miles north of the town of Deeth, 15 of which are over the road to Charleston, Nevada, but the last five must be made over an abandoned road which leaves the main road just after passing two conspicuous small buttes on the right. This spring has built up a dome or flattened cone about fifty feet in height of grayish travertine, but the water no longer flows over the top as it now escapes through a crevice some forty feet below the rim, whence the water spreads out over the nearly level ground at the base to form a very shallow pool. The edges of this are margined by a rim of travertine beyond which the water plunges over a travertine cascade to the southward into a small gulley some thirty feet in depth. The water has a low specific gravity of 1.0008 and pH of 7.4. This spring is evidently diminishing in activity.

No. 26. Near Wells, Nevada. This is reached from Wells over the road to Metropolis. It is four miles from Wells on the right hand side of the highway. Set in the flat plain at the lower edge of a steep hill of rough rhyolite, topped with tufa, is a small pool of warm water (32°–36°). The pool is surrounded by a growth of grass (Distichlis) and rushes where the water seeps into the soft muddy soil. The specific gravity of the water is 1.0018 and pH 7.9.

No. 26^a. This arises just a few feet above Spring no. 26, but issues from the rocky hillside and is quite different in composition, with a specific gravity of 1.0015, a strong odor of hydrogen sulphide, pH of 6.9, and the water contains strings of sulphur bacteria. Its temperature is about that of the other spring (33°), but the water has been led away from the pool below by means of a wooden trough.

No. 27. On returning to Wells this spring was noticed by my daughter on the south side of the highway. It is about two and one-half miles from Wells. The spring flows a considerable amount of clear hot water which runs down a natural gulley, nearly parallel with the road, to the westward, and is entirely in its original condition. It contains some filamentous green algæ and where the water has cooled to about 40° a number of insects were secured. The specific gravity of the water is 1.0017, pH 7.1 and no odor of hydrogen sulphide is noticeable.

No. 28. Ruby Valley, 65 miles south of Wells, Nevada. This large group of springs may be reached from Wells over the Valley road which leads from Wells to Eureka along the valley east of the high Ruby mountains. By road it is 65 miles from Wells. The springs are in the nearly level valley about half a mile east of the road on a low rounding knoll covered with white sinter. Although actively depositing a hard sinter, they have built up no cones, but lie close to the level of the plain. They occupy an area of three or four acres and vary greatly in size and temperature. Some are boiling and a considerable amount of gas rises in bubbles from a few. Some are shallow with muddy bottoms, while others have deep funnels in which the water is of a clear blue color. The muddy pools are not so hot and some range about 40° or lower. There is a considerable flow of water from the group, which includes perhaps thirty or forty separate springs varying in size from pools 20 feet in diameter to ones of very small dimensions. On account of the large extent and diversity of this group, it supports a large fauna and much grass, most of which is cropped by a herd of cattle which are pastured in the vicinity. The water has a specific gravity of 1.0009 and is noticeably alkaline with a pH of 9.4.

No. 29. Wendover, Utah. Directly south of the town of Wendover which is close to the Nevada-Utah state line there arises from the base of a low, rocky bluff, near the western edge of the Great Salt desert, a group of large warm springs with a temperature of 27°–28°. These supply several large ponds which are surrounded by a growth of grass and rushes. Mosquitoes breed here in supreme abundance and biting flies of the genera Tabanus and Chrysops are also abundant. The water is saline, with a specific gravity of 1.0041, but much less so than would be expected if derived from meteoric water anywhere in the vicinity as the desert is covered with crystalline salt which goes into solution when rain falls. The flow from these springs is evidently very large and their source must be through a deep fault.

No. 30. Grantsville, Utah. These are between the main highway and the southern shore of Great Salt Lake, 4.7 miles west of Grantsville which is not far west of Salt Lake City. These are only warm with a temperature of 30°, a specific gravity of 1.0207 and pH of 7.7.

No. 31. Hot Springs Station, Utah. These springs are about ten miles north of Ogden, Utah and close to the railroad station of Hot Springs. They form a small group, one of which is at the boiling temperature while the others are extremely hot. A large flow of water issues from these, some of which had been led to a bath house that burned down at some time before our visit. The water is highly saline, and deposits a large amount of ferruginous material which stains the ditches and surrounding soil where the water is led off to an extensive flat below the springs. No material was obtained here, but about a quarter of a mile further south is a much smaller spring, utilized for baths. In the overflow with a temperature of about 35°, specific gravity of 1.0146 and pH of 8.2, a number of insects and an amphipod crustacean were collected.

No. 32. Ogden Canyon, Utah. This issues from the southern wall of the canyon at its lower end where the road crosses the stream, about 3 miles from Ogden, Utah. The water flows in considerable quantity and finds its way over the rocky slope to the stream some forty feet below. The water is very hot at its source and produces a thin layer of highly ferruginous deposit. It is quite saline, with a specific gravity of 1.0063 and pH of 8.3.

No. 33. Midway, Utah. These may be reached from Provo, Utah over the road which ascends the Prova Canyon northeastward to the town of Midway which is about 25 miles from Provo at an altitude of 5500 feet. Near the town are several groups of hot springs known locally as "hot pots" as they have formed large deposits of sinter and each spring is set in a conical mound. Just northwest of the town over an acre or two there is a group of half a dozen of these mounds rising ten or twenty feet above a rounded hill covered with whitish sinter. The openings vary from ten to twenty feet in diameter, but the water lies eight or ten feet below the contracted orifice. The water was out of reach, but is evidently cool or scarcely warm, although a small amount of gas bubbles up from the depths. These springs are evidently rapidly decreasing in activity and we were told that one small dome in the yard of a house, where the water level is now several feet below the rim, had been flowing over the top fifty or sixty years ago. Nearer the town is a large conical mound at least fifty feet high with an opening some twelve feet in diameter. The water is now forty feet below, but this spring has been tapped near the base to supply water for a hotel and natatorium. The water in this crater is boiling. Northeast of the town is another group on a small sinter hillside, with a very small amount of overflowing water and on the verge of extinction. The water of these springs has a specific gravity of 1.0021 and pH of 7.4. There is no ferruginous color to the deposits.

No. 34. Monroe, Utah. These springs are directly east of the town of Monroe, at a distance of about half a mile. They issue from the western slope of a hill at the base of the high mountains that rise from the eastern edge of the narrow valley in which the town sets.

The amount of flow from these springs is not very great but they supply a natatorium after flowing down the steep slope. The water has a specific gravity of 1.0024 and pH of 9.0. No very considerable amount of deposit is produced but that which appears is of a bright ferruginous color characteristic of all the springs (Nos. 31, 32 and 34) which we visited east and south of Great Salt Lake with the exception of no. 33 at Midway which lies to the east of the Wasatch Mountains.

There are a few observations contained in the preceding account which are of geological rather than biological interest, but as certain of the phenomena exhibited by hot springs are open to more than one physical explanation, it seems worth while to call attention to some of the facts observed by us. During the year 1924 there appeared in the Journal of Geology a number of papers on hot springs which include a very full and illuminating account of the nature, origin, source of heat, temperatures and distribution of hot springs (Day and Allen '24; Adams '24; Van Orstrand '24; Meinzer '24 and Sosman '24[a], 24[b]).

Two sources of water are possible in the case of hot springs. Juvenile or magmatic water arising as steam from very great depths where it is given off by underlying magmas is a probable source in many cases and Meinzer (*loc. cit.*) believes that the thermal springs of Nevada and Utah derive their water mainly from this source. His reasons are that the springs are generally associated with faults, that they do not regularly occur closely related to volcanic rocks from which they may derive their heat, and that they cannot generally be attributed to artesian structure. The last mentioned factor is necessary if the hot water be derived from meteoric or surface water which must descend so as to come into contact with heated rocks. This second source of water is available without artesian structure before emergence as a spring in some localities, such as Lassen Park where the springs are below the source of surface water and snow and there Day and Allen ('24) believe that meteoric water supplies the springs which are very close to subterranean sources of heat in the mountain which is an active volcano. Our own observations in Lassen Park lead us to a similar conclusion.

Several observations of our own tend to confirm Meinzer's view of the magmatic origin of the waters of a number of the springs which we visited in Nevada and Utah. At Silver Peak, Nevada (Hot Spring no. 4) there are a number of hot springs and some cold ones interspersed, within a few feet of one another. In this case the cold springs arise from the mountain adjacent and the hot ones must certainly be attributed to an entirely different source. In an earlier paper ('17, p. 89) Meinzer refers to a case in the Darrough hot spring group in the Big Smoky Valley, Nevada in this region where a small spring with a temperature of 61° Fahr. arises within 100 feet of a hot spring having a temperature of 198° Fahr., indicating an entirely different source. Another evident case where it seems impossible to attribute the source of warm springs to surface or artesian water is at Wendover, Utah (Hot Spring no. 29) on the western edge of the Great Salt Desert. Here, as I have noted on another page, a surface origin seems to be precluded, but as the low temperature of these springs suggests an artesian origin, no simple theory seems to account for them.

The general belief concerning the age of the springs in the region visited is that they originated before the close of the Tertiary and that they are not now, nor have they in comparatively recent times decreased in volume. After visiting a number of these springs and traveling over the intervening territory it seems quite evident that the thermal activity of the springs is markedly on the wane. As already related there are evidences of extinct hot springs in many localities. Some of these give little indication of extensive erosion and are clearly quite recently extinct, while others remain only as mounds or masses of deposits, most certainly the products of hot-spring activity. In some cases the deposits of tufa from non-thermal sources, *e.g.*, in the vicinity of Pyramid Lake, derived from the evaporation of the former Lake Lahontan, simulate the deposits of hot springs. In this region, however, there is evidence that some of this material has been produced in hot springs of which there are remains within a very short distance of the Pyramid Lake hotel (Plate 2, fig. 1). Along the shore further northward there is also a well preserved structure which is surely the remains of a geyser, with the inner cavity still partially intact. These must certainly be of comparatively recent age as they are not far above the present level of the lake. At other localities it appears that certain hot springs now active (Nos. 6, 8, 9, 33 and the Casa Diablo hot springs mentioned under no. 5) have either decreased in activity within recent years, or give good structural evidence of reduced flow of water, or show clearly that they were formerly more extensive. At places where springs have been damaged or diverted for irrigation or other purposes, it is difficult to determine the effect which must be attributed to such changes, but taken together the evidence indicates without question in my mind that the flow of thermal water in this region is quite consistently diminishing.

In the detailed account which follows, I have included references to the zoological material which we collected in the several hot springs just described and have added certain other records which have been published by others in connection with the fauna of thermal springs of other regions. The fauna of the springs in the areas which visited had never been examined previously so that published references to them contain almost no zoological data. As will be seen, I have also discussed various other matters with reference to the distribution, adaptations and temperature relations of particular groups of animals where these have a bearing on the thermal fauna. Much of the previously published data relating to animal life in thermal springs is reviewed in my previous paper (Brues '24) and I have avoided a repetition of the references and discussion presented there.

PROTOZOA

If we compare the reactions of bacteria in relation to high temperatures (see p. 206) with those of the Protozoa among animals, the great breach which separates these two series of unicellular organisms is at once apparent. Unfortunately no data are available for the fauna of American hot springs, but Issel ('06) has reported the occurrence of a number of Protozoa in Italian thermal waters. His list includes fifteen species and is reproduced below with some changes in arrangement and nomenclature.

SARCODINA

ACTINOPODA

Actinophrys sol Ehrb. 38°.

RHIZOPODA

Amœbidæ

Amœba proteus Ehrb. 35°–45°.
Hyalodiscus limax Duj. 35°–45°.

INFUSORIA

CILIATA

Enchelynidæ

Enchelys sp. 42°–43°.

Chlamydodontidæ

Nassula elegans Ehrb. 42°–48°.
Chilodon cucullus Müll 35°–39°.

Chiliferidæ

Loxocephalus granulosus Kent 36°.
Frontonia acuminata Ehrb. 40°–50°.
Colpoda cucullus Müll 35°–39°.

Pleuronemidæ

Cyclidium glaucoma Müll 41°–51°.

Paramoeciidæ

Paramœcium aurelia Müll 38°–40°.

Plagiotomidæ

Metopus sigmoides Clap. and Lachm. 35°–45°.

Pleurotrichidæ

Oxytricha fallax Stein 32°–45°.

Euplotidæ

Euplotes patella Ehrb. 32°–35°.
Cothurniopsis imberbis Ehrb. 35°.

It will be seen that the range of none of the species exceeds 50°, with the single exception of *Cyclidium glaucoma* which he found at temperatures of 41°–51°. This is fully in accordance with the maximum temperatures observed for various forms of invertebrates. In an earlier ('01) paper Issel reported a temperature of 54.5° for protozoa but this is not repeated except as a reference in 1906. Still later ('10) he has reported *Amœba limax* at 52°, which would increase the upper thermal limit for Protozoa one degree. Some much earlier observations of Ehrenberg ('58) relating to a number of Protozoa in thermal water in Ischia offer no further data since only the extreme temperature (81°) of the spring is recorded.

For comparison with these freee-living types there are available some very recent data on the thermal resistance of certain forms parasitic in warm blooded animals. Boeck ('21) has determined experimentally the thermal death point of a number of Protozoa which occur as intestinal parasites of man. His data relate only to the resistance of the cysts which are able to withstand much higher temperatures than the active stages of the same animals. These lethal temperatures range from 64° C. to 72° C., or 147°–169° F. and thus extend well to the upper limit of tolerance of chlorophyll bearing algæ as observed by Davis ('97), Miss Tilden ('98), Setchell ('03) and myself (Brues '24) in Yellowstone Park. This would show that the encystment of Protozoa causes them to acquire a resistance to heat comparable to that noted in the spores of bacteria when compared to the vegetative forms.[1]

Boeck's observations relate to six different forms, including both Rhizopoda and Mastigophora and the lethal temperatures as determined are as follows: *Entamœba histolytica,* 68°; *E. coli,* 76°;

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[1] More recent researches cast doubt upon the accuracy of these data, as I have indicated in the postscript to the present paper (p. 228).

Iodamœba bütschlii, 64°; *Giardia intestinalis,* 64°; and *Chilomastix mesnili,* 72°. Relating to these species one other matter is of considerable interest, namely, that the greatest divergence noted is not between unrelated forms but among rather closely similar species. Thus the two species of Entamœba and the Iodamœba include among themselves the entire range of temperatures recorded for the whole series (64°–76°).

The existence of such conditions among Protozoa completely breaks down any distinction drawn between animals and plants on the basis of their resistance to high temperatures. If we attempt to explain these lethal temperatures on a physical basis, it is possible to regard them as associated with a loss of water which increases the coagulation point of albumin-like proteins within the cells. This is the explanation offered by bacteriologists with reference to the greater heat resistance of the spores of bacteria when compared with their vegetative stages. On the basis of the coagulation point of albumin we should expect that the protoplasm of the protozoan spores had its moisture content reduced to about one-fourth that of the growing organism. Whether this is actually the case it is no more possible to determine accurately in the case of Protozoa than with bacteria, although in the latter Cramer ('91) and Benecke ('12) have given evidence to show that the protoplasm of spores is actually in a state of partial dehydration. Up to the present time the only plausible explanation of these phenomena seems to be the one just outlined. It can hardly be applicable to the growing stages of bacteria, developing in water or any other liquid medium, however. Under these conditions the requisite exsiccation could not come about, especially in the case of algæ developing at high temperatures. In the case of animal life developing at 50° or thereabouts the amount of desication necessary would be slight and by no means in excess of that compatible with complete metabolic activity. That exsiccation may proceed well beyond this point without ill effects has been well established.

Delicate adjustment of certain flagellate Protozoa to high temperatures is shown by some recent work of Cleveland ('28) on Tritrichomonas. A type of these organisms derived from frogs (*T. angusta*) grows well in culture at room temperatures, but fails to grow at all at 36°, while another type (*T. fecalis*) derived from the alimentary canal of a human subject was found to develop readily at 36° and to grow even at 37°. Associated with this difference in respect to high temperatures is a morphological difference between the two. *T. angusta* is characterized by the presence of granules in the axostyle while these are lacking in *T. fecalis.* It seems therefore that unless the difference may be due to acclimatization in a homœothermic animal, that the heat-sensitive part of the animal may be the granules in the axostyle.

CŒLENTERATA

The members of this phylum are of course almost entirely marine, and among the few fresh-water types like Hydra which have an opportunity to migrate into hot springs, none appear to have done so. Among the marine cœlenterates the corals are notable on account of their restriction to the warmer seas and their relation to high temperatures of the medium in which they live is therefore of interest in connection with the thermal environment. Fortunately there are available in the literature some observations and experimental data relating to this phase of their biology.

Mayer ('17) obtained some comparative data on the temperatures fatal to a number of species of corals and reached the conclusion that those corals living in cool, relatively agitated water free from silt are more susceptible to the effects of heat, while those which live in the warmer silt-laden shallows are as a rule more resistant. These differences are clearly shown by the following records, taken from his paper, showing the temperatures which caused death in the following species:

Acrophora muricata	34.7° C.	*Mœandra areolata*	36.8° C.
Orbicella annularis	35.6° "	*Porites furcata*	36.85° "
Porites astrœoides	35.8° "	*Favia fragrum*	37.05° "
Porites clavaria	36.4° "	*Siderastes radians*	38.2° "

In the case of these cœlenterates it is therefore clear that there is a great divergence in the power of resistance to heat and that (with the exception of *Favia fragrum* which lives in both warmer and cooler water) the resistance is correlated with the temperature of their normal environment.

There can be no doubt that the temperature of the water in shallows occasionally rises very near to the lethal temperatures noted above, especially the lower ones, but it cannot be said without question that it ever becomes sufficiently high actually to kill the corals by heat.

The temperatures which inhibit the functioning of specific tissues in the scyphomedusa, *Cassiopea xamachana* have been determined experimentally by Harvey ('11) who found them to be 39.5° for muscles, 42.6° for sense-organs and 44.0° for nerves. These are closely similar to those mentioned elsewhere for the frog, but Harvey did not find that heat rigor supervened at this point. The latter was delayed until about 55° which is considerably above the point at which heat rigor appears in the frog (38°–40°) or in mammals (47°–50°) (Vincent and Lewis '01). Indeed, it is much nearer to that reported for the pigeon, which is 53° (Alcock '03).

NEMATODA

Only a single nematode worm has so far been recorded from hot springs. This is *Dorylaimus atratus* which Issel ('06) found in Italian thermal springs. This species was found in water at 40°.

Among the internal parasites of warm-blooded animals there are included also considerable numbers of lowly organized Metazoa all of which live at the temperature of their respective hosts, at or near 40° C. In only a few cases are there any accurate data concerning the reaction of these forms to higher temperatures, but on account of its great importance to the health of the human species, the nematode worm, *Trichinella spiralis,* has been very carefully studied in this respect. As is well known, the encysted larvæ of this species are frequently present in the muscle of swine and reliance may be placed upon the heat of cooking to destroy the encapsulated worms in pork which is to be eaten by man. Various observers, most recently Ransom ('19) have found that the thermal death point for the encysted stage of Trichinella is about 54°, possibly a degree or two lower on somewhat longer exposure. It is therefore quite high for animals in general, but markedly lower than those cited above for encysted Protozoan parasites of man. Moreover there is no increased resistance to heat due to encystment since decapsuled larvæ exhibit the same thermal death point. On longer exposure at lower temperatures of from 39°–40° there is a noticeable difference in the rate of killing if the decapsuled trichinæ have been immersed in pure water or in 0.7% salt solution, as in all cases they become more susceptible to heat after remaining in a hypotonic liquid. Here again therefore, it is evident that imbibition of water by the tissues lowers the thermal death point of the animal.

This change of thermal death point due to the imbibition of water in Trichinella appears to be the same phenomenon that occurs in the spores of bacteria and cysts of Protozoa where dehydration appears to be the reason for a rise in the death point, but the actual temperatures are so different in the two cases that it does not seem that the evidence from the behavior of Trichina can be applied to the others with any degree of assurance.

ECHINODERMATA

Jacobs ('19) has determined the lethal temperatures for starfish larvæ of an undetermined species obtained off the Massachusetts coast, and one consequently not normally exposed to any considerable heat. These determinations were made in connection with experiments to determine the rate of killing and individual acclimatization, but indicate also what lethal temperatures might be for prolonged exposures. Since a water temperature of 32° killed such larvæ in less than an hour, they cannot be able to withstand more than about 30° continuously, which is notably lower than the temperatures observed for tropical corals.

ROTIFERA

The members of this group of minute invertebrates are probably widely disseminated in hot springs, but their presence had seldom been recorded. A few are known from Italian thermal springs through the researches of Issel who has listed the species which he obtained in a paper published in 1906. His list of species, which is reproduced below, with some necessary changes in nomenclature, includes only two forms taken from water of over 40°, so that the group does not seem to be particularly adaptive in this respect. There are twelve, species, as follows:

Rotaria rotaria Pallas 38°.
Rotaria sp. 32–35°.
Metopidia solidus Gosse 40°.
Lepadella triptera Ehrb. 28°.
Monostyla cornuta Müll. 35°.
Distyla inermis Bryce 32–38°.
Colurus loncheres Gosse 32–35°.
Brachionus urceolaris Müll. 32–33°.
Lepidoderma ocellatum Metschn. 38°.
Chætonotus sp. 40°.
Euchlanis plicata Levand. 41°.
Notommata najas Ehrb., var. *thermalis* Issel 45°.

Among these, three at least, *Euchlanis plicata, Monostyla cornuta* and *Lepadella triptera* are fresh water species which occasionally occur in brackish water as they are reported from the saline waters of Oldesloe in Holstein by Hauer ('25). *Brachionus urceolaris* may develop in saline water also (Rumjanzew '22).

ARTHROPODA

ARACHNIDA; ACARINA

A single, remarkable species of water mite was obtained in several hot springs in Nevada and California. These were sent to Professor Ruth Marshall who has very kindly examined them and described them as *Thermacarus nevadensis* (Marshall '28).

This species was collected at the following localities:

Locality	Hot Spring No.	Temperature	Specific Gravity	pH
Bridgeport, Calif.	6	34°	1.0030	7.5
Minden, Nevada	7	45°	1.0000	9.6
Deeth, Nevada	25	43°	1.0008	7.4
Ruby Valley, 65 miles south of Wells, Nevada	28	41.8°	1.0009	9.4

So far as the collection data extend, it it seen that its tolerance for temperatures extends from 34°–45°, for specific gravity 1.0000 to 1.0030 and pH 7.4–9.6.

The mites are of large size, from 2–3 mm. in length, with broad stout body and extremely hard, thick integument. They were numerous in all of the springs where their presence was noted and were generally moving actively about on the surface of the soft mud in shallow water, usually only an inch or two in depth, near the edges of pools in still or very slowly flowing water.

The genus *Thermacarus* contains only one other species *T. thermobius* from Asia and forms the family Thermacaridæ. The old world species was collected in a hot spring near the northern end of Lake Baikal in southern Siberia in 1926 and was described by Sokolow ('27) a year later. It is therefore very surprising to find a second species in hot springs in the hot desert regions of the western United States. At the time Sokolow published his paper he knew of no other record of Hydracarina from hot springs but there is one, mentioned in my previous paper, by Plateau ('70) and only last year Uchida ('27) described an eylaid from a hot spring near Taihoku, Formosa.

This species, *Eylais thermalis* was found in water at 42°. The Asiatic Thermacarus was found in water at 45°, with a hydrogen ion concentration of 8.7, corresponding very closely with the conditions under which we collected the American species. They were found also in quiet water, but were congregated upon the filamentous blue green alga, *Phormidium laminatum.* This led Sokolow to suppose, in connection with the structure of the appendages which would seem to be adapted for clinging to the algæ, that Thermacarus is vegetarian. Most specimens of *T. nevadensis* which we found were, however, not associated with algæ, although in any of the springs such food was present in small amounts.

It appears probable that there is a much earlier reference to *Thermacarus nevadensis,* contained in a paper by Wood ('68) and cited in my previous paper. At that time I was unable to place the "spider-like animal" seen together with larvæ of Chironomus in water at 51° (124° Fahr.), but as these were seen in a hot spring in Owens Valley, California near to one of the localities at which I obtained this mite, Wood's note most probably refers to the same species. His temperature of 51° is considerably in excess of those recorded by Sokolow and myself for either the Asiatic or American species and may be slightly in error.

As Thermacarus occupies an isolated position, it is impossible to draw any conclusions concerning any relationships to other Hydracarina. Eylais is, however, known to occur in brackish water (*cf.* Viets '25).

INSECTS

As might be expected the major part of the fauna of thermal springs consists of insects belonging to several orders. The greatest variety is met among members of the order Diptera, but there are also Coleoptera belonging to several families which are commonly encountered, and the water beetles are perhaps more abundant in both individuals and species than any other insects. As their larvæ are also aquatic such forms enjoy a thermal environment in all stages. This is also true of certain Hemiptera, but the numerous Diptera, dragon flies and other types are of course aquatic only during their preparatory stages and enter a more normal environment in the imaginal condition. As will be seen, the relations of various insects to waters of high temperature are essentially the same as those of the other groups of animals already discussed, but the more detailed data bearing on these phenomena are included in connection with the several groups.

In nearly all of the hot pools which have a temperature higher than that which can be endured by insects, considerable numbers of dead beetles, flies, bugs, wasps and other insects may be seen floating on the surface of the water. No field notes were made concerning such occurrences as they are obviously individuals that have been overcome by heat either by falling into the water or in attempting to enter it from the air in the case of aquatic species. A great abundance of Hydrophilid and Dytiscid beetles was frequently noted, in many cases of species which inhabit the warm water, but have attempted to enter water too hot for them to endure. This is also true of specimens of Notonecta and Corixidæ which are active fliers. Many spiders are killed in the same way by attempting to skim over very hot water. More careful note was taken in one locality, at Hot Spring no. 8 at Steamboat Springs, Nevada, where several hot pools were dotted with insect flotsam, and a brief list was made of the several types of insects present. This is reproduced below and indicates the presence of a great variety of forms.

The series includes the following and is practically similar to that encountered at other places. *Ephemerida*, grasshopper nymphs, *Odonata*, *Trichoptera*, *Hemiptera* (Saldidæ, Corixidæ, Notonectidæ), *Coleoptera* (Carabidæ Dytiscidæ, Hydrophilidæ, Staphylinidæ, Coccinellidæ), *Hymenoptera* (ants, bees, Psammocharidæ), *Diptera* (Culcidæ, Tabanidæ, Dolichopodidæ, Ephydridæ, Sarcophagidæ, Tachinidæ). In the hot pools near Denio, Oregon and those in the Ruby Valley south of Wells, Nevada the abundance of large water beetles is very surprising, indicating a very high mortality from this cause.

ORDER ODONATA

A considerable series of nymphs of this order were secured during the course of our summer's work from a number of different hot springs. Some of these I have been able to identify specifically with certainty, but have been compelled to indicate only the generic position of others. Altogether nine species were obtained. In my previous account ('24) I have given a summary of some observations on Odonata that develop in hot springs and in brackish or saline water, but there are, in addition, a few references in a paper by Kennedy ('17) which I overlooked at that time. Those which refer to hot springs I have noted below where Kennedy records the nymphs as actually taken in the warm water. He also includes a list of species from the vicinity of the hot springs at Calistoga, Napa Co., California which breed in the more or less warm overflow. Here *Libellula saturata* occurs, a species which I have found flying about hot springs in Yellowstone Park. His records of three species from Pyramid Lake, a highly saline body of water with a specific gravity of about 1.0035 are particularly interesting. These are *Enallagma clausum* Morse, *Ischnura cervula* Selys and *Sympetrum corruptum* Hagen. The last mentioned species I found last summer in Pyramid Lake which we visited on July 26.

Argia

Nymphs of this genus were collected in a number of hot springs. At least two species are included in the series, but as I have been unable to determine the specimens, am compelled to list all in one series as follows:

Locality	Hot Spring No.	Temperature	Specific Gravity	pH
Goldfield, Nevada	3	35.4°	1.0016	8.0
Goldfield, Nevada	3	37.3°	1.0016	8.0
Denio, Oregon	15	35.2°	1.0000	8.4
Battle Mt., Nevada	23	30.0°	1.0008	8.0
Battle Mt., Nevada	23[a]	36°	1.0021	8.5
Wells, Nevada	26	32.5°	1.0018	7.9

There appear to be no previous records of any members of this genus from thermal springs.

Ischnura

Nymphs of this genus, probably representing two species, were obtained at two localities as follows:

Locality	Hot Spring No.	Temperature	Specific Gravity	pH
Wells, Nevada	26	32.5°	1.0018	7.9
Grantsville, Utah	30	30.0°	1.0217	7.7

The water in Hot Spring No. 30 which is very close to Great Salt Lake is highly saline. It will be noted that neither of these records indicate any high temperatures and it appears that the members of the genus are not adapted to living in hot or very warm water. Kennedy records the occurrence of two species of Ischnura at Golconda, Nevada in the neighborhood of the hot springs there. This is our Hot Spring No. 18, but we took no nymphs of Ischnura in these springs.

The occurrence of nymphs in Hot Spring No. 30 with a specific gravity of 1.0217 is most unexpected since this salinity is greatly in excess of that which Osburn ('06) found could be endured by unacclimatized dragon-fly nymphs. Osburn determined experimentally that a density of 1.010 was about the limit of salinity for *Libellula auripennis*. This density was taken at 75° Fahr. and would correspond to 1.0136 at 15° C. to which it must be reduced to make it comparable with the records in the present paper. Even with this correction the present record is remarkable.

Celænura

Nymphs belonging to this genus were taken at two localities, as follows:

Locality	Hot Spring No.	Temperature	Specific Gravity	pH
Osobb Valley, Nevada	21	31.6°	1.0020	8.1
Battle Mt., Nevada	23	30.0°	1.0008	8.0

It is very probable that these are *Celænura denticollis* Burm., since Kennedy ('17, p. 500 and 622) found this species breeding in the overflow from the hot springs at Golconda, Nevada (our no. 18) and at Calistoga, California. The species is common further south and its northern extension is no doubt due to the opportunity afforded by the presence of hot springs. It will be noted that it was taken only in slightly warm water at 30°–31.6°.

Lestes congener Hagen

According to Kennedy ('17, p. 621) this species breeds in the warm springs at Golconda, Nevada (our Hot Spring No. 18), and *Lestes unguiculatus* Hagen probably breeds there also according to the same observer. In the same paper he records a number of other species that occur about these same springs and some of these undoubtedly undergo their transformations there.

Libellula sp.

A number of specimens of nymphs belonging to a species of Libellula which I have been unable to identify were found in five localities as follows:

Locality	Hot Spring No.	Temperature	Specific Gravity	pH
Hot Springs, New Mexico	1	39.5°	1.0039	8.1
Battle Mt., Nevada	23	36°	1.0021	8.5
Deeth, Nevada	25	——	1.0008	7.4
Wells, Nevada	26	32.5°	1.0018	7.9
Ruby Valley, Nevada	28	41.8°	1.0009	9.4

The nymph is moderately slender, with small eyes, each only about one-tenth as wide as the head. The median lobe of the labium is produced into a broad tooth at the middle; on each side only very slightly curved, with eight short weak setæ on each side, set in minute emarginations which make the edge slightly wavy or crenulate. There are no mental setæ. Inner edge of lateral lobe regularly serrate, with seven teeth. Lateral setæ seven. Lateral spines of ninth abdominal segment very short, less than one-sixth the length of the segment.

This species will run to Libellula in Needham's recent key of genera, ('27) except that there is a faint crenulation of the edge of the median lobe of the mentum. It is certainly not *L. forensis* which has been reported as present in the neighborhood of Hot Springs at Golconda, Nevada by Kennedy, ('17) as the nymph of this species is known, (Needham '04) and differs at once by the structure of the labium. We collected at the Golconda hot springs, but no dragonfly nymphs were obtained.

Mesothemis simplicicollis Say

Nymphs of this species were taken at three localities as follows:

Locality	Hot Spring No.	Temperature	Specific Gravity	pH
Hot Springs, New Mexico	1	39.5°	1.0039	8.1
Convict Lake, Calif.	5	43°	1.0012	8.5
Denio, Oregon	15	35.2°	1.0000	8.4

They agree closely with the description given by Needham ('01) of this widespread North American form, the adult of which has been taken by Kennedy at Golconda, Nevada close to the hot springs at that place. We did not obtain nymphs at these springs, however, when we examined them.

Sympetrum corruptum Hagen

A single nymph of Sympetrum was collected in Hot Spring No. 3 near Goldfield, Nevada in water at 35.4°, specific gravity 1.0016 and pH 8.0. It agrees in the structure of the labium with *S. corruptum* as described and figured by Needham ('03) and is probably this species which occurs in this region. It develops in the highly saline and alkaline water of Pyramid Lake whence it has already been reported by Kennedy ('15). The specific gravity of the lake water is given by Kennedy as 1.0035 and is therefore very much greater than that of the present hot spring. This density is probably at the ordinary temperature of the lake, but unfortunately we did not determine the density at the time of our visit, and a much lighter one, 1.0027, is given in a report of the U. S. Geological survey made in 1870.

ORDER EPHEMERIDA

Nymphs of a mayfly belonging to the genus Siphlonurus were obtained at two places as follows:

Locality	Hot Spring No.	Temperature	Specific Gravity	pH
Osobb Valley, Nevada	21	31.6°	1.0020	8.1
Battle Mt., Nevada	23[a]	36.0°	1.0021	8.5

ORDER HEMIPTERA

Only a few species of this order are contained in the collections made, but two are of considerable interest. *Ambrysus heidemanni* which I collected in 1923 in Yellowstone Park was obtained again, at Bridgeport, California. A tropical species, *Belostoma apachum* was taken at Paradise, Nevada, and nymphs, probably of the same species were found in hot springs at Battle Mountain, Winnemucca and Wells, Nevada.

For the identification of the Hemiptera I am greatly indebted to Professor H. M. Parshley who examined a miscellaneous series and Professor H. B. Hungerford who determined the Corixidæ and Notonectidæ.

GERRIDÆ

The widespread *Gerris remigis* Say was found in Hot Spring No. 13 in water at 28° at Drakesbad in the Mt. Lassen National Park. In Europe, two species of Gerris have been reported from saline waters by Benick ('26[b]).

SALDIDÆ

Species of Salda (*sens lat.*) are common on the wet soil bordering hot springs and were collected in a number of localities. They can, of course, hardly be considered to belong strictly to the hot springs fauna.

NAUCORIDÆ

Ambrysus haldemanni Montd.

Hot Spring No. 6 in water of 34°, specific gravity 1.0030, pH 7.5, near Bridgeport, California. I have taken this species previously in

Yellowstone National Park in water at 35.5° which was highly charged with silica. It would appear, therefore, that this insect occurs generally in thermal springs of moderate temperature. One of its relatives, *Naucoris cimicoides* of Europe occurs in saline water, but no other members of the family have been found in thermal springs.

Belostoma apachum Kirkaldy

An adult was taken in Hot Spring No. 19 at Paradise, Nevada, in water at 31.8°, and nymphs, presumably of the same species were collected in four other springs. The complete data are as follows:

Locality	Hot Spring No.	Temperature	Specific Gravity	pH
Denio, Oregon	15	35.2°	1.0000	8.4
Paradise, Nevada	19	31.8°	1.0013	7.5–7.7
Winnemucca, Nevada	21	31.6°	1.0020	8.1
Battle Mt., Nevada	23	30.0°	1.0008	8.0
Wells, Nevada	26	32.5°	1.0018	7.9

This is a tropical form and this fact together with the low temperatures of the springs in which it was taken in Nevada, probably indicates that it is not a typically thermal species, but that it simply extends its range northward through the medium of thermal springs.

NOTONECTIDÆ

This family is represented by two species in our collections, taken in five different hot springs as follows:

Notonecta indica Linn.

Locality	Hot Spring No.	Temperature	Specific Gravity	pH
Goldfield, Nevada	3	35.4°	1.0020	8.0
Winnemucca, Nevada	20	36.3°	1.0014	8.6
Osobb Valley, Nevada (Nymphs only in this spring)	21	31.6°	1.0021	8.1
Battle Mt., Nevada (Adult and nymphs)	23	30.0°	1.0008	8.0
Ruby Valley, Nevada (Nymphs only in this spring)	28	33.0°	1.0009	9.4

Notonecta kirbeyi Hungerford

Battle Mt., Nevada	23	30.0°	1.0008	8.0

The data for these two species of Notonecta indicate that they are not very tolerant of high temperatures since they were not found in water above 36.3° although there are in all six records, representing five springs, as the single appearance of *N. kirbeyi* was in company with *N. indica* in Hot Spring No. 23. Also they do not appear to tolerate water of any great salinity as all of the springs in which they were found have a low specific gravity of 1.0008 to 1.0022.

In addition to the record of the European *Notonecta glauca* in a hot spring, cited in my previous paper, there is at least one other earlier record of a notonectid in a thermal spring, in an anonymous note published in 1917 in the Societas Entomologica, Jahrg. 32, p. 26. The writer records *Micronecta episcopalis* in a thermal spring near the Hungarian city of Grosswardein.

One European species, *Notonecta halophila* is a brackish water form, but has been found to be indifferent in this respect and often occurs far from the coast (Hutchison '22). Another, *N. glauca* Linn. is known to invade saline water (Benick '26^b).

In connection with the presence of hæmoglobin in Chironomus which is one of the characteristic forms occurring in hot springs it is interesting to note that hæmoglobin occurs also in certain Notonectidæ. Its presence in *Buenoa margaritacea* has been noted by Hungerford ('22) who found it localized in groups of cells associated with the trachæ in the abdomen. So far as I know, no species of Buenoa have been reported from hot springs.

CORIXIDÆ

Three species of Corixidæ in addition to an undetermined nymph were collected on the present expedition.

Trichocorixa wallengreni Stal, var.

This was found in two localities, represented in each by numerous individuals.

Locality	Hot Spring No.	Temperature	Specific Gravity	pH
Wells, Nevada	28	35.0°	1.0009	9.4
Hot Spring Sta., Utah	31	35.3°	1.0146	8.2

Arctocorixa tumida Uhler

Two lots were secured, consisting of numerous individuals from the following localities:

Locality	Hot Spring No.	Temperature	Specific Gravity	pH
Silver Peak, Nevada	4	37.2°	1.0212	8.3
Convict Lake, California	5	35.0°	1.0012	8.5

Arctocorixa wileyi Hungerford

This species was taken at a single locality and the individuals were not numerous. Denio, Oregon, Hot Spring no. 16; temperature 39°; specific gravity 1.0015; pH 8.5. The species was described from the Wasatch Mts. in Utah.

Corixid nymph

Two specimens. Twenty-nine miles south of Winnemucca, Nevada in Hot Springs no. 20; temperature 36.3°; specific gravity, 1.0014; pH 8.6.

Members of this family were thus found living in six different hot springs at temperatures from 35° to 39.6°, only the species of Arctocorixa in water of any considerable temperature (39°–39.6°). So far as the water of these springs is concerned, in two cases this is unusual. At Silver Peak, Nevada the specific gravity is quite high (1.0212) and also at Hot Springs Station, Utah (1.0146) and in both places larvæ of the brine fly, Ephydra, were present with the Corixids. Schwarz ('91) reports a species of Corixa from brackish water near Great Salt Lake, Utah, which is probably *Arctocorixa decolor*, Uhler, a western species known to occur in saline pools (Packard '71). Previous records of European and North American Corixidæ in thermal springs are cited in my earlier paper.

Some interesting observations on the occurrence of Corixidæ in brackish water in England have been made by Thorpe ('27). He finds that some species live in saline water, but that there is a great difference in their ability to survive in this medium as one species, *Corixa selecta*, is unharmed by a density of 1.040 whereas *C. sahlbergi* cannot withstand more than 1.008. In such cases, however, it is not the resistance of the adult that limits the distribution of species, but the possibility of the nymphs developing under such conditions (*cf.* Poisson '24). Thorpe concludes from his observations that Corixidæ are very sensitive to changes of salinity, although as already indicated, some are very resistant to high concentrations. Arctocorixa is represented in the saline waters of Oldesloe in Holstein by three species according to Benick ('26^b).

ORDER COLEOPTERA

The collections of beetles obtained include thirty-one species represented by adults and a smaller number of larvæ. Among the material are members of the five common families of water-beetles, the Haliplidæ, Dytiscidæ, Gyrinidæ, Hydrophilidæ and Helmidæ as well as some mud-beetles of the family Heteroceridæ. Ten of the thirty-one forms were taken at more than one locality, *i.e.*, in from two to eleven different hot springs, respectively.

In the list of these which follows, I have indicated the localities by the Hot Spring numbers which have been given to them on a previous page (p. 143), and have included the temperature and hydrogen ion concentration of the water from which the beetles were actually taken. In a few cases the temperature is not cited, where it was impossible to get it accurately because of some physical difficulty.

In the preceding table only collections made at temperatures of 30° C. (86°) Fahr.) or above are recorded as it appears that temperatures below this point are no higher than those which occur frequently in shallow pools of non-thermal origin in these hot desert regions. Above 30° there is a range of 15.7 degrees to 45.7°, the highest temperature at which any species of beetle was found. Within this rather wide range the occurrence of species may be roughly grouped as follows:

30.0–35°	17 species	42.1–43°	2 species
35.1–37.5°	10 species	43.1–44°	1 "
37.6–40°	13 "	44.1–45°	1 "
40.1–42°	5 "	above 45°	2 "

As might naturally be supposed there is a gradual reduction of species with ascending temperature. This shows very clearly in spite of the necessarily restricted number of observations and the fact that a continuous gradation of temperatures is never available under suitable conditions for observation. If these data are added to indicate the number of species which we found to invade water of gradually increasing temperatures, they form a very regular curve (Fig. 3).

Indeed, its regularity is surprising, considering the small number of observations, the great variation in the several ecological factors

List of Aquatic Coleoptera

	Spring No.	Temperature °C	Specific Gravity	pH
HALIPLIDÆ				
Peltodytes callosus Lec.	11	——	1.0016	5.7–6.5
DYTISCIDÆ				
Laccophilus decipiens Lec.	20	36.3	1.0014	8.6
Laccophilus mexicanus	28	——	1.0009	9.4
Bidessus affinis Say	28	38–42.5	1.0009	9.4
Bidessus affinis Say	9	41.0	1.0012	8.5
Bidessus affinis Say	16	39.0	1.0015	8.5
Bidessus affinis Say	26	32.5	1.0018	7.9
Cœlambus thermarum Darl.	20	38.0	1.0014	8.6
Cœlambus thermarum Darl.	23[a]	30.0	1.0021	8.6
Cœlambus bruesi Fall	28	39.2–40.0	1.0009	9.4
Cœlambus sp.	28	35.6	1.0009	9.4
Cœlambus lutescens Lec.	16	39.0	1.0015	8.5
Deronectes striatus Lec.	6	34.0	1.0030	7.5
Deronectes striatus Lec.	22	34.5	1.0014	7.8
Agabus griseipennis Lec.	23[a]	36.0	1.0021	8.5
Agabus griseipennis Lec.	16	39.0	1.0015	8.5
Rhantus sp. near *divisus* Aubé	16	39.0	1.0015	8.5
Dytiscus marginicollis Lec.	23	30.0	1.0008	8.0
Cybister sp.	21	31.6	1.0020	8.1
GYRINIDÆ				
Gyrinus maculiventris Lec.	28	33.0	1.0009	9.4
HYDROPHILIDÆ				
Ochthebius bruesi Darlington	24	38.8	1.0020	9.6
Ochthebius interruptus Lec.	26	32.5	1.0018	7.9
Helophorus lineatus Say	23	30.0	1.0008	8.0
Helophorus lineatus Say	23[a]	36.0	1.0021	8.5
Berosus infuscatus Lec.	23[a]	36.0	1.0021	8.5
Hydrophilus triangularis Say	16	39.0	1.0015	8.5
Tropisternus dorsalis Brullé	16	35.8–39.0	1.0015	8.5
Tropisternus dorsalis Brullé	19	32.2	1.0012	7.5–7.7
Tropisternus dorsalis Brullé	20	36.3	1.0014	8.6
Tropisternus dorsalis Brullé	23	30.0	1.0008	8.0
Tropisternus ellipticus Lec.	6	34–38.5	1.0030	7.5
Tropisternus ellipticus Lec.	16	35.8	1.0015	8.5
Tropisternus californicus Lec.	22	34.5	1.0014	7.8

List of Aquatic Coleoptera—continued

	Spring No.	Temperature °C	Specific Gravity	pH
Hydrobius fuscipes Linn.	23[a]	36.0	1.0021	8.5
Paracymus subcupreus Say[1]	9	44.5	1.0012	8.5
Paracymus subcupreus Say[1]	19	31.8	1.0012	7.5–7.7
Paracymus subcupreus Say[1]	28	——	1.0009	9.4
Philhydrus hamiltoni Horn	5	43.0	1.0012	8.5
Philhydrus hamiltoni Horn	9	45.5	1.0012	8.5
Philhydrus hamiltoni Horn	11	——	1.0016	5.7–6.5
Philhydrus hamiltoni Horn	12	30–35	——	5.1
Philhydrus hamiltoni Horn	28	40–41.5	1.0009	9.4
Philhydrus nebulosus Say	6	——	1.0030	7.5
Philhydrus nebulosus Say	8	33.0	1.0030	8.0
Philhydrus nebulosus Say	13	28.0	1.0021	6.7
Philhydrus nebulosus Say	14	40.2	1.0019	7.6
Philhydrus nebulosus Say	15	35.2	1.0000	8.4
Philhydrus nebulosus Say	19	31.8	1.0012	7.5–7.7
Philhydrus nebulosus Say	20	38.0	1.0014	8.6
Philhydrus nebulosus Say	23[a]	36.0	1.0021	8.5
Philhydrus nebulosus Say	24	40.5	1.0020	9.6
Philhydrus nebulosus Say	27	40.0	1.0017	7.1
Philhydrus nebulosus Say	28	35.6–38.2	1.0009	9.4
Laccobis agilis Rand.	5	43.0	1.0012	8.5
Laccobis agilis Rand.	10	38.5	1.0014	6.7
Laccobis agilis Rand.	11	——	1.0016	5.7–6.5
Laccobis agilis Rand	12	30–35	——	5.1
Laccobis agilis Rand.	22	34.5	1.0014	7.8
Laccobis agilis Rand.	24	38.8	1.0020	9.6
Laccobis agilis Rand.	27	40.0	1.0017	7.1
Laccobis agilis Rand.	28	38.0–39.5	1.0009	9.4
Laccobis agilis Rand.	33	35.8	1.0021	7.4
Laccobis agilis Rand.	34	40.2	1.0024	9.0
HELMIDÆ				
Helmis similis Horn	15	32.5	1.0000	8.4
Helmis thermarum Darlington	15	32.5	1.0000	8.4
HETEROCERIDÆ				
Heterocerus brunneus Mels.	27	40.0	1.0017	7.1
Heterocerus collaris Kies.	28	——	1.0009	9.4

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[1] I am able to record the occurrence of this species also from water at 45.7° in a hot spring northeast of Bear Lake in southeastern Idaho. These specimens were given me by Mr. A. C. Burrill who collected them, and identified by Mr. H. C. Fall.

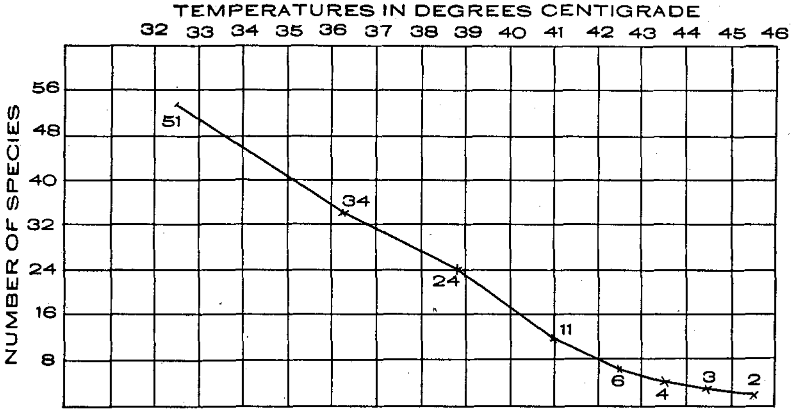

FIGURE 3. Graph illustrating the number of species of water-beetles collected in hot springs at temperatures from 32° to 46°, showing the gradual elimination of species and reduction in the fauna as the temperature increases.

such as climate, soil, altitude, composition of water, etc., and the varied types of springs, streams and pools in which the species were living. It demonstrates very clearly that temperatures above 30° or thereabouts offer a great barrier to colonization by water-beetles and that there is a successive elimination of the less highly adaptable species as the temperature rises. Most of the elimination seems to take place, at least in the western desert regions studies, between 30° and 45°.

Only five species were taken in water above 40°. They are *Bidessus affinis, Paracymus subcupreus, Philhydrus hamiltoni, Philhydrus nebulosus* and *Laccobius agilis,* and all except the Paracymus were found a number of times in water of lower temperature.

The highest temperature of 45.7° was noted in the case of *Paracymus subcupreus.* This is not so high as a few others reported from various other springs in different parts of the world, but apparently no beetles have ever been found living in water above 46°, except in some records that are possibly slightly in error.

The larvæ of a number of species of water-beetles of the families Dytiscidæ and Hydrophilade were obtained. I have been able to identify these to the genera with the aid of the excellent keys contained in Wilson's recent paper (Wilson '23) and they are listed below.

	Hot Spring No.	Temperature	Specific Gravity	pH
Hydroporus sp.	20	38.0°	1.0014	8.6
Agabus sp.	16	35.8°	1.0015	8.5
Agabus sp.	23[a]	36.0°	1.0021	8.5
Agabus sp.	28	39.5°	1.0009	9.4
Tropisternus sp.	5	35.43°	1.0012	8.5
Tropisternus sp.	6	35.°	1.0030	7.5
Tropisternus sp.	20	38.°	1.0014	8.6
Tropisternus sp.	26	32.5°	1.0018	7.9
Philhydrus sp.	27	40.0°	1.0017	7.1

It will be noted that the temperatures listed for the larvæ are considerably lower on the average than those at which adults of several species were found. It is easily possible, of course, for the adult beetles to enter water temporarily and leave it later, and it appears that some of the records for adults may be in water hotter than that in which larvæ can develop. As the number of records for the larvæ is rather small, it is however not possible to be positive on this point.

In my previous paper ('24), I gathered together all the references known to me relating to the occurrence of Coleoptera in hot springs, but since that time have discovered a few that were overlooked as well as some that have been published since that time.

In a note published nearly fifty years ago, Tournier ('79) gave some notes on a species of Laccobius which he discovered in abundance in the hot baths at Baden and described as *L. thermarius* which has since been placed as a synonym of *L. gracilis*. This species was then regularly to be found in the baths at 28°, but came in with the water from the springs at said to be 47.5–49° which was later allowed to cool. As noted above and in my previous paper, other species of Laccobius have been since discovered in the thermal fauna both in Europe and America.

The series of Cœlambus known to occur in hot springs and brackish waters has been augmented by a species lately described by Wallis ('24) as *C. salinarius* which occurs so far as known only in an alkaline lake at Baldur, Manitoba, in Canada, and Benick ('26[a]) has reported the European *C. enneagrammus* and C. *lautus* from saline water.

Aside from the aquatic beetles there are also some riparian or subaquatic species which occur occasionally on the warm or hot soil about the springs. Most noticeable among these are tiger-beetles, Cicindela.

Several forms of these were commonly observed flying about over the damp soil in which their larvæ excavate their burrows. Two species are represented in the collections. The first, *C. longilabris* Say, var. *perviridis* Schp., was abundant in Lassen Park, and in Bumpass Hell we saw them about the borders of the hot springs in considerable numbers. The second, *C. carthagena* Dej., is much more widespread in the hot springs areas at lower altitudes and is represented by three varieties, *hæmorrhagica* Lec., *bisignata* Dokh. and *pacifica* Schp. The first was collected about hot springs, No. 5, 18, 20 and 31 and seen flying about many of the others. About hot spring no. 31 at Hot Springs, Utah, we found larval burrows in great abundance in the warm soil about the borders of the stream which flows from the spring. The burrows extend into the soft soil nearly to the ground water level and at the time of our visit (Aug. 23) were inhabited by larvæ in various stages of growth. The other two varieties, which lack almost or entirely the maculations on the elytra were seen much less commonly and collected only about Hot Spring No. 5, near Convict Lake, Cal.

The European *Cicindela littoralis*, var. *nemoralis* evidently displays, at times, a similar fondness for the soil about hot springs. Many years ago, Lucas ('79) found this species very abundant along the banks of small rivulets on the Island of Eubœa in the Ægean Sea where the overflow from hot saline springs was flowing into the sea.

A few Carabidæ were collected about the borders of the hot springs, but no attempt was made to obtain a representation of this fauna which does not appear to be in any sense a thermal one. In desert regions such as these a considerable part of the water is of thermal origin and search for moist soil must lead very generally to such situations.

The following is a list of the few species collected, with the localities:

Dyschirius varidens Fall, Goldfield, Nev. (Hot Spring No. 3.)
Bembidion breve Mots., Lassen Park, Cal. (Hot Spring No. 11.)
Bembidion epphipigerum Lec., Goldfield, Nev. (Hot Spring No. 3.)
Bembidion nigripes Kby., Wells, Nev. (Hot Spring No. 28.)
Bembidion tumidum Lec., Wells, Nev. (Hot Spring No. 28.)
Bembidion flavopictum Mots., Wells, Nev. (Hot Spring No. 28.)
Bembidion dubitans Lec., Wells, Nev. (Hot Spring No. 28.)
Tachys (*Tachyura*) sp. near *incurvus* Say., Wells, Nev. (Hot Spring No. 28.)
Platynus funebris Lec., Lassen Park, Cal. (Hot Spring No. 12.)

Since the publication of my earlier paper where attention was called to the similarity of the coleopterous fauna of thermal waters and that of brackish water, a very interesting and carefully considered paper has been published by Walsh ('25) on the maritime and brackish water beetles of the British Coast. In his account Walsh groups the species into two main and several subsidiary categories. As his list is so similar to the list of hot springs Coleoptera, I have reproduced it below.

1. Brackish water species
    - *Haliplus apicalis* Thom.
    - *Philhydrus bicolor* Fabr.
    - *Berosus spinosus* Stev.
    - *Ochthebius exaratus* Muls.
2. By no means restricted to brackish water, but often occurring in fresh water also.
    - a. Found indifferently in fresh and brackish water
        - *Cœlambus parallelogrammus* Ahr.
        - *Dytiscus circumflexus* Fabr.
        - *Hygrobia (Pelobius) tarda* Hbst.
        - *Enoclerus bicolor* Payk.
        - *Helophorus intermedius* Muls.
        - *Helophorus mulsanti* Rye.
        - also under certain circumstances, *Hydroporus palustris* Linn.
        - *Agabus bipustulatus* Linn.
        - *Haliplus ruficollis* Deg.
        - *Hydrobius fuscipes* Linn.
        - *Helophorus viridicollis* Steph.
    - b. Species (in Great Britain) almost entirely restricted to brackish water, but on the continent occurring in fresh water also.
        - *Laccophilus variegatus* Germ.
        - *Philhydrus maritimus* Thoms.[1]
    - c. Species restricted to brackish water throughout Europe.
        - *Ochthebius marinus* Payk.[1]
        - *Haliplus striatus* Sharp.

A comparison of this list which contains representatives of thirteen genera, with the present list of thermal water beetles and published records of the latter shows that all but two of the genera listed above occur in hot springs and one of these (Hygrobia (= *Pelobius*)) does not occur in North America. It will be seen from these lists that the composition of the thermal and saline faunas is extremely similar in the case of the several species of water beetles taken as a whole. One noticeable difference in the two series is the occurrence of Helmidæ in thermal springs, at least in the United States and their general absence in the saline environment. This may probably be accounted for however, as suggested to me by Mr. P. J. Darlington, by the fact that these beetles generally require freely running water and such conditions are practically absent in brackish water.

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[1] According to Thorpe ('27) these two species are at home in pools liable to a density of 1.035 or more.

Within the past several years numerous observations and experiments have been made upon the effect of heat upon beetles of non-aquatic habits which are of considerable interest in relation to the fauna of thermal springs.

The larva of the scarabæid beetle, *Cotinis nitida* which lives in burrows excavated in the soil has been found by Miss Hintze ('25) to select soil of high temperature in preference to that which is cooler. Thus she found that larvæ of this species, when given a choice, selected soil at a temperature of 39–40° C., leaving almost without exception places where the temperature was noticeably lower.

Snyder and St. George ('24) have recently determined the temperatures fatal to the lyctid beetle, *Lyctus planicollis* Lec. As these determinations were made by heating the insects in large pieces of wood for a brief time, the exact temperatures of the insects themselves were not known, although doubtless noticeably below that of the chamber in which they were heated. They found that a half hour's exposure at 120° F. (49° C.) did not kill all of the larvæ but that, as might be expected without question, an additional half-hour at 130° F. (59° C.), resulted in killing all of the insects.

Welch ('24) finds that the lethal temperature for adults and larvæ of the furniture beetle, *Anobium striatum* is between 45° and 47°. An exposure of one minute at 45° proves fatal to the majority of individuals and none are able to withstand 47° for the same length of time in moist air.

Many Coleoptera develop in the bark or wood of trees, and on account of the greater amount and variety of food materials in the layers just beneath the bark, a large percentage of them lie in what may be termed the subcortical layer. When the prostrate logs in which many of the species occur are exposed to the rays of the sun they become greatly heated and the temperatures to which these forms are subjected frequently become excessive. Craighead ('21) and Graham ('21; '22) have investigated the conditions which prevail in this subcortical layer and find that the temperature may at times exceed 60° C. which is of course fatal to all insect life. Observations on a number of species living beneath bark show that 48° is in general the average temperature which is fatal to the majority, but that there is considerable variation among different species and probably also a much less variation between individuals of the same species. Graham states that adults of the buprestid beetle, *Chrysobothris dentipes* behave normally up to 51°, but above this enter a state of coma or estivation and succumb at about 52°. This larva he believes to be able to withstand even greater heat than the imagines. A bark-beetle, *Pityokeines sparsus* is more readily affected by heat, becoming inactive above 43° and dying at 47°.

Either of these temperatures (47° and 52°) is rather high, especially the latter, and certainly suggests that some acclimatization had taken place among the members of the subcortical fauna, especially in Chrysobothris which like most Buprestidæ is notoriously heliophilic in the adult state. In the case of most of these insects there is a considerable amount of moisture present beneath the bark so that conditions here compare more closely with the aquatic environment than they do where the atmospheric humidity is low and where the effect of dessication induced by heat must be taken into consideration.

Graham has later ('24) extended his work on the lethal temperatures for three species of beetles whose larvæ develop beneath the bark of logs. These are the large cerambycid, *Monochamus scutellatus*, a bark-beetle, *Ips pini* and a buprestid, *Chrysobothris dentipes*. Among these there is considerable variation as the lethal temperature was found to vary from 43° to 52°, with an average of 47°–50°. In each of the three species he found a range of about five to seven degrees between the point at which the more susceptible and more hardy individuals succumbed, so that if we make a reasonable allowance for the probability of error necessarily attendant upon such experiments there remains a sufficient individual variation to permit of a selection of individuals. We may thus expect to find an increased tolerance of heat in those species which live beneath bark where the temperature may frequently exceed the maximum death temperature by some ten degrees. That selection has not been able to proceed far, is, however, only too evident for the maxima of 49°–52° are but little above the tolerance of species which have had no opportunities for selection nor acclimatization. They are, on the other hand, far above those tolerated by animals which live in a medium that is always far below such temperatures.

ORDER TRICHOPTERA

In my earlier account, mention is made of a New Zealand caddis fly which lives in the sea, inhabiting rock pools. The habits of this species were described by Hudson ('04) and in his recent book Tillyard

('26) gives figures of the imago and larval case of this remarkable insect, *Philaniscus plebejus* Walk. which is a member of the family Sericostomatidæ.

ORDER DIPTERA

This order is better represented than any other in the thermal fauna and in my present collections six families are represented, including the Culcidæ, Chironomidæ, Stratiomyidæ, Tabanidæ, Syrphidæ and Ephydridæ.

CULICIDÆ

Larvæ of mosquitoes are usually to be found in thermal water, but so far as my experience extends, never at very high temperatures. The present collections contain three species.

Culex tarsalis Coquillett

Larvæ of this species are represented in the collections from six localities, as follows:

Locality	Hot Spring No.	Temperature	Specific Gravity	pH
Hot Spring, N. Mex.	1	39.0°	1.0039	8.1
Convict Lake, Calif.	5	35.2°	1.0012	8.8
Minden, Nevada	7	30.0°	1.0000	8.8
Battle Mt., Nevada	23	36.0°	1.0008	8.0
Battle Mt., Nevada	23^a	36.0°	1.0021	8.5
Wells, Nevada	26	34° -35.2°	1.0018	7.9
Ruby Valley, Nev.	28	35.6°	1.0009	9.4

This species has been reported twice previously from thermal springs in Yellowstone Park by Dyar ('23) and Brues ('24) and is undoubtedly one which readily adapts itself to such conditions. It is clearly a species restricted to alkaline water, however, as no specimens were taken in water of a lower pH than 7.9. Some of the records show that it develops in rather warm water also as one series was taken at 39.0°.

Aedes stimulans Walker

A single larva, apparently of this species, was obtained in Hot Spring No. 31 at Hot Springs Station, Utah in water of 35–35.3°. The specific gravity of the water in this spring is rather high (1.0146) and the pH 8.2.

Culiseta incidens Thomson

This common and widespread mosquito breeds in a variety of situations and has been reported before from thermal waters.

In 1927 we collected a single larva in Hot Spring No. 26[a] near Wells, Nevada in water at a temperature of 33.3° C., with a hydrogen ion concentration of 6.9 and charged with hydrogen sulphide. This water was not at all saline (specific gravity 1.0015).

A previous record of the occurrence of this species in thermal waters is by Dyar, who found the larvæ breeding in pools charged with hydrogen sulphide at Banff, Alberta. The same observer ('23) found the larvæ of the related *C. inornata* Williston in Yellowstone Park, breeding in "extinct hot springs, which have grown cold and dirty, with slight emission of gas." *C. incidens* has been found breeding in highly brackish water also by Hearle ('26). In this case the dirty salt pools where he found the larvæ had a density of somewhat more than half that of sea water. It would appear unquestionably to be a species which requires a low hydrogen ion concentration.

Aedes dorsalis Meigen

This species was present in almost incredible numbers at Wendover, Utah, where their presence made a study of the springs practically impossible. This is Hot Spring No. 29, of moderate temperature, 27–28°. The specific gravity is 1.0041, less than might be expected at the edge of the Great Salt Desert. This species was undoubtedly breeding there as there is no other water within many miles. It develops regularly in alkaline and saline waters as described in my previous paper ('24, p. 394).

Chironomidæ

A considerable number of larvæ and two pupæ of a species of Culicoides or some closely related genus, probably Dasyhela, were collected at Beowawe, Nevada in Hot Spring no. 24. These were taken in the overflow from one of the geysers in water at a temperature of 39.6° and the pupæ at 38.8°.

The pupæ are very similar to those of *Dasyhela longipalpis* Kieff and undoubtedly the species is closely related. The respiratory horns on the thorax are about two-fifths as long as the body, with thickened, slightly capitate tips and are finely transversely striated, due to the presence of an encircling tænidium; each bears about 21 minute stigmal openings which appear as minute black dots forming a series on the upper side of the tube. The anal protuberances are very acute and are directed laterally, perpendicular to the median line, each slightly longer than one-half the length of the last abdominal segment. The pupæ are dark reddish brown, with light yellowish, black tipped respiratory tubes.

This seems to be the first recorded instance of members of this genus in thermal waters, so far as I am aware, but one European species, *D. halophila* Kieffer has been reared from salt water (Rohde '12; Thienemann '26). Many species of Culcoides develop in fresh water and others occur in brackish water, and according to Rieth ('15) records of this genus in brackish or salt water date back to an account published in 1771 by O. F. Müller. The water from which the present specimens come showed a specific gravity of 1.0020 and hydrogen ion concentration of 9.6. In his recent paper ('26), Edwards mentions specifically two species of Culicoides *C. morium* Lutz and *maculithorax* Williston bred from the brackish water of mangrove swamps in the American tropics, and Thorpe ('27^a) finds that *C. nubeculosus* breeds in pools along the British coast below high-water mark. Likewise Rieth has described several tropical brackish water species (*C. marium, C. insignis,* etc.).

The first record of a species of this group in alkaline water is cited in my earlier paper. Packard ('71) reported a Tanypus, which was referred to Ceratopogon, *sens lat.* by Johannsen, from an alkaline pond near Clear Lake, California.

Tanypus monilis Linn or a related species. (Fig. 4; 2, 3)

The larva of *Tanypus monilis* has been described and figured by Johannsen ('05, p. 136 and plate 19, fig. 14) and again by Malloch ('15, p. 375 and plates 24, fig. 25, and fig. 7).

The specimens before me agree in most essential particulars with the characters given for this European species which is recorded from the eastern United States and from as far west as South Dakota. They are red in life however and not yellow with brown marking as described by Johannsen. I have figured the labium and antennæ and the entire larva.

The localities are as follows: Goldfield, Nevada (Hot Spring No. 3), in water at 35.4°; sp. g. 1.0022; pH 8.0; Convict Lake, California (Hot Spring No. 5) in water below 35°; sp. g. 1.0012; pH 8.8.

Tanypus sp. (Fig. 4; 4, 6)

Larvæ of a species of Tanypus quite different from the preceding were taken at Hot Springs, New Mexico (Hot Spring No. 1) in water at 39.5°; sp. g. 1.0039; pH 8.1. In the form of the labium it resembles

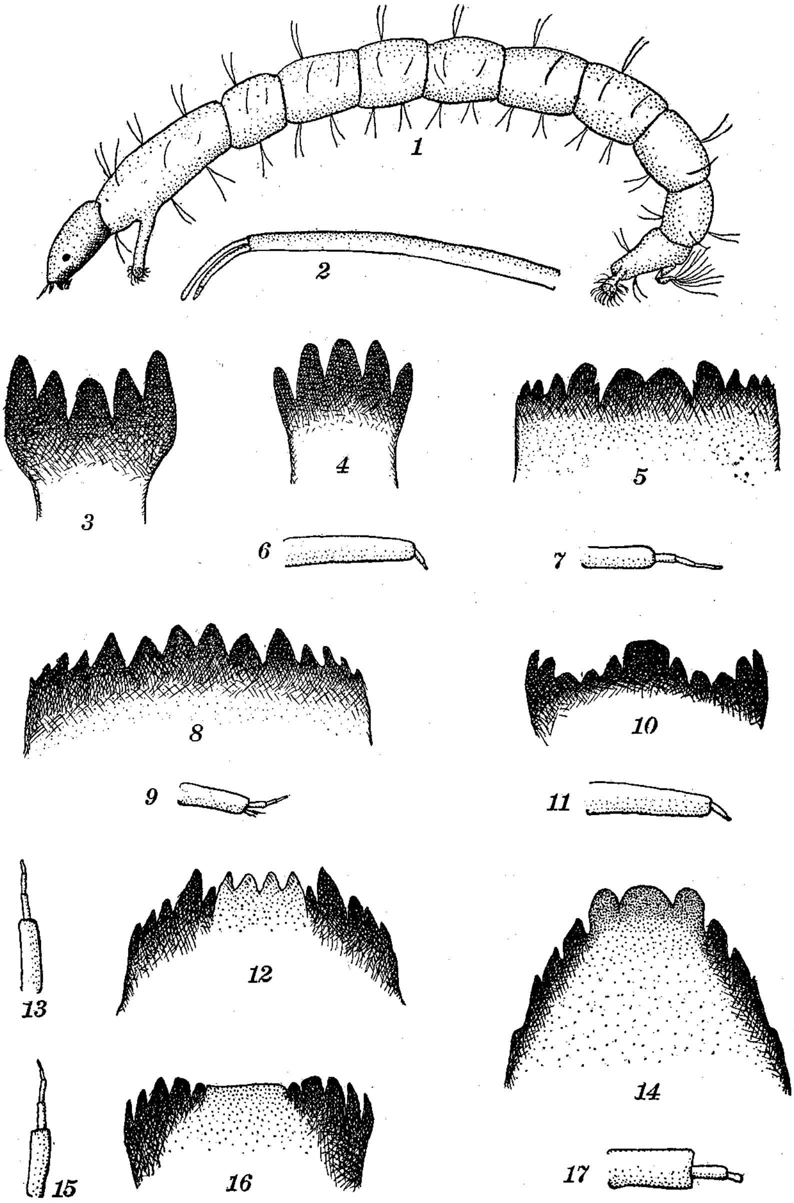

FIGURE 4. Details of larvæ of Chironomidæ. 1. *Tanypus monilis*, or related species, lateral view of larva; 2, antenna of same; 3, labium of same; 4, Tanypus sp., labium; 5, Chironomus, sp. no. 1, labium; 6, Tanypus sp., antenna; 7, Chironomus sp. no. 1, antenna; 8, Chironomus sp. no. 2, labium; 9, antennae of same; 10, Chironomus? sp. no. 3, labium; 11, antenna of same; 14, Cricotopus sp., labium; 15, Orthocladius sp., antenna; 16, labium of same; 17, Cricotopus sp., antenna.

somewhat Johannsen's figure of *Tanypus carneus* Fabr. ('05, pl. 20, fig. 6), but the antennae of the present specimens are much shorter and the head scarcely twice as wide as long. I have figured the labium and antenna.

Chironomus tentans Fabr.

One series of larvæ from Convict Lake, California, (Hot Spring No. 5) taken in water at 35° appear to be this species without reasonable doubt. They are identical with a form that I collected several years ago in thermal water in Yellowstone Park, except that there are only two pairs of ventral blood-gills instead of three.

The larvæ of *C. tentans* normally have two, but those which I figured ('24) as doubtfully this species from Yellowstone Park have an additional smaller third pair anterior to the two larger ones. Whether all represent a single species it is impossible to say.

Chironomus sp. No. 1. (Fig. 4; 5, 7)

A series of larvæ taken at Hot Spring no. 28 in the Ruby Valley, 65 miles south of Wells, Nevada, represent probably a species of Chironomus. They were taken in water at a temperature of 42°.

The mentum (Fig. 5) is quite unlike that of any American species of which the larvæ are known, but from its general form suggests the genus Chironomus. The eyes are completely divided into two parts, the anterior prolegs quite short and with pale yellow bristles. The two median teeth of the labium are usually quite distinctly paler than those on each side.

Chironomus sp. No. 2. (Fig. 4; 8, 9)

A single larva taken in Hot Spring no. 12, near Brokeoff Mountain in Lassen National Park, California resembles that of *Chironomus flavus* Johannsen but is undoubtedly not that species. The labial dentition (Fig. 8) is similar in general type, but by no means identical. The antennæ (Fig. 9) are about as long as the mandibles.

Chironomus sp. No. 3. (Fig. 4; 10)

A single larva from Beowawe, Nevada (taken in Hot Spring No. 24 at 39.5°) I cannot place with any degree of certainty. The labium is of peculiar shape (Fig. 16) and quite different from that of any species that seems to have been described.

The entire thorax and abdomen are thinly clothed with sparse, long, very delicate hairs. These are more abundant on the abdomen and on the thorax are intermixed with a small number of shorter, much stiffer hairs or bristles. The dorsal tufts of setæ on the ninth abdominal segment are attached to unusually long protuberances, fully three times as long as thick.

Chironomus sp. No. 4. (Fig. 4; 12, 13)

Larvæ of this species of blood-worm which appears to be a species of Chironomus were taken first at Hot Springs, New Mexico and later at seven other localities.

The labium of this form (Fig. 12) resembles that of certain Chironomus like *C. digitatus* in general structure and it probably belongs to this genus.

Locality	Hot Spring No.	Temperature	Specific Gravity	pH
Hot Springs, New Mexico	1	38.7°	1.0039	8.1
Goldfield, Nevada	3	37.3°	1.0022	8.0
Convict Lake, Calif.	5	43.0°	1.0012	8.5
Steamboat Springs, Nevada	8	33.0°	1.0030	8.0
Beowawe, Nevada	24	40.5°	1.0020	9.6
Wells, Nevada	26	32.5°	1.0018	7.9
Wells, Nevada	27	40.0°	1.0017	7.1
Ruby Valley, Nevada	28	40.0°	1.0009	9.4

Orthocladius sp. (Fig. 1; 15, 17)

A series of larvæ taken at Steamboat Springs, Nevada, in Hot Spring No. 8 at a temperature of 33° are apparently a species of Orthocladius. The labium is similar in form to that of an undetermined species from Illinois figured by Malloch ('15) as Orthocladius sp. B.

Species of this genus are known in Europe to occur in the sea and in brackish water along the sea coast Theinemann ('15), Kieffer ('00), Potthast ('15), Mercier ('23) and Edwards ('26).

Cricotopus sp. (Fig. 4; 14, 17)

A considerable series of larvæ and pupæ from Beowawe, Nevada are more or less like *Cricotopus varipes* Coq. as described by Johannsen ('05, p. 256, plate 25, fig. 22) and may even be this species which is known from the eastern United States and the State of Washington. They were taken in water at 38.8° in the overflow from one of the small geysers of Hot Spring no. 24. This water is strongly alkaline, with a pH of 9.6 or above and specific gravity of 1.0020.

The larvæ are 5–6 mm. in length with very dark head, the antennæ as figured (Fig. 17) and the labium (Fig. 14) with three central teeth that are quite distinctly paler than the lateral ones which are four in number on each side. The three thoracic segments are much lighter than the others (red in life and pale yellow after preservation) and the abdomen of quite dark grayish color. The anal gills are unusually small. Thoracic prolegs very short and stout, densely clothed with unusually dark spines; anal prolegs with the spines mostly dark in color.

The labium of the larva is very similar to that of a form figured by Malloch ('15, plate 29, fig. 21) as Orthocladius, but as Malloch did not rear the adult, the generic reference is doubtful.

There are several representatives of this genus known from brackish or salt water in Europe (*cf.* Thienemann, '15).

A complete discussion of marine Chironomidæ has very recently been published by Edwards ('26) in which he adds several new species in the genera Clunio and Tanytarsus and describes a most remarkable genus, Pontomyia from Samoa. In the latter the male has the wings reduced in size and the legs modified for swimming under water while the female retains a pupiform condition, without wings, mouthparts, thoracic segmentation or antennæ. Provided with only four vestigial legs she is incapable of movement. This truly marine insect was discovered by Buxton who has given an account of it as "the only known insect which is submarine in all stages" ('26), together with some interesting remarks on the colonization of the sea by insects.

A comparison of the effects of salt water upon the larvæ of the common house mosquito, *Culex pipiens* and an unidentified Chironomus larva has been undertaken by Koch and Gofferje ('20) and they find that in these fresh water species that the Chironomous larvæ withstand a salinity four or five times as great as the Culex larvæ and moreover that this relation exists with reference to both the chlorides and sulphates of sodium, potassium, magnesium and calcium. Their observations thus show conclusively that the reason for the migration of Chironomidæ into the sea and alkaline waters depends upon a primary difference in reaction toward liquids of high osmotic pressure, rather than a greater adaptability for gradual acclimatization to a saline medium. Furthermore, the same students have found that several other fresh water invertebrates, Daphnia Cyclops, and Chloë (a mayfly) are affected by salinity in approximately the same way as *Culex pipiens* which shows that the Chironomus type is unusually resistant in comparison to some other fresh water types of crustaceans and insects.

STRATIOMYIIDÆ

Fourteen lots of larvæ, representing nine species were obtained in ten of thirty-four hot spring areas from which we obtained collections. In most cases only a single species was obtained at each place although one (Hot Spring No. 10) yielded four species all belonging to different genera.

In the accompanying table the several series are listed and it will be seen that stratiomyiid larvæ were found under the most varied conditions of temperature, ranging from 28° to 50°.

Hot Spring	Temperature	S. G.	pH	Name
No. 1	38.7°	1.0039	8.1	Odontomyia sp. no. 1.
No. 4.	39–46.7°	1.0212	8.3	Odontomyia sp. no. 2.
No. 5.	35°	1.0012	8.5	Stratiomyia No. 1.
No. 8.	33°	1.0030	8.0	Odontomyia No. 4.
No. 10.	43–47°	1.0014	7.3	Odontomyia No. 5.
No. 10.	43–47°	1.0014	7.3	Stratiomyia No. 2.
No. 10.	43–47°	1.0014	7.3	Oxycera sp.
No. 10.	43–47°	1.0014	7.3	Nemotelus sp.
No. 11.	30–40°	1.0016	5.7–6.5	Stratiomyia No. 1.
No. 13.	28°	1.0021	6.7	Stratiomyia No. 1.
No. 18.	49–50°	1.0008	8.1	Odontomyia No. 3.
No. 20.	38°	1.0014	8.6	Odontomyia No. 4.
No. 22.	38°	1.0014	7.8	Odontomyia No. 2.
No. 22.	38°	1.0014	7.8	Odontomyia No. 4.

Only one species was found at the upper limit, but all the genera and many of the species were represented in one or more collections made in water at 40° of above. A considerable variety of forms were found at from 43–47° which would seem to be about the upper range of tolerance as only one species exceeded this.

As far as the specific gravity of the water is concerned, there is a considerable range of salinity, from 1.0012 in Hot Spring No. 5 to 1.0212 in Hot Spring No. 4 which is highly saline. This is not surprising, however, since several stratiomyiids are known to develop in brackish water or even in the sea (*vide* Brues, '24, p. 398).

With reference to the acidity or alkalinity of the water a considerable range was observed in the habitats of the species which we obtained. Thus while most of the series were in water of rather high hydrogen ion concentration (7.8–8.6), several were collected in more acid water with a pH ranging from 7.3 to 6.5–5.7. In the series of pools in Hot Spring No. 11, the pH ranged from 6.5–5.7, unfortunately our record does not indicate at which one the larvæ of Stratiomyia No. 1 were found. In comparison with the other data, it seems probable that they were in the more alkaline ones.

Stratiomyia sp. No. 1. (Fig. 5; 5.)

A large species, length 30–35 mm. Rather thinly hairy above and below, with the sides bare except for a patch of three bristly hairs on each segment at the lateral margin. Penultimate and antepenultimate segments unarmed, without marginal hooks. Dorsum of each body segment with a transverse row of four bristly hairs between the middle and apex and on the posterior half of the body; also with a single dorsal pair in front of the row; underside of segments with the row of four but lacking the more anterior pair. Terminal segment rather long, about four times as long as wide, quite noticeably tapered from a broad base to the middle, parallel beyond; above with a pair of bristles near the middle, set very close to the lateral margin and with another bristle at each posterior angle; below with a widely separated pair just behind the base of the anal slit, a pair behind the slit, another pair at apical fourth and a terminal pair; anal slit terminating before the middle of the segment. Body very dark, above with faint traces of narrow alternating stripes of pale and dark.

There are three series of this species. Near Convict Lake, California (Hot Spring No. 5) taken in water at 35° or below, specific gravity 1.0012 and pH 8.5; Bumpass Hell, Mount Lassen National Park, California (Hot Spring No. 11) in water at variable temperatures from 30°–40°, specific gravity 1.0016, pH 5.7–6.5 (varying with amount of H_2S); Drakesbad, Mount Lassen National Park, California (Hot Spring No. 13), in water at 28°, specific gravity 1.0021, pH 6.7.

Stratiomyia sp. No. 2

Rather small larvæ, 10–13 mm. in length. Sparsely hairy above and below; lateral protuberance of each segment with four or five slender, bristly hairs. No hooks on the penultimate or antepenultimate segment. Body hairs on segments confined, both above and below, to a central area on each segment. Terminal segment about three times as long as broad, the apex of the anal slit before the middle of the segment; above with a pair of bristles before the middle, placed one near each lateral margin; another pair beyond apical

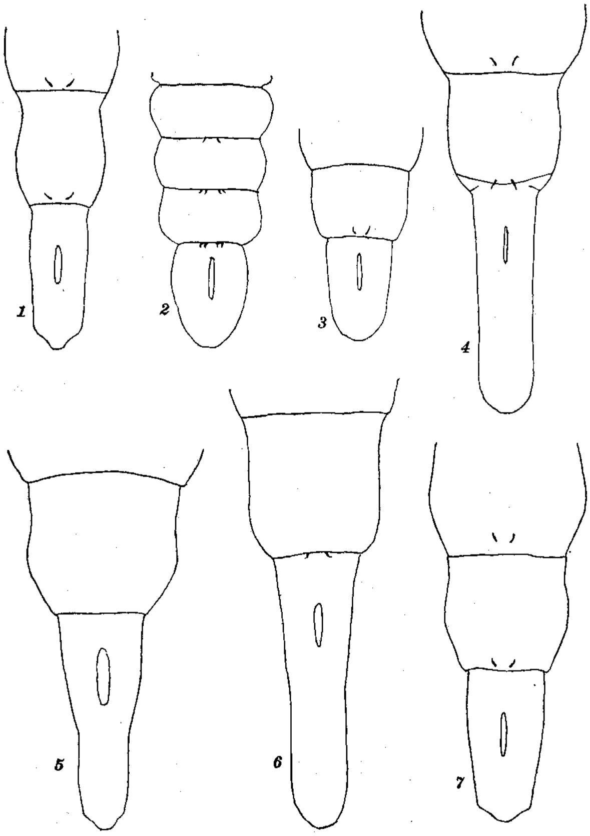

FIGURE 5. Posterior segments of body of larvæ of Odontomyia and Stratiomyia in ventral view. 1, Odontomyia sp. no. 2; 2, Odontomyia sp. no. 5; 3, Odontomyia sp. no. 3; 4, Odontomyia sp. no. 1; 5, Stratiomyia sp. no. 1; 6, Odontomyia sp. no. 4; 7, Odontomyia sp. no. 2.

third, well separated, and one at each posterior corner; below with a curved transverse row of four before the anal slit; a well separated pair beyond middle; another pair at apical third and a smaller subapical pair. Color dark gray above, with very obsolete striping of pale and dark indicated most clearly at the apices of the segments. Underside much paler.

Several specimens from Brokeoff mountain (Hot Spring No. 10) Mount Lassen National Park, California, in water at 43–47°, specific gravity 1.0014, pH 7.3.

Odontomyia sp. No. 1. (Fig. 5; 4)

A large species, the well grown larva 20–30 mm. in length. Penultimate and antepenultimate segments each with a pair of ventral hooks at the middle of the posterior edge. Terminal segment very long, about five times as long as wide (the segment is slightly widened in the figure, due to pressure in mounting); apex of anal slit at the basal third of the segment. Each segment with a rather dense patch of stiff hairs above and below, and laterally with a group of three having a very few finer hairs intermixed. Terminal segment with sparse, scattered bristly hairs; a pair at apical fourth above and a subterminal pair more widely separated behind these; a lateral bristle at middle; three pairs below, one near base, one at middle, and one at apical third. Body mottled with dark above, the markings forming seven more or less well defined longitudinal vittæ. Three specimens from Hot Springs, New Mexico (Hot Spring No. 1) at a temperature of 38.7° with specific gravity of 1.0039 and pH 8.1.

This larva will run to *Odontomyia cincta* in Johannsen's key, ('22) but the last segment is much more elongate and there is no distinct dorsal broad pale stripe while the lateral margin of the body is pale, not dark.

Odontomyia sp. No. 2. (Fig. 5; 1, 7)

A moderate sized species, the larger larvæ not over 20 mm. in length. Penultimate and antepenultimate segments each with a pair of hooks at the middle of the posterior edge. Terminal segment of moderate length, scarcely three times as long as broad. Apex of anal slit at, or just behind the middle of the segment. Body hairs on all the body segments long and rather sparse, confined to the anterior half of each segment on the fore part of the body; bristles near lateral margins of segments generally placed one in front and one behind, rather than in a group of three. Each segment below, especially the apical ones, with a distinct transverse series of four bristles behind near the middle. Apical segment with bristly hairs; a row of four in front of anal slit; one at each side of the middle of the slit; a transverse row of six just behind the slit; a subapical pair and one near each hind angle. Above, this segment bears a group of 8–10 at each side near base; several pairs on the disc basally; a median pair at basal fourth; a widely separated pair at apical fourth and one near each apical corner. Color of body quite dark, especially above, somewhat lighter laterally as the anterior corners of the segments are much paler. No distinct vittæ except for the broad darkening of the dorsum.

Numerous specimens, many of them not fully grown from Silver

Peak, Nevada (Hot Spring No. 4) in quite cool water (28°–30°) with a rather high specific gravity of 1.0212 and pH of 8.3. They were taken together with a species of Ephydra.

A second lot of many specimens is from the same place (Hot Spring No. 4) but from warm water ranging from 39.6°–46.7.°

A third lot from Brokeoff Mt., Mount Lassen National Park, California (Hot Spring No. 10) in water of 43°–47°, specific gravity 1.0014 and pH 7.3.

Another series from 26 miles south of Battle Mountain, Nevada (Hot Spring No. 22) may be the same species. The water of this spring is scarcely saline, specific gravity 1.0014, temperature 38° and pH lower (7.8).

Odontomyia sp. No. 3. (Fig. 5; 3)

Length about 15–18 mm. represented by two puparia not in a good state of preservation and not sufficiently perfect to describe in detail. The terminal segment is very short, scarcely twice as long as broad, with the apex of the anal slit just beyond the middle of the segment. Penultimate segment with a pair of hooks medially at the posterior margin; antepenultimate unarmed.

Golconda, Nevada (Hot Spring No. 18) in water at a temperature of 49°–50° (not alive and possibly killed by the heat of the water). Specific gravity of water 1.008 and pH 8.1.

These larvæ are much like those of *O. cincta* and *O. vertebrata* figured by Hart from Illinois ('95).

Odontomyia sp. No. 4. (Fig. 5; 6)

A large species, length from 25–35 mm. Penultimate segment with a pair of hooks at middle of posterior margin, antepenultimate unarmed. Terminal segment very long and slender, about five times as long as broad (somewhat broader in the figure as the mount has become flattened). Anal slit terminating well before the middle of the segment. Body quite densely clothed with bristly hairs which form a broad transverse band around the middle of each segment. Lateral margin of body with many bristly hairs, but none conspicuously larger than the others, except on the more anterior segments which bear one or two larger bristles. No rows of larger bristles on the segments above or below. Terminal segment with bristly hairs as follows: a very widely separated pair just before middle, a widely separated pair at apical fourth and one near hind angle; below with one at each side of anal slit; two pairs behind the middle, the posterior pair farther apart, a subapical pair placed near the lateral margin and one at each hind angle.

Color dark, more or less mottled dorsally and with a well-marked, narrow, median, dorsal stripe bordered by a narrow, whitish stripe on each side.

There are two series of this species; one from 29 miles south of Winnemucca, Nevada (Hot Spring No. 20) temperature 38.0°, specific gravity 1.0014, pH 8.6, and the other from 26 miles south of Battle Mountain, Nevada (Hot Spring No. 22), temperature 38.0°, specific gravity 1.0014, pH 7.8.

Another lot of larvæ from Steamboat Springs, Nevada (Hot Spring No. 8) may be the same. They are very similar, but have the terminal segment slightly shorter and broader. They came from water at 33°, specific gravity 1.0030 and hydrogen ion concentration 8.0.

Odontomyia sp. No. 5. (Fig. 5; 2)

Two very small and undoubtedly only partly grown larvæ measure 5–6 mm. in length but seem unquestionably to represent another species. The penultimate and antepenultimate segments each bear four stout hooks at the middle of the posterior margin below. They are about the size and form of Euparyphus, but the antennæ are clearly placed at the anterior angles of the head. Body nearly bare above, with only a few sparse and very minute hairs; below with a few much stouter, short, horn-like bristles that form a more or less transverse row of four or six on each of the five anterior body segments; segments behind these with a broad anterior patch of such bristles in addition. Four strong, thorn-like hooks at the middle of the posterior border of the penultimate and antepenultimate segments; also a pair on the preceding segment. Lateral edge of each body segment with several long, flexuous hairs. Terminal segment scarcely twice as long as broad, its apical fringe as long as the segment; dorsally with four bristles, one pair before middle and another before apex, these bristles placed quite close to the lateral margins. Beneath with a curved row of four before the anal slit; with apparently several pairs behind the anal slit and a subapical pair; anal slit long, its apex beyond the middle of the segment. Color dark above, not at all longitudinally striped although paler laterally; below somewhat lighter.

There are two specimens from Brokeoff Mountain, Mount Lassen National Park, California (Hot Spring No. 10) at a temperature of 43–47°, specific gravity 1.0014, pH 7.3.

Oxycera (?) sp. (Fig. 6)

Length 3–4 mm. Dark brownish above, mottled with paler, especially toward the sides; below uniformly much paler. Head with the antennæ set well behind the anterior angles of the head and about halfway between the lateral margin and the median line. Body surface shagreened as usual in the family, without hairs, except for a few stout glistening, white, delicate bristles arranged in a regular pattern. Above, the thoracic segments each bear a transverse row of four in front of the middle, extending only halfway to the sides; six basal abdominal segments each bearing a single pair placed behind the middle and nearer to the median line than to the lateral margin; penultimate segment with only a few minute bristles at apex. Below the thoracic segments bear a small number of bristly hairs, more numerous on the metathorax; each of the six basal abdominal segments bears a transverse row of small thorns or short hooked spines, usually six in number and behind these a row of about six small bristles, the armature of these segments extending only about halfway from the median line to the lateral margin; in addition along the lateral line each segment bears a single bristle; penultimate segment with two long hooks nearly as long as the segment. Terminal segment about as long as broad; middle of anal slit just before the middle of the segment. Apical bristles pectinated, scarcely as long as the segment.

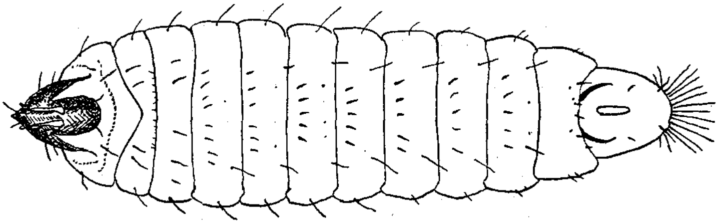

FIGURE 6. Larva of Oxycera sp., ventral view.

One series of larvæ from near Brokeoff Mt. (Hot Spring No. 10), Mount Lassen National Park, California, at a temperature of 43–47°, specific gravity 1.0014 and pH 7.3. They were taken together with Stratiomyia sp. No. 2.

These larvæ may belong to Euparyphus as they agree with the descriptions given by Johannsen of several North American species. On account of the very large hooks on the penultimate abdominal segment the species would seem to be close to the North American *E. brevicornis* Loew. They do not agree at all with figures of larvæ of the genus Oxycera given by Johannsen nor with the characters given in his table as Oxycera is said to have a terminal spiracular opening. However Lenz ('23) has described the larvæ of European species of Oxycera and these appear to be closely similar to the larva I have just described. So this is probably referable to Oxycera and not to Euparyphus. Lenz ('26) has since recorded two European species of Oxycera (*O. meigeni* and *O. trilineata*) from saline waters at Oldsloe in Holstein.

Nemotelus sp. (Fig. 7)

Length 4.5–5 mm. Body light colored above and below, but dorsally with some darker mottling which takes the form of a narrow transverse band on the second and third thoracic segments and the apical three or four abdominal ones. Those on the thorax near the middle of the segments and those on the abdomen subapical; in addition the abdominal segments each bear a pair of oblique marks which diverge from the anterior margin and together form a more or less triangular dark area with the center and posterior edge pale. Head narrow, more than twice as long as wide, with the antennæ placed well in front of the middle and considerably removed from the lateral margin. Body clothed with a number of large, isolated, bristly hairs and with some fine, recumbent pubescence. The head bears a number of short broad, more or less spatulate bristles, one in front of each antenna and three in a curved series on each side of the head; each body segment with a short curved bristle on the lateral margin, near the middle, or behind it on the posterior segments; below these on the ventral side is another, the same size and further inward, halfway

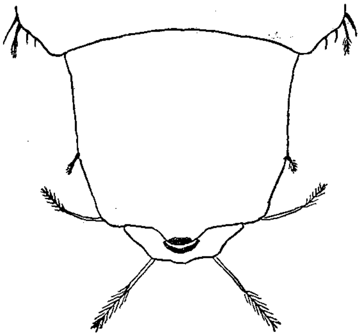

FIGURE 7. Larva of Nemotelus sp., posterior part of body

to the median line a much longer one, except on the last segment, which bears only several short ones on each side of the anal slit; each ventral segment with a transverse band of fine short pubescence. Dorsal surface apparently with a short, more or less bulbously enlarged bristle on each segment between the median line and lateral margin, but these are so obscured by a soft deposit not removable by acid that only a few are clearly to be made out. Terminal segment (fig. 7) with four long bristles at apex.

One series of larvæ were taken in Hot Spring, No. 10, near Brokeoff Mt., Mount Lassen National Park, California, at a temperature of 43–47°, specific gravity 1.0014, pH 7.3.

These are undoubtedly larvæ of a species of Nemotelus as they agree closely with the figure given by Johannsen ('22) of an European species described by Lundbeck ('07), and with a more recent description of the European *N. notatus* Zett. given by Lenz ('23).

There are at least two types of peculiar body bristles on these larvæ, those on the terminal segment and the other long ones on other parts of the body are plumose apically while many of the shorter ones are blunt apically, spatulate, dilated or almost bulbously enlarged.

To the earlier references on the occurrence of Stratiomyiidæ in thermal springs, there may be added another, contained in a brief, but interesting note by Lucas ('79). It relates to the larvæ of an undetermined species of Stratiomyia (or possibly an Odontomyia), which were obtained on the island of Eubœa off the eastern coast of Greece in hot saline springs at a temperature of 35°–40°.

Nemotelus uliginosus of Europe is known to develop in saline water whence it has been reported by Schmidt ('13).

TABANIDÆ

Six lots of larvæ belonging to this family were taken in hot springs at five different localities. The specimens vary greatly in size, but all are structurally alike and similarly colored and may be referred without question to *Tabanus punctifer* O.S. which is a very common and widely distributed species in this region. I have already recorded its occurrence in thermal springs in Yellowstone Park (Brues, '24) and it is known to breed generally in alkaline waters throughout the West (Webb and Wells '24). The adults were seen flying at a number of other places and at one of these near the hot springs at Silver Peak, Nevada, may breed in the thermal water. No larvæ were collected here however and there are cold pools and marshes near these springs from whence they may have come. Following is a transcript of the data relating to this species.

Locality	Hot Spring No.	Temperature	Specific Gravity	pH
Convict Lake, Cal.	5	43.0°	1.0012	8.5
Bridgeport, Cal.	6	36–38.5°	1.0030	7.5
Winnemucca, Nev.	20	38.0°	1.0014	8.6
Beowawe, Nev.	24	39.6°	1.0020	9.6
Hot Springs, Utah	31	35.3°	1.0146	8.2
Ogden Canyon, Utah	32	——	1.0063	8.3

There is a very considerable range in the temperatures at which this species was found, which is to be expected as it occurs abundantly in cold water. The range in specific gravity is quite considerable also, since two of the springs emit comparatively fresh water (1.0012–1.0145). The pH (8.2–9.6) is consistently high, except in one case where it is only 7.5.

No tabanid larvæ were found in the hot springs near Denio, Oregon or north of Winnemucca, but *Tabanus punctifer* is a very abundant species in the Antelope Valley where they have been found by Webb and Wells (*l.c.*) to develop together with *T. phænops* in Alkali Lake and the swampy area nearby.

SYRPHIDÆ

A single series of larvæ of a member of this family was taken in Hot Spring No. 26[a] near Wells, Nevada in water with a temperature of 33.3°, and specific gravity of 1.0015. The pH is very low, 6.9, and the water has a foul odor due to H_2S.

The larvæ are of the characteristic rat-tailed type of Eristalis and Helophilus and since they are very similar to those of *Eristalis* (*Lathryophthalmus*) *aëneus*, evidently belong to this genus. Members of this group live in the larval stages in foul water and no doubt the parent flies are attracted by the hydrogen sulphide odor which is very pronounced in this spring. As the body bristles are usually trifid, and only in part bifid, and as the anal gils have a larger number of lobes, these larvæ are probably not *E. aëneus*, and I have appended a description of the specimens before me.

Body about 9–10 mm. in length; width about 2 mm.; narrowed apical portion behind the anal gills somewhat longer, 12–15 mm.

The integument is very irregularly folded into transverse ridges which show here and there three small slender curved bristles usually arising from a small papilliform elevation. The bristles arise very close together, almost from a common base, and as they are generally trifid, form a stellate group, especially noticeable about the posterior end of the body. In some places the bristles are bifid or Y-shaped. The body anteriorly above at the sides bears numerous, closely placed, minute spines and below on each side of the anterior segment is a large convex papilla-like swelling similarly spined with hooked, claw-like spines intermixed. In a median cavity above there are the antennæ, each composed of a large, short, basal joint, a tubular second one about twice as long as thick and a minute rounded apical joint. Presumably the large first segment represents the prothorax as the spined swelling on each side below appears to be homologous with the prolegs on the six following segments. These prolegs are smaller on the next five segments, and each bears along its anterior edge a series of about eight stout hooked spines or thorns followed by a similar row of smaller ones behind which are two more or less irregular rows of much more minute and less clearly hooked spines. The prolegs of the last pair are connected by two rows of stout hooks and the arrangement of spines is reversed as well as their insertion, as these curve forward at their tips, while the others are curved backwards. Just behind the last prolegs there arise in a depression the anal blood gills each of which is provided with six digitate extensions. As in *Eristalis aëneus*, as figured by Metcalf, the posterior extension of the body is transversely wrinkled on the tapering basal portion, finely longitudinally aciculate, on the middle part while the posterior part appears to be transversely marked with extremely minute lines which represent the tænidia of the two tracheæ that extend through it. The fringed bristles of the apex are slightly damaged, but appear to be essentially as in *Eristalis aëneus.*

Rat-tailed larvæ of this type have several times been found in saline water or in the sea, and I have given several records in my earlier paper, but apparently this is the first record of their occurrence in thermal water. As there noted (*l.c.* p. 404) *Eristalis aëneus* has been bred from salt pools on the English coast.

The lethal temperature for at least one species of Syrphidæ has been determined in connection with the treatment of bulbs for the destruction of *Merodon equestris.* With this species it has been found (Mackie '27) that 43.3° (110° Fahr.) is sufficiently high to destroy the larvæ in a very short time.

EPHYDRIDÆ

Larvæ of Ephydra, possibly representing more than one species, were collected at several localities as follows:

Locality	Hot Spring No.	Temperature	Specific Gravity	pH
Silver Peak, Nev.	4	cool	1.0212	8.3
Convict Lake, Cal.	5[a]	43.0°	1.0012	8.5
Minden, Nev.	7	——	1.0000	9.6
Hot Springs Sta., Utah	31	35.3°	1.0416	8.2
Ogden Cañon, Utah	32	——	1.0063	8.3

These larvæ come from a variety of types of water and their occurrence in springs such as Nos. 5, 7 and 32 is quite unexpected since the species of Ephydra are usually restricted to water with a high degree of salinity. The water from springs Nos. 4 and 32 fulfills this expectation and possibly that from No. 32, but the springs at Minden and Convict Lake are not at all saline, in fact the former showed no salts in solution so far as could be detected with an hydrometer. At least one species of Ephydra, *E. subopaca* Lw of the eastern United States has been found by Ping ('21) to be incapable of development in fresh water. The optimum concentration of salt water for the European *Ephydra riparia* according to Thorpe ('27) is 1.026, or approximately that of sea-water.

It will be noted that all of the collections listed above came from water with a hydrogen ion concentration of 8.2 or over, and that in only one case were larvæ found in water of really high temperature (43° in Spring No. 5[a]).

In Iceland, Böving ('25) refers to the occurrence of an undetermined species of Ephydra in a hot water creek in Oräfa, at a temperature of "more than 40° C."

Some years ago Poynton ('04) gave an account of some dipterous larvæ which he found in great numbers in siliceous hot springs in New Zealand. These springs are near Lake Taupo and are known as black terraces since the flowing water deposits black sinter. In these basins the larvæ were living in masses of filamentous algæ, and Poynton noticed that the floating puparia after the emergence of the adult flies drift to the edge of the pools and serve as a basis for the deposit of the sinter which borders the pools and forms the terrace, thus the flies unwittingly aid in the development of the terraces through their discarded puparia. The larvæ are described as about half as large as those of a blow-fly, dark grayish green, and shaped curiously like a shark with its double lobed tail. The adult flies were identified for him by Hutton as *Opomyza* n. sp., but his description fits the larva of Ephydra so well, that there would seem to be little doubt that the insects belong to this or some closely related genus.

From the numerous records presented above, it is evident that Ephydra very frequently invades the hot spring environment.

MOLLUSCA

Snails were less abundantly represented in our collections than had been expected as only four species were obtained and only five localities yielded specimens. I am indebted to my friend, Mr. William J. Clench of the Museum of Comparative Zoology for the determinations.

Physa cooperi Tryon

Convict Lake, California, Hot Spring No. 5, temperature —°, specific gravity 1.0012, pH 8.5.

Physa smithiana Baker

Minden, Nevada, Hot Spring No. 7, temperature —°, specific gravity 1.0000, pH 9.6 or over.

Physa virginea Hald.

Locality	Hot Spring No.	Temperature	Specific Gravity	pH
Hot Springs, N. Mex.	1	38.7°	1.0039	8.1
Beatty, Nevada	2	38.2°	1.0016	7.2
Monroe, Utah	34	35.5°	1.0024	9.0

Lymnæa palustris desidiosa Say.

Minden, Nevada, Hot Spring No. 7, temperature —°, specific gravity 1.0000, pH 9.6 or over.

I am at a loss to account for the apparently erratic distribution of this series of molluscs as the several localities seem to have no characteristics in common. Since only five of the thirty-four hot springs visited afforded any specimens, however, it seems certain that there is some very good reason for their absence in the others. It will be seen that the specific gravities vary from fresh to 1.0039 and the hydrogen ion concentration from 7.2–9.6 (7.2–9.0 in the case of *Physa virginea*). These conditions do not, therefore, indicate barriers of a physical nature and there is good reason to believe that it cannot be an absence of lime in the other springs.

Very recently Clench ('26) has described *Physa johnsoni* from Hot Sulphur Springs at Banff, Alberta Canada. The temperature of the water in which this species was found is not recorded.

Henderson ('25) has rediscovered the Physa described by Cockerell ('89) as *Physa heterostropha cupreonitens* at the type locality near Wellsville, Colorado and from other hot springs near Nathrop, Colo., about 20 miles further up the Arkansas River found *Physa sayi warreniana* in warm water at a temperature not recorded.

Böving ('25) records small snails as plentiful in Oräfa, Iceland, in a hot water creek at a temperature of "more than 40° C." It is probable that these may have been a species of Limnæa.

VERTEBRATES

Fishes

There are very few references to the presence of fishes in hot springs and the accuracy of most of these is extremely doubtful. There is no need to repeat them here, as I have cited them in my previous paper.

We found fish in only one of the hot springs areas visited during the summer but the circumstances under which they occurred are very interesting. At Hot Springs, New Mexico (Hot Spring No. 1) there is a small stream which runs through a ditch that carries off the combined flow of several of the springs at this place. It flows rapidly and is of an almost perfectly uniform temperature of from 38.7–39.5°. Here, where the water averages 39.5°, we saw considerable numbers of a small cyprinid fish, *Notropis lutrensis* (Baird and Girard). The fish are regularly present in the stream as we were informed by some small boys who told us of their occurrence when they saw us collecting aquatic insects in the water at the head of the stream. The fish were actively swimming about and behaving in a perfectly normal manner. This water has a pH of 8.1. *Notropis lutrensis* is a widely distributed species inhabiting the basins of several rivers in the Southwestern states from South Dakota southward (Evermann and Kendall ('94) and Jordan and Evermann ('96) and is especially abundant in the Rio Grande basin.

The temperature recorded above agrees closely with the data.

relating to several European fishes and confirms my previous opinion that 40° represents a close approximation to the critical point of heat endurance for fishes. This is quite high, considering the fact that many forms undoubtedly succumb at temperatures between 30 and 40°.

Issel ('06) has reported *Gasterosticus aculeatus* at 29–30° in Italian hot springs and a variety of *Barbus callensis* (var. *figuigensis*) has been described by Pellegrin ('13) from artesian water at the Figuig Oasis in Morocco in water at 30°.

In several of the thermal springs of North Africa *Cyprinodon fasciatus* C. V. has been found in water as high as 40° or thereabouts. Cyprinodont fishes are quite commonly represented in saline or brackish water.

AMPHIBIA

A single frog was collected during the course of the summer. This has been very kindly identified for me by Mr. Arthur Loveridge as *Hyla regilla* Baird and Girard. It was taken in Hot Spring No. 16 at Denio, Oregon, but as the spring is about six miles south of the town it is really located in the extreme northern part of Nevada. The pool in question is surrounded by a small marshy area with coarse grass and has a temperature of 35.8°; specific gravity 1.0015 and pH 8.5. As this group of springs is separated by several miles of desert country from any non-thermal water the species undoubtedly breeds regularly in the warm springs.

The frog and the tadpoles of toads have been the object of a considerable amount of investigation with respect to the effect of high temperatures. As mentioned in my former paper ('24) they show a very definite reaction to temperatures at or near 40° C., which not only cause death but induce definite changes in the muscles and nervous tissues. At that time I overlooked an important paper by Cameron and Brownlee ('15) describing experiments of the same kind. They were able to confirm in the main the observations of the older investigators, showing that in the case of late winter and early spring individuals of *Rana pipiens* in Canada, the resistance to high temperatures was even less than has been found previously. In this case they found that an internal temperature of 39.5°, corresponding to 44° external temperature was completely fatal in 25–30 minutes. After exposures of an hour, 29°–30° internal temperature was fatal if the frogs were immersed in water but 32°–33° was necessary if they were in air saturated with water vapor for the same length of time. This difference appears to be constant and they cannot account for it. It may, however, I think be readily explained on the basis of a greater heating of the superficial tissues when immersed in water. They found also that temperatures of one or two degrees above 18° C. proved fatal within a few days. This is not in accordance with former observations on other frogs and the authors think that this may be due to the fact that the material studied had recently emerged from the hibernation period.

These observations accord very closely with those of Maurel and Lagriffe ('00); Vernon ('99) and the ones of Alcock ('03) who showed that 40° is fatal to frog nerve, and of Vincent and Lewis ('01) and Vrooman ('07) who found that temperatures of 38°–40° induce a heat rigor in frog muscle through a coagulation of proteins present in the muscle fibre during life. After such coagulation, recovery is of course impossible, so that the lethal temperature limit is a definite point which may vary only through a change in the coagulability of these proteins. That such variations exist seems probable in the light of the acclimatization phenomena and they may even be periodic with reference to the seasons of hibernation and activity.

Since my previous account appeared, I have seen another reference to the occurrence of batrachians in brackish water. Rose ('25), in a paper on the habits of some South African frogs, states that the Plathander frog (*Xenopus lævis*) can survive in brackish water, and he determined experimentally that a concentration of about 10 grams to the litre of water was sufficient to kill them after an immersion of four days.

Professor Pearse has also called my attention to a paper of his own (Pearse '11) which I had previously overlooked. This relates to a species of Rana in the Philippines which he found developing in the water of salt flats that contained sodium chloride at the rate of 14.8 grams per litre. Tadpoles were numerous in this medium which contained numbers of both young and nearly metamorphosed individuals.

The ability of a least one frog to withstand salt water was known to Darwin when he wrote the Origin of Species as he refers (6th edition, p. 414) to a species of frog from India which is not killed by salt water, in connection with his comments on the lack of batrachians in insular faunas.

PLANT LIFE

Bacteria

The occurrence of various algæ in thermal springs is so general that it has attracted the attention of numerous naturalists and I have given a brief outline of the observations made on these thermal plants in my earlier paper. These date from the latter part of the eighteenth century when their presence was noted in hot springs in Europe, to the more explicit and careful observations of recent botanists such as Setchell ('04) in the hot springs of Yellowstone Park, which I was able to confirm from studies in the same locality.

There is also a very extensive and, of course, more recent literature relating to the existence of a great variety of bacteria in hot springs. These are of very general biological interest because they represent a type of thermophilous microörganisms which have also been found to be very generally distributed in other environments and commonly under conditions which would not suggest that they were really thermophilous. These microörganisms are mainly bacteria, but include some yeasts and other fungi whose optimum temperatures for development are very noticeably higher than those of other types of living organisms. Their optima are indeed above or quite near the lethal temperatures for other bacteria and other types of animal and plant life in general.

Our knowledge of this group of organisms begins with papers by Van Tiegham ('81), Globig ('87) and a study by Miquel ('88) who isolated and described *Bacillus thermophilous* which he found to be quite generally distributed in sewage, other water, in the air and in the alimentary canal of man and other animals, and it has since been found in soil. This organism grows at temperatures between 42° and 70°, but its optimum lies between 65° and 70°. Its discoverer was unable to account for its presence in places where it must develop very slowly at temperatures below those at which he was able to cultivate it. Recognizing that this organism multiplied and flourished at temperatures which coagulate protoplasm, Miquel quite naturally ascribed to it some peculiar type of protoplasm which was not affected by temperatures fatal to all other forms of life, except the spores of bacteria which were known at that time to survive heat far above the coagulation point of the protoplasm of growing organisms.

A very considerable flora of similar bacteria was made known through the researches of Cohn ('93), Mac Fayden ('94), Miss Tsiklinsky ('03[a]), Miehe ('07; '19), Kruyff ('10), Nègre ('13[a]) and other recent workers.

These show in general the same relations to temperature; their optimum range for growth lies well above that for normal animals and plants and the growth of some species is not inhibited by temperatures between 70° and 80°.

Similarly, the bacterial flora of hot springs includes a variety of forms which behave in the same way and are in some cases identical. These have been obtained in numerous localities and studied by a series of bacteriologists; Certes and Garrigon ('86), Karlinski ('95), Teich ('96), Spallicci ('97), Myioshi ('97[a]) ('97[b]), Tsiklinsky ('02 and '03), Falcioni ('07), Nègre (13[b]), *et al.*

In at least one case they have been found alive in water at 83° and the presence of bacterial growths in pools in Yellowstone Park as high as 89° indicates that some species are able to develop at this temperature. Schmidt and Weis ('02) also refer to a blue green alga, (p. 144), living in water above 90°. On the other hand the bacteria found in hot springs of comparatively low temperature are not necessarily thermophilous forms as some of them are restricted to the lower temperatures at which animal life occurs.

The wide distribution of these thermophilous bacteria precludes any assumption that they are the results of acclimatization to excessively thermal environments since such high temperatures occur consistently in nature only in hot springs or in the fumaroles and hot soil associated with volcanic phenomena. Miehe and others have shown that fermenting hay and the dung of animals when greatly heated in the process of fermentation develop an actively growing flora of thermophilous organisms.

Aside from their marked ability to withstand excessively high temperatures or even to multiply under conditions, some thermal bacteria seem to require an unusual amount of heat for their existence. Three species of Bacterium, *B. ludwigii, B. calfactor* and *B. ilidzensis capsulatus* which were isolated by Karlinski ('95) from thermal springs at Ilidze in Bosnia, occur normally in water as hot as 80°.

Of these, *Bacterium ludwigii* was found by its discoverer to be unable to survive at a temperature below 50°.

There is also a great mass of data relating to bacteria which shows that the thermal death point rises with an increase in the osmotic pressure of the culture medium. That is to say, a hypotonic liquid such as distilled water which tends to increase the amount of water in the organisms, lowers the thermal death point, while an increased osmotic pressure raises the death point. Such an affect is easily seen in cultures of the abundant *Micrococcus aureus.* This species is not killed by a temperature of 65° in water, physiological salt solution nor broth containing 16% sucrose on an exposure of half an hour, but when the temperature is raised to 70° or 75° there is a noticeable drop in the surviving bacteria, more pronounced in the distilled water; finally at 80°, all in water are killed, but a few survive in the two other media (Robertson '27). The same was noted for *Streptococcus* *thermophilus*, but here the difference is more pronounced as a treatment for the same period in distilled water and physiological salt solution showed no survival in water at 70°, and large numbers of individuals alive in the salt solution.

That this is a generally prevalent condition is shown by studies on numerous other bacteria by various workers and also in the case of spores the same effect has been observed by Viljoen ('26) in their resistance to heat. The phenomenon appears to be due without question to the higher coagulation point of albumin or similar substances when partially desiccated.

One type of environment presents a suitable substratum for the growth of microörganisms at high temperature. This is the bark and subcortical tissues of logs which regularly reach temperatures of 60° or more under the influence of the sun's rays. When sufficiently moist they rapidly disintegrate through the action of various mircoörganisms and fungi. This then furnishes one suitable type of substratum and may account for the presence of certain thermophilous forms which occur in soil and in the alimentary tract of animals.

With due consideration to the several possibilities it is difficult to evade the conclusion that the resistance to heat of thermophilous bacteria is a primordial trait retained in these microörganisms from previous ages when conditions of excessive heat were much more prevalent and extensive than they are at the present time. It is no doubt such considerations as these which led Mereschkowsky ('10; '20) to consider that there are two distinct types of protoplasm in living organisms, distinguishable physiologically by their resistance to heat, and corresponding so far as bacteria are concerned to a series including the psychrophilic and mesophilic forms as distinguished from the thermophilic types. Since the maximum growth temperatures for the former are near 43–45° and the minimum for the latter are approximately the same, there is a rather clear cut division into two groups possible on the basis of temperature alone.

Algæ and Higher Plants

Among the primitive algæ which occur in thermal springs by no means all are able to exist at the higher temperatures known to support plant life. One blue-green alga, *Mastigocladus laminosus*, which is widely distributed in thermal springs of Europe, Asia and North America is almost as sensitive to heat as the non-thermal types of plants and animals. This species grows normally in water having a temperature of 52° or less and has been shown experimentally to thrive in cultures (Löwenstein '03) between 19.3° and 51° C., but not above that point. It is quite evidently a form which has migrated into the thermal environment from the freshwater flora and its temperature relations are fundamentally different from those of Phormidium and similar types of blue-green algæ that occur in water of very high temperature.

That the upper temperature limits for the growth of higher plants, such as grasses, growing in the hot soil about thermal springs are in general similar to those reported by Löwenstein for Mastigocladus is indicated by observations made by Mrs. Brues, but as yet unpublished.

THE NATURE OF HEAT SUSCEPTIBILITY

To account for the great differences in tolerance to high temperature as well as to certain poisons, Mereschkowsky ('10) conceived the rather startling theory that there exist two types of protoplasm in living organisms. This innovation has received scant recognition at the hands of biologists, but as it appears to be quite plausible in connection with some of the facts outlined in the present account, I think it deserves some further discussion. He assumed in brief that animal cells and those of higher plants represent symbiotic associations while the protoplasm of bacteria and most fungi is of a single primordial type. The animal cell is a single symbiotic association with a "Micrococcus" which forms the nucleus while the plant cell is a double symbiosis of protoplasm, nucleus and "Haplobacteria" which become the chloroplasts. Thus a mycoplasma of simple type is present in bacteria and Cyanophyceæ, and a complex, symbiotic type is characteristic of the higher plants and of all animals. One of the main distinctions between the mycoplasma and the animal type or amœboplasma lies in the great resistance to high temperatures exhibited by the former.

The distinction based on the resistance to high temperatures would appear to be broken down by the resistance of certain encysted Protozoa to high temperatures[1], and as a matter of fact among bacteria alone we find both heat-susceptible and heat resistant types. Recent studies also tend to disprove Mereschkowshy's conception of the structure of the bacterial cell.

More recently another type of intracellular symbiosis of bacteria or fungi with insects and other animals has been shown by a number of workers (*cf.* Buchner '21) to be quite widespread, but this is so clearly an independent and specialized development that it has no bearing on Mereschkowshy's conception. However, the latter was revived a few years ago by Wallin ('23; '27) with reference to the mitochondria which he regards as microörganisms living symbiotically in the cells of both plants and animals. In this conception, the mitochondria are the only symbionts, giving rise to the chloroplasts of the higher plants and a single or binary symbiosis exists in both animal and plant cells. At first blush this theory appears to accord very well with some of the facts observed in thermal animals and plants. The presence of mitochondria does not appear to have been demonstrated in bacteria and the blue-green algæ, and these two groups of organisms are those which exhibit the greatest resistance to heat. They are in fact the only organisms in hot springs which may have persisted in such an environment from the early stages in the earth's history, since I have already shown that the animal life of hot springs includes only a series of highly specialized forms derived from fresh water. Thus what we may regard as the primordial types of organisms in hot springs are the groups apparently destitute of mitochondria. Furthermore, the mitochondria are very susceptible to heat and their disintegration appears visibly to take place at temperatures close to those which are fatal to organisms provided with them, that is in the neighborhood of 48–50°. We would appear, therefore, to have here visible evidence of a disintegration of mitochondria coincident or very nearly so to the onset of irreversible changes induced by heat which indicate the threshold of destruction of living tissues by heat. In the absence of other evidence it is therefore quite probable that the heat suceptibility of organisms is due to its effect upon their mitochondria, especially as these bodies are extremely susceptible to other chemical and physical changes. Even in their staining reactions, they behave very differently from the chromatin and other constituents of the cell, and as Zirkle ('28) has shown, their staining properties are greatly changed by differences in the hydrogen ion concentration of fixing agents.

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[1] See postscript, p. 228.

Such a conclusion, does not, however, entail any necessarily symbiotic relationship on the part of the mitochondria although it does very strongly suggest that it is the thermal sensitivity of these granules which determined the reaction of tissues or entire organisms to heat. Some support to this assumption is given by reactions of certain flagellate Protozoa to heat described on a previous page (p. 161). Accompanying the irreversible changes in animal tissues which lead to injury or death from heat, there are always certain visible changes of a physical nature which result in heat rigor. These appear to be in the nature of a coagulation or change in the fluidity of the protoplasm or of certain tissues. These temperatures, as already pointed out are closely coincident with the temperatures at which the mitochondria are destroyed.

BODY TEMPERATURE

In my earlier paper ('24) there is included a number of references to the body temperatures of non-aquatic insects, but I failed to touch upon one important matter in connection with the body temperature of the social hive-bee which is of great interest.

This highly social insect has become modified in several particulars with reference to community life. In common with other social insects it has developed, quite independently, however, a sterile worker caste like that of the termites and ants and in connection with the development of extensive colonies has found it necessary to occupy a domicile to house the thousands of individuals which form each family. With this massing together of individuals an opportunity has presented itself to this poecilothermic animal to acquire some of the advantages enjoyed by warm-blooded animals. Thus in the winter when the air temperature and consequently that of the nest or hive becomes greatly lowered, the bees institute a series of calisthenic exercises which serve to raise the temperature of their bodies above that of the surrounding medium. Enclosed in a hollow tree or artificial hive the temperature of the swarm promptly rises and the bees are able to maintain a temperature far more uniform than that of other non-gregarious insects, and by so simple an artifice to become at least partially warm-blooded animals.

The body temperature of the hive-bee has received careful investigation by Brünnich ('19), the temperature of the colony has been studied by Gates ('14) and the temperature of the honey-bee cluster in winter by Philipps ('14).

Brünnich determined the body temperature with a minute thermocouple, and found that it sometimes reached an extremely high level. Thus he found the temperature of the worker reaching 39.6°, that of the drone 48.4° and that of the developing brood 45.2° at times when the outside air was far lower (12–17° C. or 53–62° Fahr.). In the honey-bee cluster, Philipps never found the winter temperature to rise above 33° and even in warmer parts of the year Gates found that it did not rise above 36°. The body temperatures of the bees and brood would therefore appear to be much higher than that of the hive and clusters. It really seems, however, that further data should be collected to determine whether the body temperatures of bees are normally so high as those found by Brünnich.

The hive-bee appears to be entirely exceptional among insects and probably among pœcilothermic animals in general in regulating the temperature of its body and dwelling by concerted action during cold weather. Such activities do not seem to have been recorded among ants and termites as the latter passively allow themselves to succumb to the benumbing effects of the cold at the approach of the cold season. Their body temperatures during the hibernating season are undoubtedly slightly higher than those of solitary insects due to the heat produced by metabolism in a number of individuals closely confined or huddled together, but there can be but little rise in temperature compared with that produced in the beehive. Similarly in masses of hibernating beetles, such as certain coccinellids which collect in great masses during the winter season, the temperature of the mass is known to be considerably above that of the surrounding soil or air.

The normal body temperatures of birds are of particular interest in connection with the thermal environment, since these animals maintain temperatures considerably above those encountered among mammals. According to Wetmore ('21) who has presented a very extended discussion of the temperature in birds, it seems probable that there has been a progressive rise in body temperature in the evolutionary development of the birds. There appear to be numerous exceptions to this, but in general the lower temperatures are encountered in the more primitive families and the higher ones in the more advanced families. He says (*loc. cit.*): "Thus grebes, the totipalmate groups (Anhingidæ, Phalacrocoracidæ and Pelicanidæ) and herons are in general low in average body temperature, while gulls, shorebirds, pigeons and cuckoos are high." It is evident from the extensive tables given by Wetmore that temperatures of 108°–109° Fahr. (42.2°–42.7° C.) are normal for many birds and that 110° Fahr. (43.3° C.) is quite normal for a few forms. Higher temperatures have been at times reported, but some of these have no doubt been due to febrile disturbances induced possibly by avian malaria or other infectious diseases. The highest single reading which Wetmore believes to be valid is 112.7° Fahr. for an individual of the western wood pewee (*Myiochanes richardsonii*) this is however only 1.7° above the mean normal for this species. It seems indeed, rather surprising that higher temperatures should not have been occasionally encountered in sick birds. I think one may be quite justified, in the absence of higher records among birds averaging 108°–109°, to believe that these animals live so near the upper lethal temperature limit that they are unable to withstand so great a rise in temperature as may be endured by mammals like man where a rise of 5°–6° Fahr. occurs commonly without serious sequælæ.

The Brackish Water Fauna in Relation to That of Thermal Waters

Buxton has recently expressed the opinion ('26) that the colonization of the sea by insects has been held in check not by the physical factors in the environment, such as currents, tides, waves, etc., but by the salinity of the sea and "the presence of a variety of inorganic salts; it may be that magnesium salts or sulphates or other substances are incompatible with the life of certain aquatic insects." Since a few insects have been able to overcome such barriers, he believes that other matters, such as the presence of different kinds of enemies and the lack of the ordinary diet of phytophagons insects act as barriers and that it is the failure of insects to overcome more than one type of barrier that has doomed them to practical exclusion from the sea.

The dearth of insects in the sea has long been recognized and has received much comment, and the scarcity of marine insects is in fact almost pronounced as the lack of sponges and Cœlenterates in fresh water.

Naturally, most observers have seized upon the one constant peculiarity of the sea as the determining factor and have assumed that the salinity of the water is the exclusive, or at least the only influence worthy of consideration, and there is much to support this view.

There has in the past been much speculation as to why it is that some fresh water animals are able to exist temporarily or permanently in brackish or salt water and *vice versa* why certain marine forms may adapt themselves to estuarine or freshwater existence. Bert ('73; '83). Loeb ('03), Quinton ('04) Fredericq ('04); '22) have dealt with various matters relating to these phenomena which appear to be rather complex and to depend upon several factors. A change of medium causes either a loss of weight when fresh water animals are removed to water of greater salinity, or a gain in weight when marine animals are transferred to water of reduced salinity. The ability to survive such changes of medium is by no means uniform among various invertebrates, but varies within wide limits in the case of both fresh water and marine species. The body fluids of marine invertebrates are of practically the same concentration and composition as sea water, and they quickly change in response to changes in the medium to the point of osmotic equilibrium. On the other hand, fresh water forms have a lower osmotic pressure than that of marine ones, and they are able to maintain this, preventing a further dilution of the blood. Adolph ('25) believes from experiments of his own and of others that the possibility of survival of fresh water invertebrates in brackish or salt water depends directly upon the osmotic pressures of their body fluids. As this varies considerably, from about 12 to 50% that of sea water, we should expect their resistance to salinity to vary in a corresponding degree. Thus he found that the crustacean *Gammarus fasciatus*, is able to survive in sea water while some other forms (a flatworm and Paramœcium) can withstand only one-third or less the concentration of sea water. This is particularly interesting since Gammarus is one of the common types found in waters of greatly varying salinity. Duval ('24) has similarly noted a great difference in in the composition of the body fluids in crabs (cf. Brues '27^b) and Kemp ('15) has noted great differences among crabs in their ability to invade estuarine waters.

That these experimental data may explain the great delicacy of adjustment of various insect larvæ to slight differences in salinity of the medium cannot be granted, however. They show that the limiting factors may vary within wide limits and this forms a physiological basis for an understanding of the readiness with which some fresh water animals may invade marine or estuarine or saline habitats. As we have seen such types among insects are very clearly restricted to particular groups. For the change from marine to freshwater life which we must grant occurred originally in the case of all fresh water groups, and has taken place more recently in scattered cases there seems at present to be no adequate explanation. The great similarity between the fauna of thermal waters and that of brackish water to which I called attention in my former paper ('24) is again fully demonstrated by the collections discussed in the present account and serves to show that aside from temperature the most important single characteristic of thermal waters which determines the nature of its fauna is the presence of salts in solution. Almost universally these are present in sufficient amounts to increase the osmotic pressure of the medium very appreciably.

The Temperature Range of Individual Species

A large mass of data has been presented in the preceding account indicating the actual temperatures at which a number of species have been observed to occur, and these indicate clearly the upper temperature limits of life for individual species and groups of related forms. Since many of these thermophilous organisms occur generally at the lower temperatures characteristic of non-thermal waters, they demonstrate that the thermal environment for such organisms is simply an extension of a more ordinary one. In the case of many bacteria and blue-green algæ it is equally evident that there are species of these groups which are restricted to higher levels of temperature at least so far as growth is concerned. Among animals, the number of thermophilous species dependent upon abnormally high temperatures for their existance is very small and at the present state of our knowledge it is difficult to pick out with certainty any number of forms which are thus restricted. This is due mainly to the small size of the springs that support the thermal fauna which prevents the occurrence of graded temperature series under comparable conditions.

At only one place in the world has it been possible to follow the changes in composition of the thermal fauna over a considerable range in water from a single source. This has been done at the thermal springs of Meskhoutine, or Hammam Meskhoutine in northern Africa. The French naturalist, Blanchard ('03) published a very interesting account of the fauna of these springs which had previously been considered by several others and has since been dealt with in detail by Seurat ('22). These springs are of very large size, yielding between 400 and 500 gallons of water per minute or about 100,000 litres per hour at a high temperature between 78° and 95°. Descending the stream arising from these springs where the water has cooled to 60° there are many dark green algæ, but no animal life. At 51° the crustacean, *Cypris balnearia* first appears. At 45° this Cypris becomes very abundant and frogs jump into the water when disturbed, but do not remain there voluntarily. At 44° Cypris is still abundant while the fresh-water crab, *Telphusa fluviatilis* and tadpoles of the frog, *Rana esculenta* appear. Adult frogs are numerous, entering the water and staying there quite willingly. At 43° Cypris becomes more rare; annelids of the genus Näis appear while frogs and their tadpoles remain abundant. At 39° the Cypris has disappeared and according to Blanchard occurs only between 43°and 51°. Below this temperature the fauna gradually becomes normal just as I have indicated in a composite way with reference to the water beetle fauna of our western hot springs (see p. 178).

From Blanchard's observations there is no doubt that *Cypris balnearia* is restricted to a definite range of temperatures above 40°. To this may be added also the acarine genus *Thermacarus* mentioned on a previous page. Here two species are concerned one from Eastern Asia and another from the Western United States, both found only in hot springs and furthermore representing the only known species of a genus which constitutes by itself a distinct family of hydracarine mites.

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POSTSCRIPT.

It will be noted by the reader that no consideration of the crustacean fauna of hot springs is included in the foregoing account. Rather extensive collections were made, but it has not been possible to work over this material in time for inclusion in the present report.

In connection with the effects of heat upon encysted Protozoa, more recent work shows that these animals cannot survive the temperatures reported by Boeck which I have quoted on p. 160. This investigator tested the living condition of the cysts by their staining reactions, and this method does not appear to be reliable. Consequently, these encysted Protozoa do not form an exception to the general relations of animal life to high temperatures.

PLATE I

Above

View of the siliceous sinter terrace at Beowawe, Nevada, looking toward the southeast from the desert floor. The geysers and hot pools are on the nearly level upper surface. The regular flow of water is indicated by dark streaks on the slope caused by the presence of algæ.

Below

View from upper part of same terrace, looking westward over the valley. The springs at the foot of the terrace are visible near the center of the photograph with the darker, green area which they water.

PLATE II

Above

Basins of part of a group of extinct hot springs near the western shore of Pyramid Lake, Nevada. These rise above the level of the surrounding surface from eight to twelve feet.

Below

Hot Spring No. 7 at Minden, Nevada. The springs are in the foreground and on the slope at the right.

PLATE III

Above

Large pool on upper surface of deposits from Hot Spring No. 21 at the northern end of the Osobb Valley, 72 miles south of Winnemucca, Nevada. The surface of this mound rises about 60 feet above the floor of the desert valley.

Below

Overflow from Hot Spring No. 23[a], 37 miles south of Battle Mountain, Nevada. The spring lies beyond the photograph on the left.

PLATE IV

Above

Mud pot or steaming vent from which there is no flow of water, in the Devil's Kitchen, near Drakesbad, Mount Lassen National Park.

Below

Edge of pool formed by overflow from Hot Spring No. 17, twenty-one miles south of Denio, Oregon.

PLATE V

At Left

Part of mound built up by Hot Spring No. 19 at Paradise, Nevada. The internal structure is visible as the spring has been tapped to supply water for irrigation purposes.

At Right

One of the small overflow streams on the slope of the terrace at Beowawe, Nevada (Hot Spring No. 24). The dark color is due to the presence of algæ.

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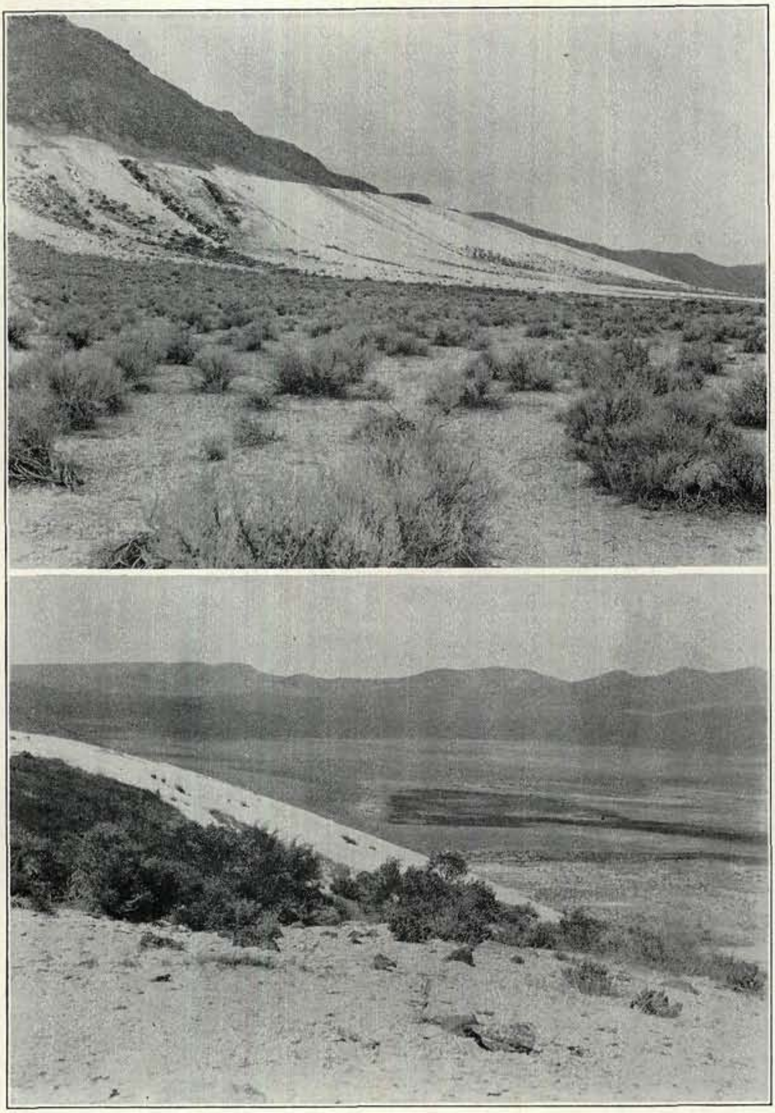

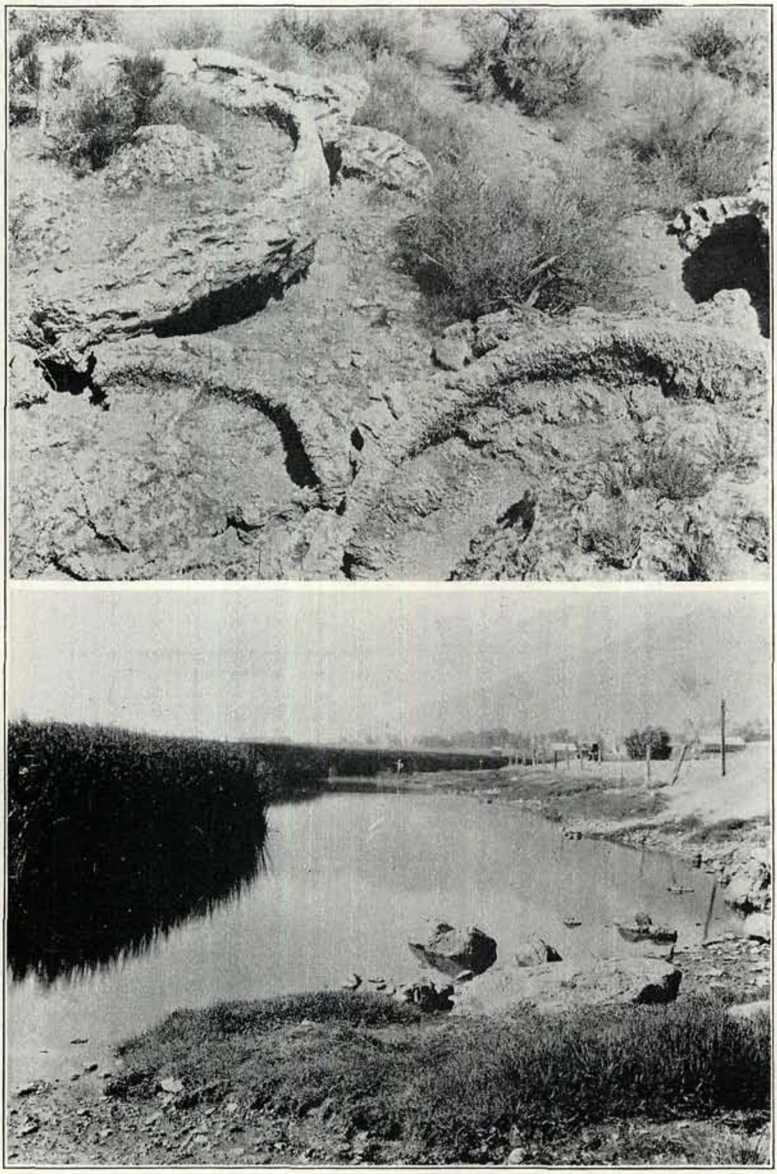

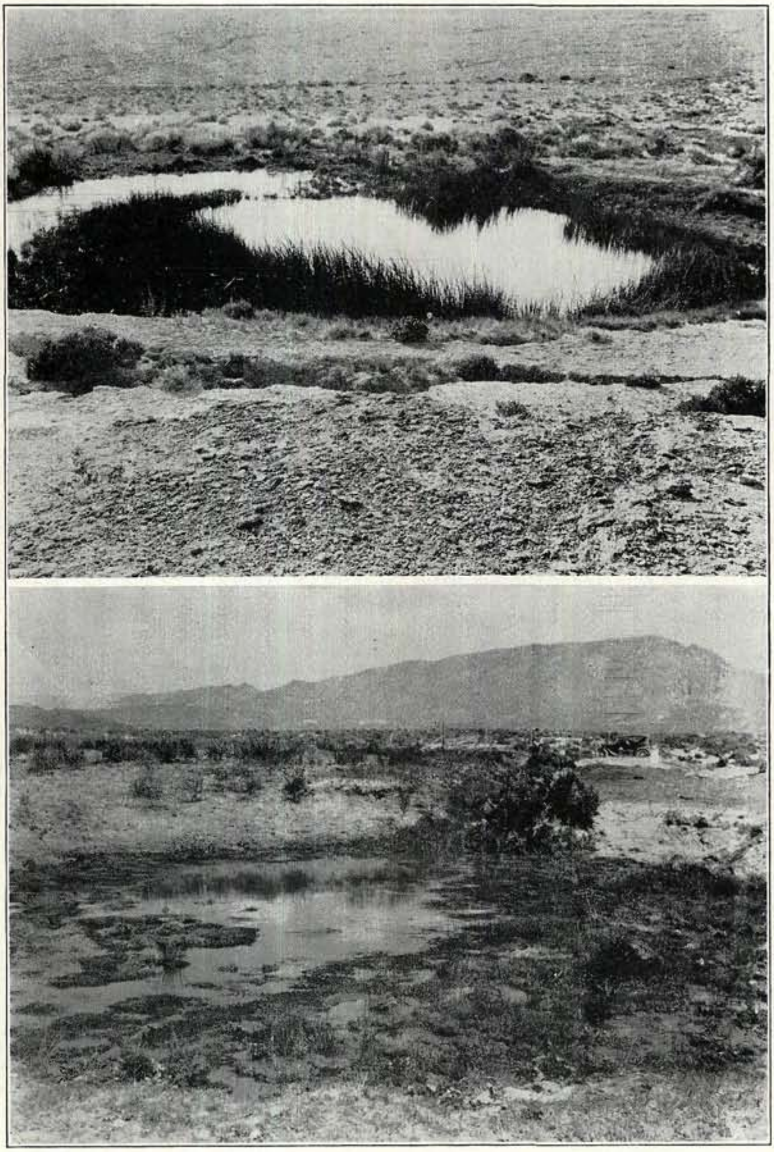

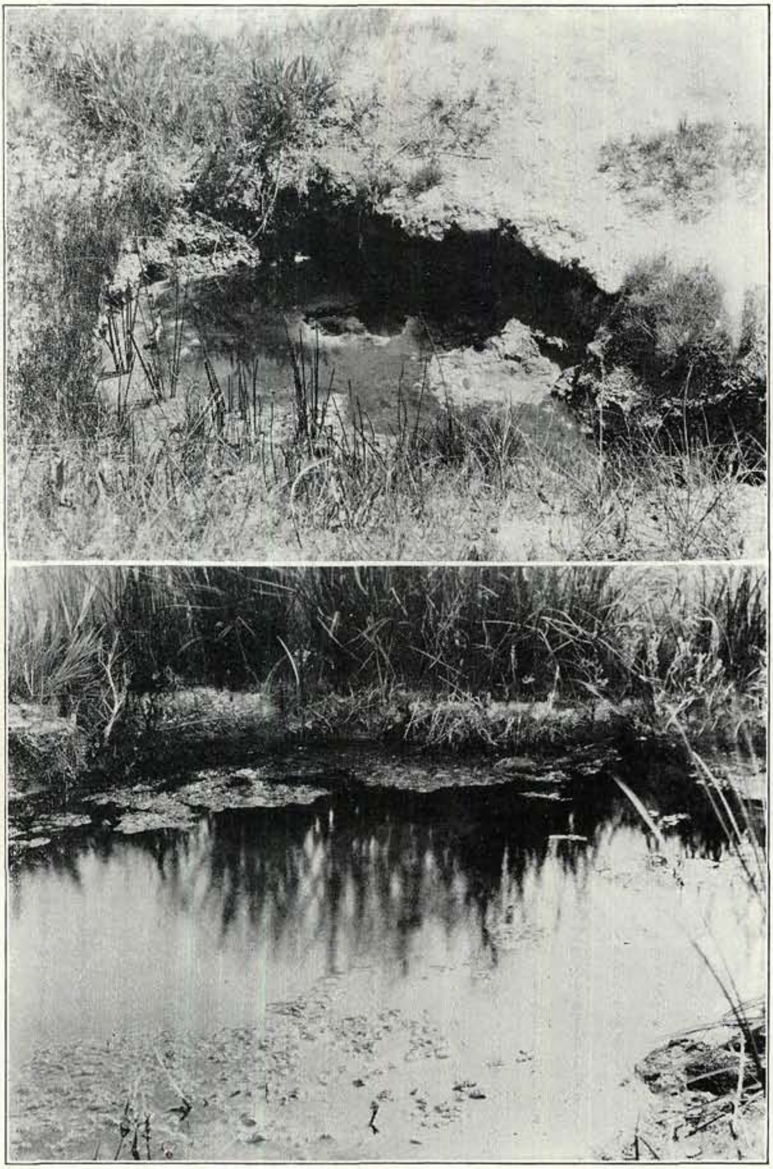

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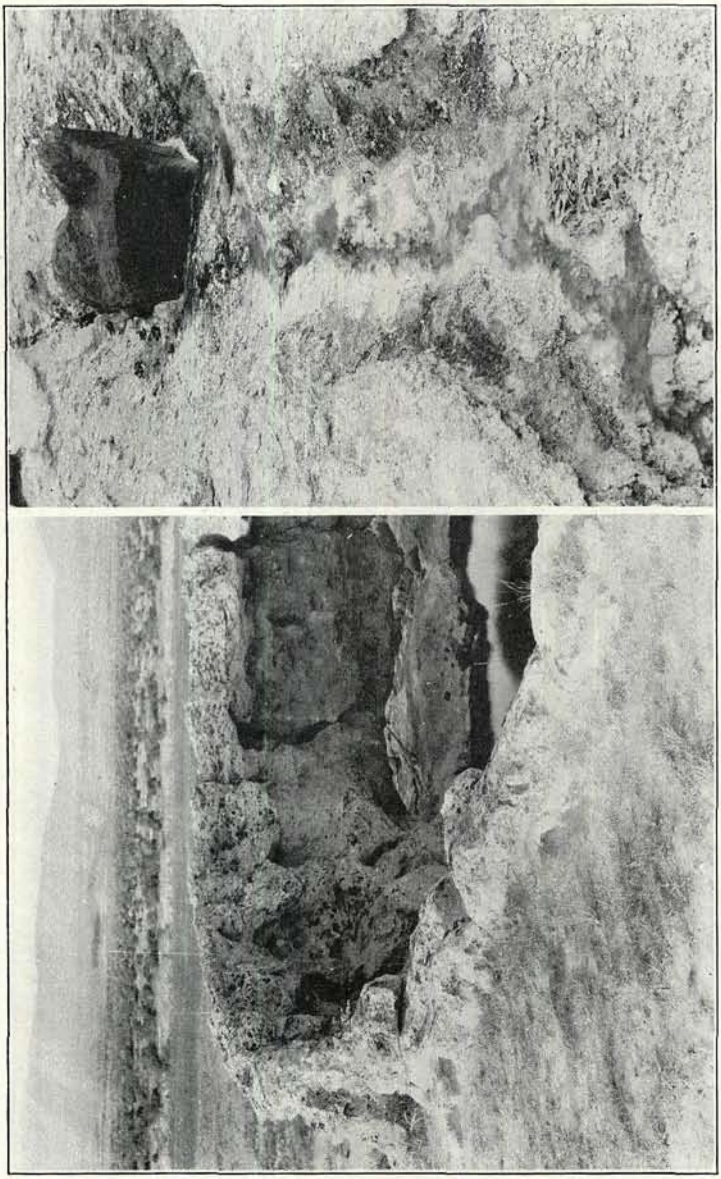

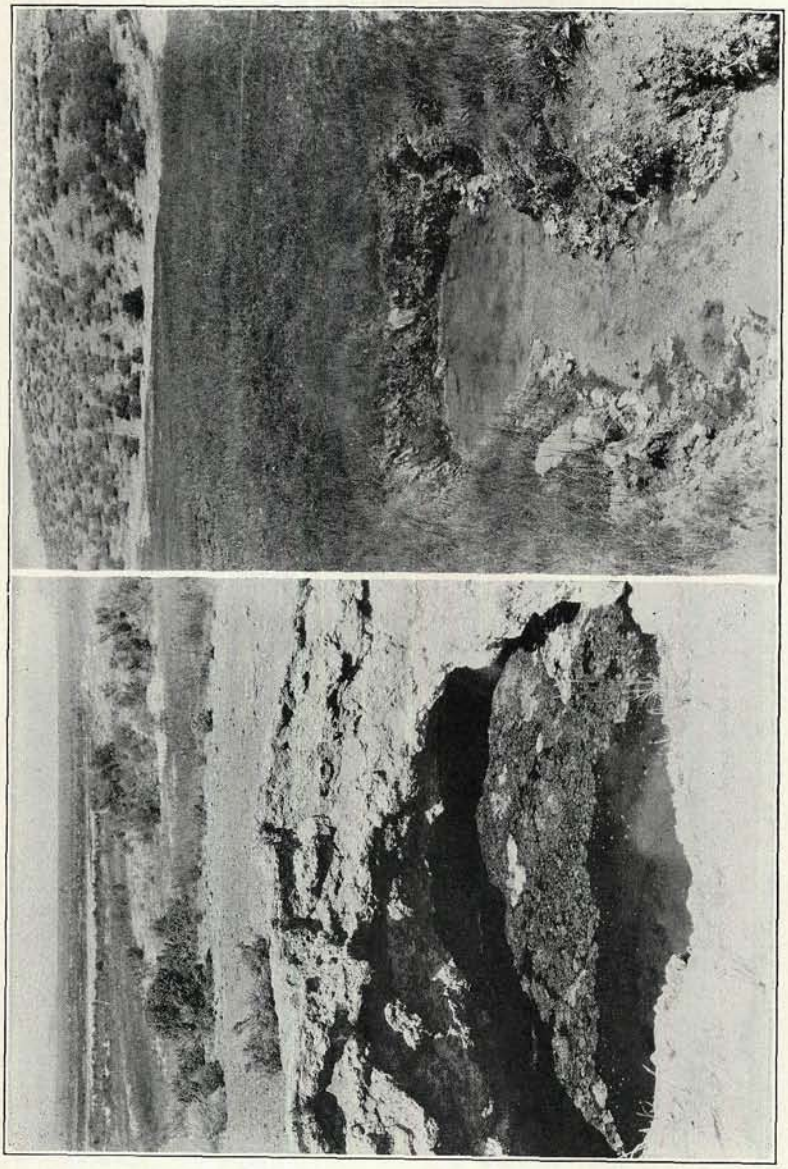

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A NEW EQUATION OF STATE FOR FLUIDS.

By James A. Beattie and Oscar C. Bridgeman.

A NEW EQUATION OF STATE FOR FLUIDS.*

(Contribution from the Research Laboratory of Physical Chemistry, Massachusetts Institute of Technology, Cambridge, Mass., No. 200.)

By James A. Beattie and Oscar C. Bridgeman.†

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1. INTRODUCTION.

The functional relationship between the pressure, volume and temperature of a fluid, commonly called the equation of state, has been the subject of many papers since the investigations of Boyle and Charles led to the formulation of the perfect gas law. It was soon recognized that this simple equation represents the behavior of actual gases to the first approximation only, as large differences between observed and calculated values occur except in the region of low pressures. Many modifications have been suggested in order to represent more accurately the relation between the measured pressures, volumes and temperatures of a gas or liquid, and of these the best known are the equations of state proposed by van der Waals,[1] Lorentz,[2] Clausius,[3] and Dieterici.[4]

---

* Communicated by Professor Frederick G. Keyes.

† National Research Fellow in Chemistry.

$$p = \frac{RT}{V - b} - \frac{a}{V^2} \qquad \text{(van der Waals)}$$

$$p = \frac{RT}{V^2}[V + B] - \frac{A}{V^2} \qquad \text{(Lorentz)}$$

$$p = \frac{RT}{V - b} - \frac{a}{T\,(V + c)^2} \qquad \text{(Clausius)}$$

$$p = \frac{RT}{V - b} e^{-\frac{a}{VRT}} \qquad \text{(Dieterici)}$$

which represent second order approximations.

As more accurate experimental data became available, still further or third order modifications were introduced. To this class belongs the equation of state of Keyes[5] which, for a non-associating gas, has the form

$$p = \frac{RT}{V - \beta e^{-\alpha/V}} - \frac{A}{(V + l)^2} \qquad \text{(Keyes)}$$

This equation has been extensively compared with experimental data and applied to the calculation of various thermodynamic properties of fluids.[6] It reproduces the measured pressures very satisfactorily for specific volumes greater than about 15 cubic centimeters per gram.

Much of the pressure-volume-temperature data on gases has resulted from the investigations of Kamerlingh Onnes and his associates at the University of Leiden, and from those of Holborn, Schultze and Otto at the Physikalisch-Technischen Reichsanstalt in Berlin. Neither of these groups of workers use any of the theoretical equations of state now existent but express their results in the form of empirical expansions,

$$pV = A + \frac{B}{V} + \frac{C}{V^2} + \frac{D}{V^4} + \frac{E}{V^6} + \dots \qquad \text{(Leiden)}$$

$$pV = a + bp + cp^2 + dp^4 + \dots \qquad \text{(Berlin)}$$

There is one such expression for each isotherm so that the parameters A, B, C, ... and a, b, c, ... are functions of the temperature. A and a can be associated with RT but each of the other parameters require a four or five term equation in order to represent the data over the experimentally studied temperature range.

Thus it appears that the simple equations of state with a more or less theoretical background do not reproduce the experimental measurements over a wide range of pressure and temperature with sufficient accuracy for all requirements, whereas the empirical equations require a large number of terms in order to represent a comparatively large field, and consequently are not convenient for use in thermodynamic calculations.

Practically all theoretical equations of state express p as an explicit function of V and T, and may be written in the generalized form

$$p = T\,\psi(V) - \phi(V) - F(V, T) \tag{1}$$

In the equations of van der Waals, Lorentz and Keyes (for a non-associating gas), $F(V, T) = 0$; Clausius places $\phi(V) = 0$; while the equation proposed by Dieterici, when the exponential is expanded, contains all three of these functions.

In the present paper, the authors propose the equation of state[7]

$$p = \frac{RT(1-\varepsilon)}{V^2}[V + B] - \frac{A}{V^2} \tag{2}$$

in which

$$A = A_0\left(1 - \frac{a}{V}\right)$$

$$B = B_0\left(1 - \frac{b}{V}\right)$$

$$\varepsilon = \frac{c}{VT^3}$$

This equation is of the general form of (1) and the functions $\psi(V)$, $\phi(V)$ and $F(V, T)$ can be obtained by rearrangement of (2) into the form

$$p = \frac{R}{V^2}[V + B]\,T - \frac{A}{V^2} - \frac{Rc}{V^3}[V + B]\frac{1}{T^2} \tag{3}$$

In the preceding equations, the gas constant R has the same value on the molal basis for all gases, and A_0, a, B_0, b, and c are constants whose values depend upon the kind of gas under consideration.

It is believed that this equation of state has the following desirable properties: easy and unique evaluation of the constants from the pressure-volume-temperature data, accurate reproduction of the experimental pressures and their derivatives over the entire measured temperature range and over a wide range of densities, simplicity of mathematical form, and adaptability of the volume and temperature functions to further expansion, if necessary.

In the following sections a theoretical basis for this equation is given, a detailed method for obtaining the constants from the pressure-volume-temperature data is outlined, a study of the accuracy of evaluation of the constants is made, and a comparison is given between the observed pressures and those calculated from the equation for the ten gases helium, neon, argon, hydrogen, nitrogen, oxygen, air, carbon dioxide, methane and ether, together with a comprehensive bibliography of the compressibility data. Finally some suggestions concerning the use of the equation in thermodynamic calculations are included.

2. General Experimental Conditions imposed upon an Equation of State.

In thermodynamics it is taken as axiomatic that for a given mass of any isotropic system in which mechanical work only is concerned, there exists a relation of the form $f(p, V, T) = 0$ between the variables which define the state of the system. Theory demands no unique form for this function nor does it specify which of the variables, or combinations of them, is to be regarded as dependent. However, kinetic theory has developed mainly from a consideration of pressure as the dependent variable, and the majority of equations of state are in accord with this. On the other hand, pressure and temperature are the usual properties measured and varied in experimental procedure and, from this viewpoint, are more naturally the independent variables. Unfortunately no adequate theory seems to be existent for developing volume as an explicit function of pressure and temperature, and until such theory appears, it seems necessary to express theoretical equations of state in the form $p = f\ (V, T)$.

The laws of thermodynamics impose no restrictions on the form of the equation of state but kinetic theory and well-established experimental evidence do so. One of these restrictions is concerned with the form which the equation assumes in the limit as the pressure (or density) is decreased at any constant temperature, while another applies to the limiting form as the temperature is increased at any constant density. In this paper, the region immediately around the critical point is not considered and no reference will be made to the relations demanded of equations at this point. Also the possibilities of dissociation of the simple molecular species and of chemical reaction between different species in a mixture are excluded.

It appears to be well established that, as the pressure on a gas is reduced at constant temperature, the pressure-volume product approaches the ideal gas value, namely

$$\operatorname*{Limit}_{p \doteq 0} pV = RT$$

and this would seem to be true regardless of the temperature, excluding as mentioned above, the effect of dissociation. But it is also true that a considerable number of the thermodynamic derivatives do not reduce in the limit to the form given by the perfect gas law. For example, in the case of all actual gases

$$\operatorname*{Limit}_{p \doteq 0} \left(\frac{\partial pV}{\partial p} \right)_T \gtreqless 0 \text{ if } T \lesseqgtr T_B$$

where T_B is the Boyle temperature, whereas the limit of this derivative is zero under all conditions for a perfect gas. In this respect, a real gas becomes more imperfect as the temperature increases above the Boyle point.

A most outstanding phenomenon in connection with all real gases is the close approach to linearity of the pressure-temperature lines of constant density, the so-called isometrics. This fact has long been known,[8] and has led some investigators[9] to measure pressures for the same set of densities on each isotherm. At high densities, the curvature of the isometrics is very well defined, but with decreasing density, it approaches zero as a limit. On the basis of his experimental data, Young[10] considered that

$$\left(\frac{\partial^2 p}{\partial T^2} \right)_V \lesseqgtr 0 \text{ if } V \gtreqless V_c$$

where V_c is approximately the critical volume, and it is now definitely established that in the gaseous region, the curvature is negative.

Further, the experimental data seem to show that, at any density the curvature of the isometrics becomes less with increasing temperature, and hence,

$$\operatorname*{Limit}_{T \doteq \infty} \left(\frac{\partial^2 p}{\partial T^2} \right)_V = 0$$

or in other words, the isometrics approach linearity at high temperatures.[11]

In general, equations of state can be written in the form of relation (1)

$$p = T\,\psi(V) - \phi(V) - F(V, T) \qquad (1)$$

which can be rearranged or expanded into the form

$$pV = RT + \frac{1}{V} f_1(T) + \frac{1}{V^2} f_2(T) + \frac{1}{V^3} f_3(T) + \ldots\ldots \qquad (4)$$

Equation (1) is more convenient for the study of properties at constant volume, whereas (4) is more adapted for use at constant temperature. The various conditions imposed on equations of state can now be summarized by reference to (1) and (4). For all actual gases, the following relations can be written.

I. At constant temperature—from Equation (4)

(a) $\underset{1/V \doteq 0}{\text{Limit}}\ pV = RT$

(b) $f_1(T) \neq 0$, otherwise $\underset{1/V \doteq 0}{\text{limit}} \left(\dfrac{\partial pV}{\partial p}\right)_T = 0$ for all temperatures.

II. At constant volume—from Equation (1)

(c) $\underset{T \doteq \infty}{\text{Limit}}\ F(V, T) = 0$, *i. e.* $\underset{T \doteq \infty}{\text{limit}} \left(\dfrac{\partial^2 p}{\partial T^2}\right)_V = 0$ for all volumes.

(d) $\underset{1/V \doteq 0}{\text{Limit}}\ F(V, T) = 0$, *i. e.* $\underset{1/V \doteq 0}{\text{limit}} \left(\dfrac{\partial^2 p}{\partial T^2}\right)_V = 0$ for all temperatures.

(e) $F(V, T) > 0$, *i. e.* $\left(\dfrac{\partial^2 p}{\partial T^2}\right)_V < 0$

Relations (a) and (b) illustrate the isothermal conditions that, for the calculation of pV, the equation of state for real gases must approach that for an ideal gas as the density is decreased, but not for the calculation of certain derivatives. Equation (e) indicates that the curvature of the isometrics of a gas is always negative, whereas (c) and (d) show that the isometrics approach linearity at high temperatures, and also with decreasing density. Hence (c) and

(e) are equivalent to the statement that with decreasing temperature, increasingly larger amounts must be added to the actual pressures to make the resulting isometrics linear.

Equation (1) can be put into the form

$$p' = p + F\,(V,\,T) = T\,\psi\,(V) - \phi\,(V) \tag{5}$$

where p' can be designated as the 'corrected pressure' since it varies linearily with the temperature at constant density. The right hand side of (5) is evidently the equation of the line to which the original isometric is asymptotic.

Equation (1) purports to be a generalized equation of state and is based on the considerations outlined in this section. In the following section the kinetic theory is used as the basis for suggestions regarding the specific form of the volume and temperature functions.

3. THEORETICAL BASIS FOR THE PROPOSED EQUATION OF STATE.

The kinetic theory is founded on the hypothesis that a gas consists of continually moving molecules or atoms, whose velocities are distributed according to certain laws; from which it is shown that the pressure and density of the gas are uniform throughout the interior. From the thermodynamic and experimental viewpoints, however, it is the pressure at the boundary which is of interest, and in general this will differ from that in the interior. In the case of actual gases, for given conditions of temperature and density, the pressure in the interior will depend, among other things, upon the forces which act between the molecules and upon their dimensions, and at the boundary this pressure will be modified by the field of force set up by the action of these intermolecular forces. It is customary to consider the effects of the forces in the interior and at the boundary as independent, so that the net pressure is that existent in the body of the gas diminished by an amount determined by the field at the boundary, although rigorously these two effects are perhaps not independent. Following the usual custom, the measured pressure of a gas can be written as the difference of two terms, one of which arises from the kinetic energy possessed by the gas, and the other from its potential energy,

$$p = p_{\text{kin.}} - p_{\text{coh.}}$$

where $p_{\text{kin.}}$ is the kinetic pressure and $p_{\text{coh.}}$ is the so-called cohesive pressure. These two terms will be considered separately.

(*a*) *Kinetic Pressure.*

A number of methods are available for the calculation of this pressure, among which may be mentioned the method of general dynamics,[12] the method of collisions,[12] the method of the mean free path,[13] and that employing the Clausius virial.[14] In the case of an ideal gas, namely one in which the molecules are infinitesimally small, perfectly elastic, and in which there are no intermolecular forces, these methods lead readily to the result that $p = \rho RT$ where ρ is the density of the gas. A simple method, due to Phillips,[15] of obtaining this relation is to consider an imaginary plane in the interior of a vessel containing a perfect gas and to evaluate the time rate of transfer of momentum, I_i, across unit area of this plane. It is easily shown $I_i = \rho RT$. In the case of an actual gas, the transfer of momentum is not the same, and the theoretical deductions of Clausius,[16] Lorentz,[17] van der Waals,[18] Boltzmann,[19] Jeans,[20] van Laar,[21] etc. have all shown that it is greater than for a perfect gas. On general considerations, Phillips has pointed out that the transfer of momentum is greater for any gas in which intermolecular forces exist regardless of whether these forces consist of repulsion or attraction, regardless of the law of such forces, and further independent of the size of the molecules, which may be infinitesimal.

The method which Phillips[22] proposed for the evaluation of the volume function ψ (V) of Equation (1) can be applied with slight modifications to the present case. Consider a thin section of the fluid between parallel planes which are in the interior of the fluid and which are a distance dx apart, where dx is the x-component of the distance travelled in the time dt by a molecule of class A (*i. e.* those possessing velocities between u and $u + du$). The kinetic pressure is equal to the total rate of transfer of momentum across unit area of one of these planes (called the reference plane) due to the passage of molecules of all classes, the arithmetic sum of the momenta of the molecules being taken regardless of the direction of passage through the plane.

Consider now the molecules of class A contained in a cylinder of unit area and length dx, the two ends of which lie in the parallel planes; their total momenta are $\nu_1 mu^2 dt$ where ν_1 is the number of molecules of this class in unit volume. If, in the time dt, ν_1 molecules of class A pass through the end of the cylinder lying in the reference plane, the contribution to the pressure would be $\nu_1 RT$ and when summated for all typed of molecules the result for the kinetic pressure would be ρRT.[23] In the case that the molecules of the gas attract or repel each other (due to the existence of intermolecular forces) the passage of momentum is greater than this value. For, during the time, dt, which must be long enough for a large number of molecules to cross the reference plane so as to give an average value, a certain fraction of the molecules which have already passed through the plane will be reflected and pass through again, due to their interaction with other molecules. This reflection will occur across all boundaries of the cylinder equally in both directions and so will not affect the distribution in the cylinder. But the momentum of the reflected molecules must be included in any method for the calculation of the pressure. Hence, due to the reflection of the molecules through the reference plane (which may be any plane in the interior of the fluid) the value ρRT must be increased, in the ratio of the number of molecules which actually pass through the boundary to the number in the cylinder. If r is the average value of the "reflection" for all classes of molecules, there is obtained for the true rate of transfer of momentum (I_a) the relation

$$I_a = \rho RT\,(1 + r) \tag{6}$$

If the density of the gas is small so that the molecules act independently as reflectors, the fraction reflected is proportional to the density, and hence $r = \rho B$ so that Equation (6) becomes

$$I_a = p_{\text{kin.}} = \frac{RT}{V^2}[V + B] \tag{7}$$

This expression was first deduced by Lorentz[24] from the Clausius virial. If the density is such that this simple assumption of independent reflection is not valid, and the reflecting power per molecule is interfered with by those around it, B will not be constant but will be dependent upon the density, and the simplest form for this second order correction is a linear function of the density. Under these conditions

$$B = B_0\,(1 - b\rho) \tag{8}$$

and (7) becomes

$$p_{\text{kin.}} = \frac{RT}{V^2}\left[V + B_0\left(1 - \frac{b}{V}\right)\right] \tag{9}$$

In the treatment thus far, the assumption has been made that the time of encounter between the molecules is independent of the kinetic energy, or in other words, of the temperature. That this is not strictly valid can be seen from general considerations. When two slowly moving molecules encounter one another, there is a tendency for them to move under the influence of each other for an appreciable length of time due to the intermolecular forces between them. This has the effect of decreasing the number of independent aggregates in the system, and hence simulates association or aggregation. At low temperatures the number of such slowly moving molecules is greater than at higher temperatures and a correction must be made for this. Clausius[25] attempted to account for this effect by making the cohesive pressure term vary as the reciprocal temperature. Boltzmann,[26] Gibbs,[27] and Keyes and Taylor[28] have all treated aggregation by a modification of the kinetic pressure term, as have also Boynton and Bramley.[29] The variation of the average time of encounter with the temperature has the same effect as a change in the average molecular weight of the gas and hence the gas constant R may be considered to depend upon the temperature and density, since the latter also affects the number of encounters and hence the number of molecules which can be considered as independent. In a former publication, the authors,[30] using a method somewhat analogous to that employed by Keyes and Taylor, modified R in the following manner:

$$R' = R\left(1 - \frac{m}{V}e^{n/T}\right)$$

where m and n are constants, R the usual perfect gas constant and R' is its analogue in the equation of state of an aggregating gas. This relation was shown to reproduce the experimental data in a satisfactory manner, but was not convenient due to the use of an exponential function of the temperature. A complete treatment of the relation of the degree of aggregation to the temperature has not been given, but in general it can be assumed that to the first approximation the variation in the number of independent aggregates due to the effect on the average time of encounter is directly proportional to the density and inversely proportional to some power of the absolute temperature, so that R' becomes

$$R' = R\left[1 - f(V, T)\right] = R\left[1 - \frac{c}{VT^n}\right] \qquad (10)$$

From a study of the experimental data, it has been found that $n = 3$ gives a very satisfactory reproduction of the measurements, and the introduction of this modification into Equation (9) leads to the complete expression for the kinetic pressure,

$$p_{\text{kin.}} = \frac{RT\,(1 - c/VT^3)}{V^2}[V + B_0\,(1 - b/V)] \tag{11}$$

(*b*) *Cohesive Pressure.*

It has been pointed out by Phillips[31] that whatever may be the law of force between the molecules, the potential energy will be directly proportional to the density for a given mass of gas, provided only that the law of force is independent of the density and that the force decreases with distance with sufficient rapidity for integrals having infinite limits to converge. Under these conditions the potential energy may be written

$$E = A\rho = \frac{A}{V}$$

and differentiation with respect to the volume gives the cohesive pressure

$$p_{\text{coh.}} = -\frac{\partial E}{\partial V} = \frac{A}{V^2} \tag{12}$$

which is the usual expression obtained by van der Waals and Lorentz.

In general, the forces between electrical systems are dependent upon the dielectric constant of the medium. Therefore Phillips considers that, if the molecules can be considered as substantially rigid electrical systems, the molecular forces which give rise to the cohesive pressure are affected by the dielectric constant k of the gas, and since this varies with density it must be introduced into (12). The cohesive pressure is therefore given by the relation

$$p_{\text{coh.}} = \frac{A_0}{kV^2} \tag{13}$$

The dielectric constant of a gas is related to the density by the Lorentz equation

$$\frac{k-1}{k+2} = \frac{C}{V}$$

$$k = \frac{V + 2C}{V - C}$$

Substitution of this value of k into (13) and neglecting all powers of density higher than the first in the expansion gives

$$p_{\text{coh.}} = \frac{A_0}{V^2}\left(1 - \frac{3C}{V} + \ldots\right)$$
$$= \frac{A_0}{V^2}\left(1 - \frac{a}{V}\right) \qquad (14)$$

The preceding method used for evaluating the variation of the quantity A (of Equation 12) with the density could have been applied to B, and would have given the same form of relation as assumed in (8).

Combination of (11) and (14) gives the complete equation of state,

$$p = \frac{RT\,(1 - c/VT^3)}{V^2}[V + B_0\,(1 - b/V)] - \frac{A_0}{V^2}(1 - a/V) \qquad (15)$$

which is Equation (2) written in extended form.

In the deduction of (15), general physical considerations have been presented rather than detailed computation founded on the assumption of certain laws for the distribution of velocities and for the forces acting between the molecules. The first reason for this choice is that the utility of an equation of state depends largely upon its ability to reproduce the measured pressure-volume-temperature data and to give accurate values for the thermodynamic derivatives, and hence in the evaluation of the specific form of functions used much consideration was given to the representation of the experimental measurements. In the second place, (15) differs from the Lorentz equation of state, which has been deduced in a rigorous manner from the general laws of kinetic theory, only in that the three second order constants a, b and c have been introduced.

The proposed equation rests upon the two main assumptions that the kinetic and cohesive pressures can be treated separately and that the law of force (the specific form of which does not enter into the treatment) shall be such that it will diminish rapidly with distance. Both of these assumptions are common to the treatment of a large number of kinetic theory problems.

4. The Proposed equation of State from the Standpoint of the Experimental Restrictions.

It remains to be seen whether or not the equation of state (15) conforms to the experimental conditions which were outlined in

Section 2. When Equation (15) is written out in full and the terms rearranged, there results for the virial form

$$pV = RT + \frac{\beta}{V} + \frac{\gamma}{V^2} + \frac{\delta}{V^3} \tag{16}$$

where

$$\beta = RTB_0 - A_0 - Rc/T^2$$

$$\gamma = -RTB_0b + A_0a - RB_0c/T^2$$

$$\delta = RB_0bc/T^2$$

For a given mass of a gas, β, γ and δ are functions of the temperature only and hence are constants at a given temperature.

I. Conditions at Constant Temperature. From a consideration of Equations (16) and (4) it can be seen that at constant temperature

$$\underset{1/V \doteq 0}{\text{Limit}}\, pV = RT$$

and that $\beta = f_1(T)$, $\gamma = f_2(T)$ and $\delta = f_3(T)$. Thus conditions (a) and (b) are fulfilled. Also by differentiation of (16) there results

$$\left(\frac{\partial pV}{\partial p}\right)_T = V + p\left(\frac{\partial V}{\partial p}\right)_T$$

$$= \frac{\beta + 2\gamma\rho + 3\delta\rho^2}{RT + 2\beta\rho + 3\gamma\rho^2 + 4\delta\rho^3}$$

where $\rho = 1/V$. Hence

$$\underset{1/V \doteq 0}{\text{Limit}} \left(\frac{\partial pV}{\partial p}\right)_T = \frac{\beta}{RT}$$

$$= B_0 - \frac{A_0}{RT} - \frac{c}{T^3}$$

so that there is only one temperature at which this derivative is zero—the Boyle temperature—since this cubic in $1/T$ has only one real root.

For a study of the equation of state at constant volume, (15) may be written in the form

$$p = T\psi(V) - \phi(V) - \frac{1}{T^2}\Gamma(V) \tag{17}$$

where

$$\psi(V) = \frac{R}{V^2}[V + B] = \frac{R}{V^2}[V + B_0\,(1 - b/V)]$$

$$\phi(V) = \frac{A}{V^2} = \frac{A_0}{V^2}(1 - a/V)$$

$$\Gamma(V) = \frac{c}{V}\,\psi(V) = \frac{cR}{V^3}[V + B_0\,(1 - b/V)]$$

II. Conditions at Constant Volume. By a comparison of (17) with (1), it is seen that $\frac{1}{T^2}\,\Gamma(V) = F(V,\,T)$. Hence as the temperature increases at constant volume, or as the volume decreases at constant temperature, this term approaches zero as a limit, thus fulfilling conditions (*c*) and (*d*), whereas if $c > 0$ this is also true of (*e*).

5. DETERMINATION OF THE VALUES OF THE CONSTANTS FROM THE EXPERIMENTAL DATA.

It is of major importance that the form of an equation of state be such that the values of the constants may be determined easily and uniquely from the experimental pressure-volume-temperature measurements. With the exception of *c*, each of the constants of the equation presented in this paper appears as the slope or intercept of a straight line when the data are treated in a suitable manner. In order to illustrate the method of determining the values of the constants, a detailed description for the gas oxygen is given.

The compressibility of oxygen has been studied by Holborn and Otto[32] over the temperature range from 0° to 100° C. and to 100 atmospheres, by Kamerlingh Onnes[32] and his associates from 0° to −117° C. and to 60 atmospheres, and by Amagat[32] from 0° to 200° C. and from 100 to 3000 atmospheres. Amagat's pressure measurements do not extend into the region covered by either the Berlin or Leiden investigations, and hence his data are not considered in the present paper, but will be treated in a later publication.

The values of the constants of the proposed equation of state can be most conveniently determined from those data which list the pressure for the same set of densities at each temperature. Many investigators present the results of their measurements in the form of isothermals, and they can be converted to isometrics by a modification of a method introduced by Andrews.[33]

On a given isotherm, the quantity $(pV - RT)$ is calculated for each measured density, and a linear equation passed through the points of zero density [where $(pV - RT)$ is also zero] and the highest measured density. The deviations of the measured values of $(pV - RT)$ from those calculated by the linear equation are plotted against the corresponding densities on a large scale. By use of the linear equation and the deviation plot, it is possible to obtain a series of values of pV, and hence of pressure, for a set of evenly spaced densities, the same densities being used at each temperature. In Table III are listed the smoothed data on oxygen which were obtained by this method. The pressures given in this table will be called the "observed pressures."

Throughout the present paper the units used are atmospheres for pressure, degrees Kelvin for temperature, and liters per mole for volume (or moles per liter for density). The position of the ice-point on the Kelvin scale is taken as 273.13°.

(a) *The F (V, T) Function.*

From Equation (5), it can be seen that if the form of function chosen for $F\ (V,\ T)$, namely

$$F(V,\, T) = \frac{c\psi(V)}{VT^2}$$

is accurate from the standpoint of both temperature and volume variations, there will be one value of c, which will cause the "corrected pressure"

$$p' = p + \frac{c\psi(V)}{VT^2} = T\psi(V) - \phi(V)$$

to vary linearly with the temperature for every isometric. In order to evaluate p' it is necessary to know $\psi(V)$ which is the slope $\left(\frac{\partial p'}{\partial T}\right)_V$, and since p' is not known until the additive term $F(V,\, T)$ has been evaluated, several approximations are required.

A provisional value for c is first obtained from a consideration of a single isometric, which should be of sufficiently high density to depart markedly from linearity, and which should also cover as long a temperature range as possible. For oxygen the isometric 3.5 moles per liter was used. A value is assumed for c and the term $F\ (V,\ T)$ evaluated at the temperatures T_2 and T_1 from the relation

$$F(V, T) = \frac{c}{VT^2}\left[\frac{p_2 - p_1}{T_2 - T_1}\right]_V = \frac{c}{VT^2}\left(\frac{\Delta p}{\Delta T}\right)_V$$

where p_2 and p_1 are the observed pressures at the highest and lowest temperatures T_2 and T_1 on the isometric. The results are added to p_2 and p_1, giving p'_2 and p'_1 to the first approximation. The calculation is repeated using the slope $\left(\frac{\Delta p'}{\Delta T}\right)_V$ resulting from the two values of p' just obtained, and this process continued, using the extreme temperatures only, until no further change appears in $\left(\frac{\Delta p'}{\Delta T}\right)_V$. With this slope, a set of provisional values of p' at each temperature on the isometric is computed. The slope of the best line through these data is then used in the computation of the final values of the corrected pressure, which are callod $p'_{\text{obs.}}$ in Tables I and II. Since the whole term $F(V, T)$ is usually small, several approximations are in general sufficient, and the actual amount of work is not large. The complete details of this calculation for oxygen are given in Table I. It should be noted that the number of approximations used in Table I is greater that necessary. Since $F(V, T)$ increases with decreasing temperature, the slope of the (p', T) curve is always less than that of the (p, T) curve, and hence the number of operations can be materially reduced by using, in the first approximation, a slope which is smaller than that given by $\left(\frac{\Delta p}{\Delta T}\right)_V$.

When the provisional value of c, which straightened the test isometric, is found, the values of p' for all of the isometrics are computed and tested for linearity. If it is considered that all of the isometrics are satisfactorily linear the provisional value of c is accepted as the final value; if not a small adjustment is made in the value of c and the calculation repeated.

In Table II, the effect of a variation in the value of c is shown. When c is increased, the numerical magnitude of the curvature of p' with T is decreased, aand if c becomes too large the curvature may even pass through zero and become positive.

Tables III and IV show the effect of the term $F(V, T)$ in the equation of state. In Table III the observed pressures of oxygen are compared with those calculated from a straight line through the extreme temperatures for each isometric. It will be noticed that the curvature is always negative and that its numerical magnitude increases with the density of the isometric. In Table IV are given the values of the corrected pressures p', which were computed from the observed pressures of Table III by the method given above, using the value 4.8×10^4 for c. They are compared with the values calculated from the equation of the best straight line[34] through each isometric.

When combining the data of Holborn and Otto with those of Kamerlingh Onnes and his associates, it was found that in the case of all gases studied, the measurements of the former gave isometrics having greater curvature than those of the latter, and thus the value of c represents in each case a compromise between the two sets of measurements.

TABLE I.

Details of the Method for Determination of the Value of *c*.

Calculation of p' from an Assumed Value for c, and Test of the Linear Variation of p' with T along an Isometric.

$$p' = p + F(V, T) = T\psi(V) - \phi(V)$$

$$F(V, T) = \frac{c\psi(V)}{VT^2}\,; \quad \psi(V) = \left(\frac{\partial p'}{\partial T}\right)_V$$

The value of $\left(\frac{\partial p'}{\partial T}\right)_V$ is obtained by several successive approximations.

To determine if a given value of c is satisfactory, select a test isometric

(1) which is of sufficient density for large deviations from linearity to occur, and

(2) which extends over as wide a temperature range as possible.

For oxygen the isometric $\rho = 3.5$ mols per liter was chosen.

Assume $c = 4.8 \times 10^4$ in units of atmospheres, liters per mole, and degrees Kelvin ($T°\,K = t°\,C + 273.13$).

Temp. °K	156.14	157.12	159.15	163.16	170.67	193.10	233.08	273.13	323.13
$p_{\text{obs.}}$	33.00	33.37	34.10	35.58	38.25	46.21	59.97	73.64	90.41

$$\psi_1(V) = \left(\frac{\Delta p}{\Delta T}\right)_V = \frac{90.41 - 33.00}{323.13 - 156.14} = 0.3438$$

$$F_1(V, T) = \frac{4.8 \times 10^4 \times 3.5 \times 0.3438}{T^2} = \frac{57760}{T^2}$$

$F_1(V, T)$	2.37								0.55
p'_1 (1st approx.)	35.37								90.96

$$\psi_2(V) = \left(\frac{\Delta p_1'}{\Delta T}\right)_V = 0.3329$$

$$F_2(V, T) = \frac{55930}{T^2}$$

TABLE I (continued).

Temp. °K	156.14	157.12	159.15	163.16	170.67	193.10	233.08	273.13	323.13
$F_2(V, T)$	2.29								0.54
p_2' (2nd approx.)	35.29								90.95

$$\psi_3(V) = 0.3333$$

$$F_3(V, T) = \frac{56000}{T^2}$$

$F_3(V, T)$	2.30	2.27	2.21	2.10	1.92	1.50	1.03	0.75	0.54
p' (final)	35.30	35.64	36.31	37.68	40.17	47.71	61.00	74.39	90.95

The best straight line through these values is

$$p' = 0.33313\ T - 16.67 \qquad (A)$$

$$\left(\frac{\partial p'}{\partial T}\right)_V = 0.33313$$

$$F(V, T) = \frac{55966}{T^2}$$

No further approximations are necessary and p' (final) becomes p' (obs.)

p' (calc. from A.)	35.34	35.67	36.35	37.68	40.19	47.66	60.98	74.32	90.97
$p'_{\text{obs.}} - p'_{\text{calc.}}$	−0.04	−0.03	−0.04	±0.00	−0.02	0.05	0.02	0.07	−0.02

TABLE II.

DETAILS OF THE METHOD FOR DETERMINATION OF THE VALUE OF c.

SELECTION OF A PROVISIONAL VALUE OF c.

$$p'_{obs.} = p_{obs.} + F(V, T).$$

$p'_{obs.}$ is calculated by the method given in TABLE I.
$p'_{calc.}$ is calculated from the best linear equation through the values of $p'_{obs.}$
The isometric is the same as that used in TABLE I: 3.5 moles per liter.

Temp. °K	156.14	157.12	159.15	163.16	170.67	193.10	233.08	273.13	323.13
$p_{obs.}$	33.00	33.37	34.10	35.58	38.25	46.21	59.97	73.64	90.41
				Assume $c = 4.0 \times 10^4$					
$F(V, T)$	1.92	1.90	1.85	1.76	1.61	1.26	0.86	0.63	0.45
$p'_{obs.}$	34.92	35.27	35.95	37.34	39.86	47.47	60.83	74.27	90.86
				$p' = 0.33499\, T - 17.31$					
$p'_{calc.}$	35.00	35.32	36.00	37.35	39.86	47.38	60.77	74.19	90.94
$p'_{obs.} - p'_{calc.}$	−0.08	−0.05	−0.05	−0.01	0.00	0.09	0.06	0.08	−0.08
				Assume $c = 4.8 \times 10^4$					
$F(V, T)$	2.30	2.27	2.21	2.10	1.92	1.50	1.03	0.75	0.54
$p'_{obs.}$	35.30	35.64	36.31	37.68	40.17	47.71	61.00	74.39	90.95
				$p' = 0.33313\, T - 16.67$					
$p'_{calc.}$	35.34	35.67	36.35	37.68	40.19	47.66	60.98	74.32	90.97
$p'_{obs.} - p'_{calc.}$	−0.04	−0.03	−0.04	0.00	−0.02	0.05	0.02	0.07	−0.02

TABLE II (continued).

Temp. °K	155.14	157.12	159.15	163.16	170.67	193.10	233.08	273.13	323.13
				Assume $c = 5.6 \times 10^4$					
$F(V, T)$	2.67	2.63	2.57	2.44	2.23	1.74	1.20	0.87	0.62
$p'_{\text{obs.}}$	35.67	36.00	36.67	38.02	40.48	47.95	61.17	74.51	91.03
				$p' = 0.33152\,T - 16.08$					
$p'_{\text{calc.}}$	35.68	36.01	36.68	38.01	40.50	47.94	61.19	74.47	91.04
$p'_{\text{obs.}} - p'_{\text{calc.}}$	−0.01	−0.01	−0.01	0.01	−0.02	0.01	−0.02	0.04	−0.01

For this isometric 5.6×10^4 is the best value for c. When the other isometrics were considered it was found that this was too high and hence 4.8×10^4 was chosen as the final value for c.

TABLE III.

Curvature of the Isometrics of Gaseous Oxygen.

Comparison of the Observed Pressures with those Calculated on the Assumption of Linear Isometrics.

For each isometric the first line gives the linear equation passed through the extreme temperatures, the second line gives the observed pressure, the third the pressure calculated from the equation, and the fourth the observed minus the calculated pressure.

Temp. °C.	−116.99	−116.01	−113.98	−109.97	−102.46	−80.03	−40.05	0	0	20	50	100
Observer	L	L	L	L	L	L	L	L	B	L	B	B
Density moles/liter					Pressures in atmospheres							
0.5					$p = 0.04212\,T - 0.43$							
	6.15	6.19	6.28	6.45	6.77	7.71	9.40	11.09	11.09	11.93	13.19	15.29
	6.15	6.19	6.27	6.44	6.76	7.70	9.39	11.07	11.07	11.92	13.18	15.29
	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.01	0.01	0.00

TABLE III (continued).

Temp. °*C.* Observer Density moles/liter	−116.99 L	−116.01 L	−113.98 L	−109.97 L	−102.46 L	−80.03 L	−40.05 L	0 L	0 B	20 L	50 B	100 B
					Pressures in atmospheres							
1.0					$p = 0.08641\ T - 1.70$							
	11.79	11.87	12.06	12.41	13.07	15.01	18.48	21.95	21.95	23.67	26.26	30.54
	11.79	11.88	12.05	12.40	13.05	14.99	18.44	21.90	21.90	23.63	26.22	30.54
	0.00	−0.01	0.01	0.01	0.02	0.02	0.04	0.05	0.05	0.04	0.04	0.00
1.5					$p = 0.13296\ T - 3.83$							
	16.93	17.05	17.35	17.90	18.93	21.92	27.27	32.61	32.61	35.25	39.22	45.78
	16.93	17.06	17.33	17.86	18.86	21.84	27.16	32.49	32.49	35.14	39.13	45.78
	0.00	−0.01	0.02	0.04	0.07	0.08	0.11	0.12	0.12	0.11	0.09	0.00
2.0					$p = 0.18171\ T - 6.77$							
	21.60	21.77	22.17	22.94	24.35	28.47	35.79	43.09	43.09	46.68	52.10	61.03
	21.60	21.78	22.15	22.88	24.24	28.32	35.58	42.86	42.86	46.49	51.95	61.03
	0.00	−0.01	0.02	0.06	0.11	0.15	0.21	0.23	0.23	0.19	0.15	0.00
2.5					$p = 0.23273\ T - 10.53$							
	25.81	26.04	26.55	27.54	29.36	34.69	44.06	53.39	53.42	57.97	64.91	76.31
	25.81	26.04	26.51	27.44	29.19	34.41	43.71	53.04	53.04	57.69	64.67	76.31
	0.00	0.00	0.04	0.10	0.17	0.28	0.35	0.35	0.38	0.28	0.24	0.00
3.0					$p = 0.28591\ T - 15.04$							
	29.60	29.89	30.51	31.74	33.98	40.60	52.10		63.59		77.67	91.64
	29.60	29.88	30.46	31.61	33.76	40.17	51.60		63.05		77.35	91.64
	0.00	0.01	0.05	0.13	0.22	0.43	0.50		0.54		0.32	0.00

TABLE III (continued).

Temp. °*C*.	−116.99	−116.01	−113.98	−109.97	−102.46	−80.03	−40.05	0	0	20	50	100
Observer	L	L	L	L	L	L	L	L	B	L	B	B
Density moles/liter					Pressures in atmospheres							
3.5					$p = 0.34379\,T - 20.68$							
	33.00	33.37	34.10	35.58	38.25	46.21	59.97		73.64		90.41	
	33.00	33.34	34.03	35.41	37.99	45.71	59.45		73.22		90.41	
	0.00	0.03	0.07	0.17	0.26	0.50	0.52		0.42		0.00	
4.0					$p = 0.40188\,T - 26.72$							
	36.03	36.49	37.33	39.07	42.21	51.54			83.58		103.14	
	36.03	36.42	37.24	38.85	41.87	50.88			83.05		103.14	
	0.00	0.07	0.09	0.22	0.34	0.66			0.53		0.00	
4.5					$p = 0.46765\,T - 34.30$							
	38.72	39.25	40.21	42.23	45.86	56.60			93.43			
	38.72	39.18	40.13	42.00	45.51	56.00			93.43			
	0.00	0.07	0.08	0.23	0.35	0.60			0.00			
5.0					$p = 0.53081\,T - 41.80$							
	41.08	41.67	42.76	45.07	49.22	61.42			103.18			
	41.08	41.60	42.68	44.81	48.79	60.70			103.18			
	0.00	0.07	0.08	0.26	0.43	0.72			0.00			

$T = t°\,C + 273.13$

L = Leiden data. B = Berlin data (Holborn and Otto)

TABLE IV.

THE CORRECTED PRESSURES, p', OF GASEOUS OXYGEN.

Comparison of the Observed Corrected Pressures along an Isometric with those calculated from a Linear Equation.

For each isometric the first line gives the best linear equation through the data, the second line gives the observed corrected pressures, the third the values calculated from the equation, and the fourth the observed minus the calculated values.

$$p'_{\text{obs.}} = p_{\text{obs.}} + \frac{4.8 \times 10^4}{VT^2}\left(\frac{\partial p'}{\partial T}\right)_V$$

Temp. °C.	−116.99	−116.01	−113.98	−109.97	−102.46	−80.03	−40.05	0	0	20	50	100
Observer	L	L	L	L	L	L	L	L	B	L	B	B
Density moles/liter	Pressures in atmospheres											
0.5	$p' = 0.04198\ T - 0.36$											
	6.19	6.23	6.32	6.49	6.80	7.74	9.42	11.10	11.10	11.94	13.20	15.30
	6.19	6.24	6.32	6.49	6.80	7.75	9.42	11.11	11.11	11.95	13.20	15.30
	0.00	−0.01	0.00	0.00	0.00	−0.01	0.00	−0.01	−0.01	−0.01	0.00	0.00
1.0	$p' = 0.08581\ T - 1.44$											
	11.96	12.04	12.22	12.56	13.21	15.12	18.56	22.01	22.01	23.72	26.30	30.57
	11.96	12.04	12.22	12.56	13.21	15.13	18.56	22.00	22.00	23.71	26.29	30.58
	0.00	0.00	0.00	0.00	0.00	−0.01	0.00	0.01	0.01	0.01	0.01	−0.01
1.5	$p' = 0.13157\ T - 3.22$											
	17.32	17.43	17.72	18.26	19.26	22.17	27.44	32.74	32.74	35.36	39.31	45.85
	17.32	17.45	17.72	18.25	19.24	22.19	27.45	32.72	32.72	35.35	39.29	45.87
	0.00	−0.02	0.00	0.01	0.02	−0.02	−0.01	0.02	0.02	0.01	0.02	−0.02

TABLE IV (continued).

Temp. °C.	−116.99	−116.01	−113.98	−109.97	−102.46	−80.03	−40.05	0	0	20	50	100
Observer	L	L	L	L	L	L	L	L	B	L	B	B
Density moles/liter	Pressures in atmospheres											
2.0	$p' = 0.17919\,T - 5.66$											
	22.31	22.47	22.85	23.59	24.94	28.93	36.11	43.32	43.32	46.88	52.26	61.15
	22.32	22.49	22.86	23.58	24.92	28.94	36.11	43.28	43.28	46.87	52.24	61.20
	−0.01	−0.02	−0.01	0.01	0.02	−0.01	0.00	0.04	0.04	0.01	0.02	−0.05
2.5	$p' = 0.22863\,T - 8.73$											
	26.94	27.15	27.63	28.57	30.30	35.43	44.57	53.76	53.79	58.29	65.17	76.51
	26.97	27.19	27.66	28.57	30.29	35.42	44.56	53.72	53.72	58.29	65.15	76.58
	−0.03	−0.04	−0.03	0.00	0.01	0.01	0.01	0.04	0.07	0.00	0.02	−0.07
3.0	$p' = 0.27987\,T - 12.41$											
	31.25	31.52	32.10	33.25	35.36	41.68	52.84		64.13		78.06	91.93
	31.29	31.56	32.13	33.25	35.36	41.63	52.82		64.03		78.02	92.02
	−0.04	−0.04	−0.03	0.00	0.00	0.05	0.02		0.10		0.04	−0.09
3.5	$p' = 0.33313\,T - 16{,}67$											
	35.30	35.64	36.31	37.68	40.17	47.71	61.00		74.39		90.95	
	35.34	35.67	36.35	37.68	40.19	47.66	60.98		74.32		90.97	
	−0.04	−0.03	−0.04	0.00	−0.02	0.05	0.02		0.07		−0.02	
4.0	$p' = 0.38784\,T - 21.42$											
	39.08	39.51	40.27	41.87	44.77	53.54			84.58		103.85	
	39.14	39.52	40.30	41.86	44.77	53.47			84.51		103.90	
	−0.06	−0.01	−0.03	0.01	0.00	0.07			0.07		−0.05	

TABLE IV (continued).

Temp. °*C*. Observer	−116.99 L	−116.01 L	−113.98 L	−109.97 L	−102.46 L	−80.03 L	−40.05 L	0 L	0 B	20 L	50 B	100 B
Density moles/liter	Pressures in atmospheres											
4.5	$p' = 0.44482\ T - 26.76$											
	42.66	43.14	44.00	45.84	49.16	59.18			94.72			
	42.69	43.13	44.03	45.82	49.16	59.13			94.73			
	−0.03	0.01	−0.03	0.02	0.00	0.05			−0.01			
5.0	$p' = 0.50286\ T - 32.48$											
	46.03	46.56	47.52	49.60	53.36	64.66			104.80			
	46.04	46.53	47.55	49.57	53.34	64.62			104.87			
	−0.01	0.03	−0.03	0.03	0.02	0.04			−0.07			

$T = t°C + 273.13$

(*b*) *The* $\psi(V)$ *and* $\phi(V)$ *Functions.*

It can be seen by Equation (5)

$$p' = T\psi(V) - \phi(V)$$

that for each isometric, the quantities $\psi(V)$ and $\phi(V)$ are the slope and the intercept on the p' axis of the (p', T) line. The linear equations expressing the relations between the corrected pressures and the temperatures for the isometrics of oxygen are given in Table IV, and the slopes and intercepts of these lines are the "observed values" of $\psi(V)$ and $\phi(V)$ given in Tables V and VI.

Since

$$\psi(V) = \frac{R}{V^2}[V + B], \quad \text{where } B = B_0\left(1 - \frac{b}{V}\right);$$

then

$$B = \frac{V^2\psi(V)}{R} - V$$

and from R, V and the observed value of $\psi(V)$, a value of B can be calculated for each isometric. The quantities B_0 and B_0b are the intercept on the B axis and the slope respectively of a straight line through a graph of the calculated values of B against $\frac{1}{V}$.

Also

$$\phi(V) = \frac{A}{V^2}, \quad \text{where } A = A_0\left(1 - \frac{a}{V}\right)$$

and hence

$$A = V^2\phi(V)$$

and from V and the observed value of $\phi(V)$ obtained from Table IV, a value of A can be calculated for each isometric. On a plot of A against $\frac{1}{V}$, the quantities A_0 and A_0a appear as the intercept on the A axis and the slope of a straight line through the points.

The graphs of the B and A functions for oxygen are given in Figures I and II, and the details of the calculations, in Tables V and VI. Since the graphs are indirect as regards $\psi(V)$ and $\phi(V)$, it is necessary to calculate these functions with the chosen values of the constants, and compare with the observed values, adjusting the constants if necessary to give the best representation over the entire density region considered. The form of these functions is such that the most

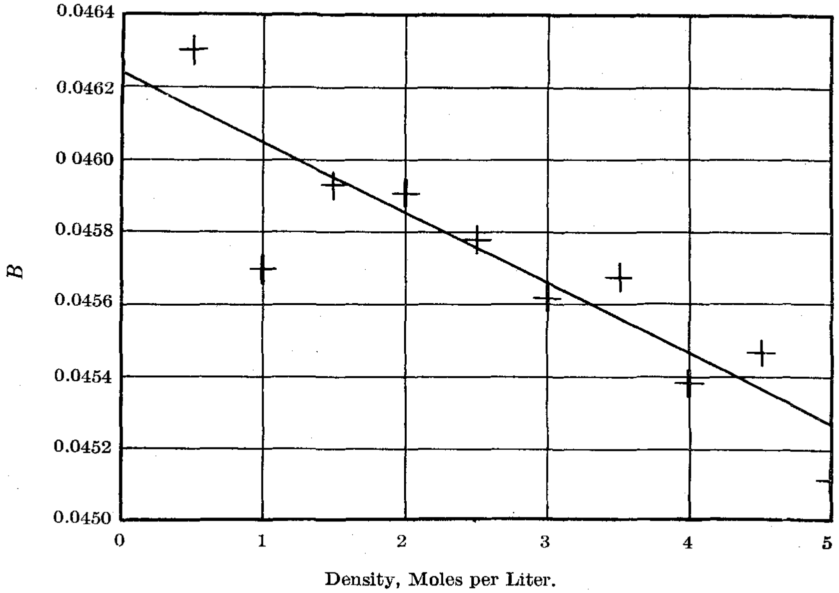

FIGURE 1. The *B* Plot.

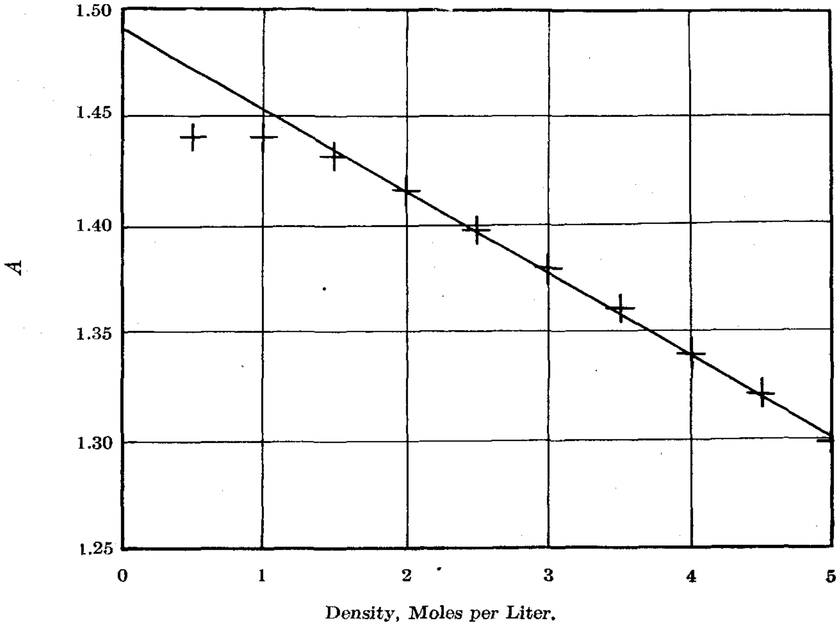

FIGURE 2. The *A* Plot.

weight should be placed on the points in the region of high density when drawing the straight lines on the B and A plots. However, as the density of an isometric increases, the temperature range usually becomes less, and the accuracy of obtaining the functions $\psi(V)$ and $\phi(V)$ from the experimental data decreases. The experimental data on oxygen extends to a density of 9 moles per liter, but above 5 moles per liter the temperature range is only about 15° C. Hence for the determination of the values of the constants only the region below a density of 5 was used, the comparison of the observed and calculated pressures being given however over the entire density range.

The constants of the equation of state of oxygen are given in Table IX, and the comparison of the calculated with the observed pressures in Tables XV and XX. The average deviation over the whole field is 0.05 atmospheres or 0.12%, the maximum density considered being 9 moles per liter.

TABLE V.

THE $\psi(V)$ FUNCTION

$$\psi(V) = \frac{R}{V^2}[V + B]; \quad B = B_0\left(1 - \frac{b}{V}\right)$$

$$\psi(V)_{\text{obs.}} = \left(\frac{\partial p'}{\partial T}\right)_V \text{(from Table IV)}; \quad B_{\text{obs.}} = \frac{V^2\psi(V)_{\text{obs.}}}{R} - V$$

	B		$\psi(V)$		
Density moles/liter	Observed	Calculated	Observed	Calculated	Obs.-Calc. $\times 10^5$
0.5	0.0463	0.046143	0.04198	0.04198	0
1.0	.0457	.046045	.08581	.08584	−3
1.5	.04593	.045948	.13157	.13157	0
2.0	.04591	.045851	.17919	.17917	2
2.5	.04578	.045754	.22863	.22862	1
3.0	.04562	.045656	.27987	.27990	−3
3.5	.04568	.045559	.33313	.33301	12
4.0	.04539	.045462	.38784	.38793	−9
4.5	.04547	.045364	.44482	.44465	17
5.0	.04512	.045267	.50286	.50317	−31

$$B_{\text{calc.}} = 0.04624\left(1 - \frac{0.004208}{V}\right)$$

$$\psi(V)_{\text{calc.}} = \frac{0.08206}{V^2}[V + B_{\text{calc.}}]$$

TABLE VI

THE $\phi(V)$ FUNCTION

$$\phi(V) = \frac{A}{V^2}; \quad A = A_0\left(1 - \frac{a}{V}\right)$$

$\phi(V)_{\text{obs.}} = -p'$ at 0°K (from Table IV); $A_{\text{obs.}} = V^2\phi(V)_{\text{obs.}}$

	A		$\phi(V)$		
Density moles/liter	Observed	Calculated	Observed	Calculated	Obs.-Calc.
0.5	1.44	1.4720	0.36	0.37	−0.01
1.0	1.44	1.4529	1.44	1.45	−0.01
1.5	1.431	1.4338	3.22	3.23	−0.01
2.0	1.415	1.4147	5.66	5.66	±0.00
2.5	1.397	1.3956	8.73	8.72	+0.01
3.0	1.379	1.3765	12.41	12.39	+0.02
3.5	1.361	1.3574	16.67	16.63	+0.04
4.0	1.339	1.3383	21.42	21.41	+0.01
4.5	1.321	1.3192	26.76	26.71	+0.05
5.0	1.299	1.3001	32.48	32.50	−0.02

$$A_{\text{calc.}} = 1.4911\left(1 - \frac{0.02562}{V}\right)$$

$$\phi(V)_{\text{calc.}} = \frac{A_{\text{calc.}}}{V^2}$$

6. THE EFFECT OF SMALL VARIATIONS IN THE VALUES OF THE CONSTANTS.

It is perhaps of interest to discuss the allowable variation in the values of the constants of the equation of state in order that the pressure may be calculated to any desired accuracy. Consider the equation in the form

$$p = T\psi(V) - \phi(V) - F(V, T)$$

Now since

$$T\psi(V) = \frac{RT}{V^2}[V + B], \quad B = B_0\left(1 - \frac{b}{V}\right)$$

the error $\Delta[T\psi(V)]$ and fractional error $\dfrac{\Delta[T\psi(V)]}{T\psi(V)}$ produced by the variations δB_0, δb, and δR are given by the following equations:

$$(a) \quad \Delta[T\psi(V)] = \frac{RT}{V^2}\left(1 - \frac{b}{V}\right)\delta B_0 = \frac{RT}{V^2}\frac{B}{B_0}\delta B_0$$

$$\frac{\Delta[T\psi(V)]}{T\psi(V)} = \frac{B}{V + B}\,\frac{\delta B_0}{B_0}$$

$$(b)\quad \Delta[T\psi(V)] = -\frac{RT}{V^3}B_0\,\delta b$$

$$\frac{\Delta[T\psi(V)]}{T\psi(V)} = -\frac{bB_0}{V(V+B)}\,\frac{\delta b}{b}$$

$$(c)\quad \Delta[T\psi(V)] = \frac{T\psi(V)}{R}\,\delta R$$

$$\frac{\Delta[T\psi(V)]}{T\psi(V)} = \frac{\delta R}{R}$$

and since

$$\phi(V) = \frac{A}{V^2},\quad A = A_0\left(1 - \frac{a}{V}\right)$$

the error and fractional error in $\phi(V)$ produced by the variations δA_0 and δa are

$$(d)\quad \Delta\phi(V) = \frac{\phi(V)}{A_0}\,\delta A_0$$

$$\frac{\Delta\phi(V)}{\phi(V)} = \frac{\delta A_0}{A_0}$$

$$(e)\quad \Delta\phi(V) = -\frac{A_0}{V^3}\,\delta a$$

$$\frac{\Delta\phi(V)}{\phi(V)} = -\frac{A_0 a}{VA}\,\frac{\delta a}{a}$$

Also

$$F(V,\,T) = \frac{c\psi(V)}{VT^2}$$

and the effect of a variation δc is given by the equations

$$(f)\quad \Delta F(V,\,T) = \frac{F(V,\,T)}{c}\,\delta c$$

$$\frac{\Delta F(V,\,T)}{F(V,\,T)} = \frac{\delta c}{c}$$

In Table VII are given the allowable variations and fractional variations in the several constants which will each cause an error of 0.1% in the calculation of the pressure of oxygen at the densities 1

TABLE VII.

Allowable Variations in the Constants of the Equation of State which will each Cause a Variation of 0.1% in the Calculated Pressure of Oxgyen.

These values divided by 6 give the allowable variations which would together produce a total error of 0.1% in the calculated pressure.

$t = 100°C.$

	$\rho = 1$ mole/liter			$\rho = 6$ moles/liter		
Symbol	Value	Absolute error	% Error	Value	Absolute error	% Error
R	0.08206	0.000079	0.097%	0.08206	0.000066	0.081%
A_0	1.4911	0.0318	2.13	1.4911	0.0061	0.41
a	0.02562	0.02079	81.1	0.02562	0.00058	2.27
B_0	0.04624	0.00102	2.20	0.04624	0.00018	0.38
b	0.004208	0.02192	521.	0.004208	0.00061	14.6
c	4.8×10^4	4.96×10^4	103.	4.8×10^4	0.69×10^4	14.5
B	0.046045			0.045073		
$\psi(V)$	0.085838			0.625513		
$T\psi(V)$	32.029			233.398		
A	1.4529			1.26189		
$\phi(V)$	1.453			45.428		
$F(V,T)$	0.030			1.294		
P	30.546	0.031	0.10	186.676	0.187	0.10

TABLE VIII.

Allowable Variations in the Constants of the Equation of State which will each Cause a variation of 0.1% in the Calculated Pressure of Oxgyen.

$t = -100°C.$

Symbol	$\rho = 1$ mole/liter			$\rho = 6$ moles/liter		
	Value	Absolute error	% Error	Value	Absolute error	% Error
R	0.08206	0.000072	0.088%	0.08206	0.000046	0.056%
A_0	1.4911	0.0133	0.89	1.4911	0.0019	0.13
a	0.02562	0.00872	34.0	0.02562	0.00018	0.69
B_0	0.04624	0.00093	2.01	0.04624	0.00012	0.26
b	0.004208	0.01997	475.	0.004208	0.00043	10.
c	4.8×10^4	0.46×10^4	9.5	4.8×10^4	0.046×10^4	0.95
B	0.046045			0.045073		
$\psi(V)$	0.085838			0.625513		
$T\psi(V)$	14.861			108.295		
A	1.4529			1.2619		
$\phi(V)$	1.453			45.428		
$F(V,T)$	0.137			6.010		
P	13.271	0.013	0.10	56.857	0.057	0.10

mole per liter and 6 moles per liter and at 100° C; and in Table VIII the same information is given for −100° C. As the density increases or as the temperature decreases, the allowable variations in the values of the constants become smaller. Thus as mentioned in the preceding section when determining the constants from the experimental data, most attention is given to the high density data when drawing the straight lines on the B and A plots; and the value of c depends largely on the low temperature high density measurements where the function $F(V, T)$ is large.

7. Discussion of the Experimental Data.

The references to all of the data considered in this paper are listed in the Bibliography, which gives the temperatures in degrees centigrade (unless otherwise noted) of the isotherms studied by each investigator as well as the pressure range in atmospheres over which the measurements extended.

General Considerations. For all gases for which comparison is possible—helium, neon, argon, hydrogen, nitrogen, oxygen, and air—the data of Holborn and Otto, when converted into constant volume form, gave isometrics with greater curvature than those of Kamerlingh Onnes and his collaborators; in fact, at higher temperatures, above 100° C., the curvatures of the isometrics of Holborn and Otto for some gases increased with temperature. Hence in obtaining a value for the equation of state constant c, which depends directly upon the curvature of the isometrics, the mean between the curvatures of the two sets of data was used.

Holborn and Otto, and Holborn and Schultze in general made measurements at about 25, 50, 75, and 100 atmospheres on each isotherm. The temperatures varied several hundredths of a degree, and were reduced by the experimentators to even values. Besides presenting the original data, they also gave an equation for each isotherm expressing pV in the form of a power series in p as well as smoothed tables listing, for each temperature, the pV product for even intervals of pressure. Pressures were expressed in terms of meters of mercury and the unit of pV was the value at 0° C. and one meter of mercury.

Kamerlingh Onnes and his collaborators measured at unevenly spaced temperature intervals, and sometimes several isotherms for the same gas differed by only a few hundredths or tenths of a degree. In other cases low pressure data were given for one temperature, and higher pressure measurements at a closely nearby temperature. For some isotherms, equations were given expressing pV as a function of the density. The unit of pressure was the atmosphere, and pV was expressed in terms of the value at 0° C. and one atmosphere.

Penning (Leiden Comm. 166) has pointed out that the earlier data obtained at Leiden with the "divided piezometer" contains an unknown volume error due to the soldered connection between the glass and the steel capillaries, which connect the two parts of the piezometer. This error may be small and probably does not effect the results until the density of the gas becomes large.

Two isotherms which differed by only a few hundredths of a degree were reduced to the temperature of one of them or to an intermediate temperature. When smoothing the data to even densities as described in Section 5, the appropriate values of T were used in evaluating for each isotherm the quantities $(pV - RT)$, which were then treated as though they were derived from the same isotherm. The appropriate value of T was also used in the calculation of pV at the even densities from the smoothed differences $(pV - RT)$. The temperature scales of the various experimenters differed somewhat, and the values of T used for the calculations presented in this paper were obtained in all cases by adding 273.13 to the centigrade temperatures listed by the observers.

The data of Amagat were not used except for gaseous carbon dioxide. For no other gas considered in the present paper did he give accurate measurements below 100 atmospheres, and since practically all of the data of Leiden and the Reichsanstalt were below this pressure, there was not sufficient overlapping for a good correlation.

For the seven gases mentioned above, the equations of state rest on a combination of the data of Holborn and Otto and Kamerlingh Onnes and his co-workers, about equal weight having been given to both sets of measurements. In some cases—hydrogen, nitrogen, and air—the equations so obtained were used to calculate values for comparison with the data of other observers.

In general, when an equation was passed through the data of either of the above two sets of measurements alone, the deviations for this group between observed and calculated pressures were about one-half of the deviations obtained by using an equation based on both groups. This would seem to indicate that the consistency of a set of data taken by one observer is usually greater than the absolute accuracy of the data. It is believed that by combining all the latest data, the best average values for the constants were obtained.

Helium, Neon, and Argon. Only the measurements made at Leiden and the Reichsanstalt were used. In averaging the deviations for helium, the isotherms below −252° C. were omitted, as the pressures were so low that a disproportionately large percentage deviation resulted. The deviations in atmospheres were about the same magnitude as those for the rest of the field.

Hydrogen. The equation is based on the Leiden and Reichsanstalt measurements. The temperatures of the Onnes and Braak isotherms were reduced to the Kelvin scale by Penning (Leiden Comm. 165(b)). The low temperature data of Crommelin and Swallow could not be correlated with the rest of the field as their isometrics had positive curvature.

The lowest pressure measured by Bartlett (at 0° C.) and by Verschoyle (at 0° and 20°) was about 50 atmospheres. The data of these observers on hydrogen were not used in the present paper but will be considered in a later publication covering the high pressure range.

There is quite an extensive set of measurements by Witkowski, and the equation derived from the Leiden and Reichsanstalt data was used to calculate values for comparison with his measurements. The agreement was good and his deviations are averaged with the others, although most of his points show positive deviations from the equation.

Nitrogen. The equation based on the Leiden and Reichsanstalt measurements agreed quite well with the 0° C. isotherm of Bartlett, and the 0° and 20° isotherms of Verschoyle. This is the accepted equation for nitrogen.

The data of Smith and Taylor could not be correlated with the measurements of the other investigators. A special equation was passed through the Smith and Taylor data, and it showed that their measurements are very consistent, except for the region below a density of 5 moles per liter, which depends on a single isometric.

Oxygen. The data of Holborn and Otto and of Kuijpers are in good agreement. The measurements of Nijhoff and Keesom covered such a short pressure range that they were not considered.

Air. The measurements of Holborn and Schultze, Witkowski, Koch and Penning were all used in the determination of the constants. The −130° to −145° isotherms of Witkowski were omitted due to the impossibility of correlating them with the rest of the field.

Carbon Dioxide. Only the 0 to 100 atmosphere data of Amagat were used for determining the values of the constants.

The measurements of Andrews, which were expressed relative to air, were transformed to absolute values using the Reichsanstalt data on air. The temperatures of Andrews' isotherms varied several tenths of a degree from point to point and each reading was corrected to a mean value of the temperature. The 21.5° isotherm had only one point in the gas region, and the 100°, 63.75°, and 48.1° isotherms show extreme variations from Amagat's data and hence are omitted from the average. The equation based on Amagat's values was used in calculating Andrews' data.

Methane. The measurements of Keyes and Burks supersede those of Keyes, Smith and Joubert and the equation is derived for the former data alone. Amagat's older measurements were not considered.

Ether. The data of Beattie alone were used. The original measured pressures, and not the smoothed values, are given. The densitites were converted from grams per c.c. to moles per liter, otherwise no change was introduced.

8. DISCUSSION OF THE COMPARISON OF THE EQUATION WITH THE MEASUREMENTS.

In Table IX are given the values of the constants for the ten gases obtained by the method outlined in Section 5. The data used for the determination of the constants are critically examined in Section 7.

The comparisons of the observed pressures (smoothed to even values of the density but not smoothed for temperature) with those calculated from the equation of state are given in Tables X to XIX and a summary of the average deviations and average percentage deviations is given in Table XX.

In the tables for each gas, the observed pressures and the observed minus the calculated pressures are listed. The average deviation in atmospheres and percent are given for each isometric and for the whole field considered. With very few exceptions, all of which are mentioned as footnotes to the tables or in Section 7, all of the recent data, which cover any considerable temperature and density range, were considered and included in the averages.

The entire experimental temperature range was included in the comparison for all gases except Amagat's data above 100° C. on carbon dioxide. For several of the gases the entire measured density range was not used. In some instances the data in the region of higher density were fragmentary, in other cases the temperature range was small. In the Bibliography are given the complete range of pressures measured on each isotherm and comparison with the tables will show what densities are omitted in the latter.

Comparisons of observed and calculated pressures for the ten gases were made at 1777 points and the general average deviation was 0.08 atmospheres or 0.18%. It is considered that the equation reproduces the measured pressures very satisfactorily.

The A function for hydrogen is practically independent of the density, i.e. the value of the constant a is almost zero. However the representation of the experimental data was somewhat better when a negative value was used for this constant. It is believed that this is due to a slight trend in the experimental measurements, and that the value of a for hydrogen should be positive, as it is for all other gases.

It can be seen that the value of b for the monotomic gases is zero. It is of interest to note that the constant α in the Keyes' equation of state was also found[35] to be zero for these gases. Although the forms of these two equations of state are quite different, the constant α of Keyes' equation may be considered to correspond approximately to b.

9. USE OF THE EQUATION OF STATE IN THERMODYNAMIC CALCULATIONS.

For the theoretical discussion the most suitable form of the equation of state is

$$p = T\psi(V) - \phi(V) - F(V, T) \qquad (A)$$

where

$$\psi(V) = \frac{R}{V^2}[V + B]; \quad B = B_0\left(1 - \frac{b}{V}\right)$$

$$\phi(V) = \frac{A}{V^2}; \quad A = A_0\left(1 - \frac{a}{V}\right)$$

$$F(V, T) = \frac{c\psi(V)}{T^2}$$

For use in certain thermodynamic calculations however, the forms (B) and (C) given below, have also been found convenient.

$$p = T\psi(V) - \phi(V) - \frac{\Gamma(V)}{T^2} \qquad (B)$$

where

$$\Gamma = c\psi(V)$$

$$p = RT + \frac{\beta}{V} + \frac{\gamma}{V^2} + \frac{\delta}{V^3} \qquad (C)$$

where

$$\beta = RTB_0 - A_0 - \frac{Rc}{T^2}$$

$$\gamma = -RTB_0b + A_0a - \frac{RB_0c}{T^2}$$

$$\delta = \frac{RB_0bc}{T^2}$$

For the calculation of pressures along an isometric or an isothermal, the forms (*A*) and (*C*) respectively are the most convenient; and in general for isometric or isothermal calculations these two equations give the desired result most directly.

In some cases where it is desired to keep the temperature and pressure functions separated, the form (*B*) in which all the functions $\psi(V)$, $\phi(V)$ and $\Gamma\ (V)$ depend upon volume alone is more suitable.

In a series of papers, the authors propose to use the equation of state presented in the present article, for the calculation of various thermodynamic quantities.

10. SUMMARY.

There is presented a new equation of state for fluids

$$p = \frac{RT(1-\varepsilon)}{V^2}[V+B] - \frac{A}{V^2} \qquad A = A_0\left(1-\frac{a}{V}\right)$$

$$B = B_0\left(1-\frac{b}{V}\right)$$

$$\varepsilon = \frac{c}{VT^3}$$

which has the desirable properties: (*a*) ease and uniqueness of determination of the values of the constants for a substance from the experimental pressure-volume-temperature measurements, (*b*) simplicity of mathematical form, (*c*) accuracy of representation of the pressure-volume-temperature data over wide ranges of temperature and density, (*d*) provision for extension of the volume and temperature functions A, B and ε without alteration of the general form of the equation.

A rational basis for this equation of state is presented, using a method introduced by Phillips, which considers that both of the terms A and B are due to the mutual interactions of the molecules composing a gas. The method is a very general one and does not involve the assumption of any definite law of force. The term ε is introduced to take into account the effect of density and temperature on the time of encounter of the molecules.

A summary is given of the general experimental facts known about the isotherms and isometrics of a gas, and the new equation of state is studied from the standpoint of this information. It is shown (1) that along an isometric the curvature $\left(\frac{\partial^2 p}{\partial T^2}\right)_V$ is always negative, but approaches zero as the temperature increases or as the density of the isometric decreases, (2) that along an isotherm the pressure approaches $\frac{RT}{V}$ as the density decreases, but $\left(\frac{\partial pV}{\partial p}\right)_T$ at infinitely low pressures is zero at only one temperature being negative below and positive above this value.

The complete detailed method used for the determination of the values of the constants from any set of pressure-volume-temperature data is given, with the necessary graphs and tables. The data on oxygen is used for this example, which is carried out to a numerical solution for the five adjustable constants A_0, a, B_0, b, and c. The method is unambiguous, and the values of the constants are uniquely obtained.

The errors produced in the calculation of pressure by small variations of the value of the constants are discussed, and the numerical values given for oxygen.

An extensive bibliography of the compressibility data on the ten gases helium, neon, argon, hydrogen, nitrogen, oxygen, air, carbon dioxide, methane, and ethyl ether is given and critically examined.

In Tables IX to XIX are presented the detailed comparisons of pressures calculated from the equation of state with the observed pressures and the results are summarized in Table XX. The comparison is made at 1777 points for the ten gases with an average deviation of 0.08 atmospheres or 0.18%. It is believed that in most of the cases the consistency of a set of data taken by one observer is greater than their absolute accuracy, and that the equation of state reproduces the measurements to within the experimental error.

The more convenient forms of the equation of state for thermodynamic calculations are briefly discussed. In future papers, the authors will apply the equation to the high pressure data on gases and liquids, and present the results of the calculations of various experimentally measured thermodynamic quantities.

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11. Comparison of the Observed with the Calculated Pressures for Ten Gases

TABLE IX.

Values of the Equation of State Constants for Ten Gases.

$$p = \frac{RT(1-\varepsilon)}{V^2}[V+B] - \frac{A}{V^2}$$

$$A = A_0\left(1 - \frac{a}{V}\right)$$

$$B = B_0\left(1 - \frac{b}{V}\right)$$

$$\varepsilon = \frac{c}{VT^3}$$

Units

Pressure	Atmospheres
Volume	Liters per mole.
Density	Moles per liter
Temperature	Degrees Centigrade ($T°\ K = t°\ C + 273.13$)

Gas	R	A_0	a	B_0	b	c	Molecular weight.
He	0.08206	0.0216	0.05984	0.01400	0.0	0.0040×10^4	4.
Ne	0.08206	0.2125	0.02196	0.02060	0.0	0.101×10^4	20.2
A	0.08206	1.2907	0.02328	0.03931	0.0	5.99×10^4	39.91
H_2	0.08206	0.1975	−0.00506	0.02096	−0.04359	0.0504×10^4	2.0154
N_2	0.08206	1.3445	0.02617	0.05046	−0.00691	4.20×10^4	28.016
N_2*	0.08206	1.1440	0.01788	0.04314	−0.01608	5.60×10^4	28.016
O_2	0.08206	1.4911	0.02562	0.04624	0.004208	4.80×10^4	32.
Air	0.08206	1.3012	0.01931	0.04611	−0.01101	4.34×10^4	28.964
CO_2	0.08206	5.0065	0.07132	0.10476	0.07235	66.00×10^4	44.000
CH_4	0.08206	2.2769	0.01855	0.05587	−0.01587	12.83×10^4	16.0308
$(C_2H_5)_2O$	0.08206	31.278	0.12426	0.45446	0.11954	33.33×10^4	74.077

*Values of the constants for Smith and Taylor's data alone.

TABLES X to XIX.

COMPARISON OF THE OBSERVED PRESSURES WITH THOSE CALCULATED FROM THE EQUATION OF STATE.

See Table IX for the values of the constants and the units used.

For each isotherm the observed pressures are given in the first horizontal line, and the observed minus the calculated pressures in the second. The latter values subtracted from the former give the pressures calculated from the equation of state.

Under the column headed Ref. there is given the reference to the data from which the "observed pressures" were obtained, the following abbreviations being used.

A	Amagat
An	Andrews
B	Beattie
Br	Bartlett
K	Koch
K and B	Keyes and Burks
L	Data from Leiden Laboratory
R	Data from Reichsanstalt (Holborn and Schultze; Holborn and Otto)
S and T	Smith and Taylor
V	Verschoyle
W	Witkowski

TABLE X.—HELIUM.

Density, moles/liter			0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
Temp. °*C*	Ref.		Pressure, atmospheres.								
400	R	obs	27.76	55.80	84.13	....	....	....	....	....	....
		obs–calc	−0.05	−0.19	−0.42	....	....	....	....	....	....
300	R	obs	23.64	47.53	71.67	96.07	....	....	....	....	....
		obs–calc	−0.04	−0.14	−0.31	−0.55	....	....	....	....	....
200	R	obs	19.52	39.26	59.22	79.41	99.82	....	....	....	....
		obs–calc	−0.02	−0.09	−0.20	−0.34	−0.53	....	....	....	....
100.35	L	obs	15.41	30.98	46.73	62.67	....	....	....	....	....
		obs–calc	−0.02	−0.08	−0.16	−0.27	....	....	....	....	....
100	R	obs	15.40	30.97	46.73	62.67	78.80	95.11	....	....	....
		obs–calc	−0.01	−0.06	−0.12	−0.21	−0.31	−0.45	....	....	....
50	R	obs	13.34	26.83	40.49	54.31	68.30	82.46	96.79	....	....
		obs–calc	−0.01	−0.04	−0.08	−0.13	−0.20	−0.27	−0.36	....	....
20	L	obs	12.10	24.34	36.73	49.26	61.93	....	....	....	....
		obs–calc	−0.01	−0.03	−0.07	−0.12	−0.20	....	....	....	....
0	R	obs	11.27	22.68	34.23	45.92	57.75	69.72	81.84	94.12	....
		obs–calc	−0.01	−0.03	−0.05	−0.09	−0.13	−0.18	−0.24	−0.29	....
0	L	obs	11.27	22.68	34.22	45.90	57.71	....	....	....	....
		obs–calc	−0.01	−0.03	−0.06	−0.11	−0.17	....	....	....	....
−37.40	L	obs	9.73	19.59	29.57	39.67	....	....	....	....	....
		obs–calc	−0.01	−0.01	−0.01	−0.03	....	....	....	....	....
−50	R	obs	9.21	18.54	27.98	37.53	47.20	57.00	66.92	76.97	87.16
		obs–calc	0.00	−0.01	−0.02	−0.04	−0.06	−0.08	−0.10	−0.11	−0.11
−70.32	L	obs	8.38	16.86	25.45	34.15	42.96	51.87	....	....	....
		obs–calc	0.00	0.00	0.01	0.01	0.01	0.01	....	....	....
−100	R	obs	7.15	14.38	21.70	29.12	36.63	44.23	51.94	59.74	67.64
		obs–calc	0.00	−0.01	−0.01	−0.01	−0.02	−0.02	−0.01	−0.01	0.00
−103.64	L	obs	7.00	14.09	21.27	28.54	35.90	43.35	50.89	....	....
		obs–calc	0.00	0.01	0.01	0.02	0.03	0.03	0.04	....	....
−142.01	L	obs	5.41	10.90	16.45	22.08	27.77	33.53	39.36	45.26	51.23
		obs–calc	0.00	0.01	0.01	0.03	0.04	0.06	0.07	0.08	0.08
−150	R	obs	5.08	10.23	15.43	20.70	26.03	31.42	36.88	42.41	48.01
		obs–calc	0.00	0.00	0.00	0.00	0.00	0.00	−0.01	0.00	0.00
−183	R	obs	3.72	7.48	11.28	15.12	19.01	22.95	26.93	30.95	35.02
		obs–calc	0.00	0.00	0.00	−0.01	−0.01	−0.01	−0.01	−0.02	−0.03
−183.32	L	obs	3.71	7.46	11.26	15.11	19.00	22.93	26.91	30.94	35.02
		obs–calc	0.00	0.01	0.02	0.04	0.05	0.06	0.07	0.08	0.10
−201.51	L	obs	2.95	5.94	8.96	12.02	15.11	18.23	21.39	24.58	27.80
		obs–calc	0.00	0.00	0.00	0.02	0.02	0.02	0.03	0.03	0.02
−205.31	L	obs	....	....	....	....	....	....	....	....	....
		obs–calc	....	....	....	....	....	....	....	....	....
−208.0	R	obs	2.69	5.40	8.14	10.90	13.70	16.53	19.39	22.29	25.22
		obs–calc	0.00	0.00	0.00	−0.01	−0.01	−0.01	−0.01	−0.01	−0.01
−212.06	L	obs	....	....	....	....	....	....	....	....	....
		obs–calc	....	....	....	....	....	....	....	....	....
−216.56	L	obs	2.33	4.69	7.07	9.48	....	....	....	....	....
		obs–calc	0.00	0.00	0.01	0.02	....	....	....	....	....

TABLE X.—Helium.

Pressure, atmospheres.

5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
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....	....	....	....	....	....	....	....	....	....	....
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....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
97.48	....	....	....	....	....	....	....	....	....	....
−0.10	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
75.66	83.79	92.03	100.38	....	....	....	....	....	....	....
0.03	0.08	0.13	0.19	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
53.68	59.43	65.26	71.17	77.16	83.23	89.39	95.65	101.99	....	....
0.01	0.02	0.05	0.08	0.13	0.18	0.24	0.33	0.42	....	....
39.15	43.33	47.56	51.85	56.19	60.59	65.05	69.58	74.17	78.82	83.53
−0.03	−0.03	−0.03	−0.02	−0.02	−0.01	0.00	0.03	0.06	0.09	0.13
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	45.35	48.67	....	....	....	....
....	....	....	....	....	−0.06	−0.07	....	....	....	....
28.18	31.17	34.20	37.27	40.38	43.53	46.72	49.96	53.24	56.57	59.94
−0.01	−0.03	−0.03	−0.04	−0.04	−0.05	−0.05	−0.05	−0.04	−0.04	−0.03
....	....	....	....	....	40.76	43.72	....	....	....	....
....	....	....	....	....	−0.05	−0.08	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....

TABLE X.—Continued.

Density, moles/liter			0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
Temp. °*C*	Ref.		Pressure, atmospheres.								
−217.41	L	obs	....	....	....	....	....	....	....	....	....
		obs–calc	....	....	....	....	....	....	....	....	....
−225.01	L	obs	....	....	....	....	....	....	....	....	....
		obs–calc	....	....	....	....	....	....	....	....	....
−252.63	L	obs	....	....	....	....	....	....	....	....	....
		obs–calc	....	....	....	....	....	....	....	....	....
−252.8	R	obs	0.83	1.66	2.49	3.32	4.15	4.98	5.81	6.64	7.48
		obs–calc	0.00	0.00	0.00	0.00	0.00	0.00	0.00	−0.01	−0.01
−258.0	R	obs	0.62	1.23	1.83	2.43	3.03	3.62	4.21	4.79	5.38
		obs–calc	0.00	0.00	0.00	0.01	0.02	0.03	0.04	0.05	0.07
−258.34	L	obs	....	....	....	....	....	....	....	....	....
		obs–calc	....	....	....	....	....	....	....	....	....
Average deviation (atm.)			0.009	0.036	0.075	0.103	0.112	0.092	0.086	0.070	0.044
Average % deviation			0.051	0.110	0.153	0.197	0.186	0.153	0.146	0.126	0.098

Note: The values below −252° C are omitted in the averages.

TABLE XI.—NEON.

Density, moles/liter			0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
Temp. °*C*	Ref.		Pressure, atmospheres.								
400	R	obs	27.80	55.99	84.57	....	....	....	....	....	....
		obs–calc	−0.05	−0.18	−0.38	....	....	....	....	....	....
300	R	obs	23.67	47.67	72.01	96.70	....	....	....	....	....
		obs–calc	−0.04	−0.12	−0.25	−0.42	....	....	....	....	....
200	R	obs	19.54	39.33	59.41	79.77	100.42	....	....	....	....
		obs–calc	−0.02	−0.09	−0.16	−0.26	−0.38	....	....	....	....
100	R	obs	15.40	30.99	46.78	62.79	79.02	95.49	....	....	....
		obs–calc	−0.01	−0.05	−0.11	−0.16	−0.21	−0.25	....	....	....
20	L	obs	12.10	24.33	36.71	49.24	61.93	74.78	87.80	101.00	....
		obs–calc	0.00	−0.01	−0.02	−0.03	−0.04	−0.05	−0.04	−0.03	....
0	R	obs	11.27	22.65	34.17	45.82	57.61	69.54	81.63	93.86	106.26
		obs–calc	0.00	−0.02	−0.02	−0.04	−0.05	−0.06	−0.05	−0.06	−0.05
0	L	obs	11.26	22.64	34.14	45.77	57.55	69.48	81.58	93.84	....
		obs–calc	−0.01	−0.03	−0.05	−0.09	−0.11	−0.12	−0.10	−0.08	....
−50	R	obs	9.20	18.48	27.85	37.32	46.88	56.55	66.32	76.20	86.19
		obs–calc	0.00	0.00	0.00	0.01	0.01	0.03	0.04	0.05	0.07
−100	R	obs	7.13	14.30	21.53	28.80	36.14	43.53	50.98	58.49	66.07
		obs–calc	0.01	0.01	0.03	0.04	0.07	0.09	0.11	0.13	0.15
−103.01	L	obs	6.99	14.01	21.07	28.17	35.33	42.56	49.86	57.24	64.72
		obs–calc	−0.01	−0.03	−0.05	−0.08	−0.09	−0.09	−0.08	−0.05	0.02
−141.22	L	obs	5.41	10.82	16.24	21.67	27.11	32.58	38.08	43.62	49.20
		obs–calc	0.00	−0.01	−0.03	−0.04	−0.06	−0.07	−0.07	−0.06	−0.04
−150	R	obs	5.05	10.11	15.17	20.24	25.32	30.41	35.52	40.64	45.78
		obs–calc	0.00	0.01	0.02	0.03	0.05	0.06	0.08	0.09	0.10

TABLE X.—Continued.

Pressure, atmospheres.

5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
....	....	....	....	....	37.09	39.79	....	....	....	....
....	....	....	....	....	−0.07	−0.09	....	....	....	....
....	....	....	....	....	....	....	....	....	....	43.81
....	....	....	....	....	....	....	....	....	....	−0.18
....	....	....	....	....	12.60	13.46	....	....	....	....
....	....	....	....	....	−0.19	−0.23	....	....	....	....
8.32	9.17	10.01	10.85	11.70	12.55	13.41	14.27	15.14	16.01	16.89
−0.02	−0.02	−0.03	−0.06	−0.08	−0.11	−0.15	−0.19	−0.24	−0.29	−0.36
5.96	6.53	7.10	7.67	8.25	8.83	9.40	9.97	10.54	11.12	11.71
0.08	0.08	0.08	0.08	0.09	0.10	0.10	0.09	0.07	0.06	0.06
....	....	....	....	....	8.63	9.18	9.72	....	....	....
....	....	....	....	....	0.17	0.17	0.16	....	....	....
0.036	0.040	0.060	0.083	0.063	0.070	0.088	0.137	0.173	0.065	0.113
0.056	0.075	0.093	0.113	0.103	0.132	0.155	0.163	0.190	0.090	0.207

Total average deviation (atm.) 0.071
Total average % deviation 0.133

TABLE XI.—NEON.

Pressure, atmospheres.

5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
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....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
96.30	106.53	....	....	....	....	....	....	....	....	....
0.09	0.12	....	....	....	....	....	....	....	....	....
73.71	81.43	89.22	97.10	105.05	....	....	....	....	....	....
0.16	0.18	0.20	0.22	0.23	....	....	....	....	....	....
72.31	80.01	....	....	....	....	....	....	....	....	....
0.13	0.28	....	....	....	....	....	....	....	....	....
54.83	60.51	66.25	72.06	77.94	....	....	....	....	....	....
−0.01	0.04	0.11	0.20	0.32	....	....	....	....	....	....
50.95	56.14	61.36	66.61	71.89	77.21	82.58	87.98	....	....	....
0.11	0.11	0.10	0.10	0.08	0.06	0.04	0.01	....	....	....

TABLE XI.—Continued.

Density, moles/liter			0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
Temp. °C	Ref.		Pressure, atmospheres.								
−182.5	R	obs	3.70	7.38	11.03	14.66	18.27	21.85	25.42	28.98	32.52
		obs–calc	0.00	0.01	0.01	0.03	0.04	0.04	0.05	0.06	0.05
−182.6	L	obs	3.70	7.37	11.00	14.62	18.21	21.78	25.34	28.88	32.41
		obs–calc	0.00	0.01	0.00	0.00	0.00	0.00	0.00	−0.01	−0.02
−200.08	L	obs	2.97	5.90	8.78	11.62	14.42	17.18	19.90	22.59	25.26
		obs–calc	0.00	0.01	0.01	0.01	0.02	0.02	0.01	0.01	0.01
−207.9	R	obs	2.65	5.25	7.79	10.29	12.73	15.13	17.49	19.80	22.08
		obs–calc	0.00	0.01	0.02	0.04	0.04	0.05	0.06	0.06	0.07
−208.10	L	obs	2.64	5.22	7.75	10.23	12.66	15.05	17.39	19.68	21.94
		obs–calc	0.00	0.00	0.00	0.01	0.02	0.03	0.03	0.02	0.01
−213.08	L	obs	2.43	4.81	7.12	9.38	11.58	13.73	15.83	17.88	19.88
		obs–calc	0.00	0.00	0.01	0.03	0.03	0.04	0.04	0.04	0.04
−217.52	L	obs	2.24	4.42	6.53	8.57	10.55	12.47	14.34	16.15	17.91
		obs–calc	−0.01	0.00	0.00	−0.01	−0.02	−0.02	−0.03	−0.04	−0.06
Average deviation (atm.)			0.008	0.032	0.062	0.074	0.073	0.064	0.053	0.053	0.053
Average % deviation			0.070	0.139	0.150	0.187	0.185	0.172	0.153	0.137	0.142

TABLE XII.—ARGON.

Density, moles/liter			0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
Temp. °C	Ref.		Pressure, atmospheres.								
400	R	obs	27.83	56.09	84.80	113.96	....	....	....	....	....
		obs–calc	−0.01	−0.05	−0.11	−0.23	....	....	....	....	....
300	R	obs	23.65	47.59	71.83	96.42	....	....	....	....	....
		obs–calc	0.00	−0.01	−0.04	−0.05	....	....	....	....	....
200	R	obs	19.46	39.05	58.78	78.70	98.83	....	....	....	....
		obs–calc	−0.01	−0.02	−0.04	−0.04	−0.02	....	....	....	....
150	R	obs	17.38	34.80	52.29	69.88	87,60	105.46	....	....	....
		obs–calc	0.00	0.00	0.00	0.02	0.05	0.09	....	....	....
100	R	obs	15.28	30.52	45.75	60.99	76.26	91.59	107.01	....	....
		obs–calc	0.00	−0.01	0.00	0.01	0.03	0.06	0.11	....	....
50	R	obs	13.19	26.26	39.22	52.10	64.92	77.71	90.48	103.25	....
		obs–calc	0.00	0.01	0.02	0.02	0.03	0.06	0.08	0.10	....
20.39	L	obs	11.97	23.79	35.46	47.00	58.44	....	....	....	....
		obs–calc	0.02	0.08	0.14	0.21	0.30	....	....	....	....
0	R	obs	11.09	21.96	32.63	43.13	53.47	63.67	73.77	83.77	93.67
		obs–calc	0.00	−0.01	−0.01	−0.01	−0.02	−0.03	−0.03	−0.03	−0.06
0	L	obs	11.10	22.03	32.79	43.34	53.68	63.88	....	....	....
		obs–calc	0.01	0.06	0.15	0.20	0.19	0.18	....	....	....
−50	R	obs	8.99	17.65	26.01	34.10	41.92	49.50	56.86	64.02	71.00
		obs–calc	0.00	−0.02	−0.04	−0.05	−0.08	−0.11	−0.14	−0.18	−0.22
−57.72	L	obs	8.67	17.04	25.08	32.85	40.38	47.65	54.66	61.42	....
		obs–calc	0.00	0.04	0.06	0.10	0.17	0.24	0.28	0.28	....

TABLE XI.—Continued.

Pressure, atmospheres.

5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
36.05	39.57	43.08	46.59	50.09	53.59	57.09	60.59	....	....	....
0.04	0.02	−0.01	−0.05	−0.10	−0.17	−0.25	−0.35	....	....	....
35.93	39.45	42.96	46.48	49.99	53.52	57.05	60.60	....	....	....
−0.03	−0.05	−0.07	−0.09	−0.13	−0.16	−0.21	−0.26	....	....	....
27.89	30.49	33.07	35.63	38.17	40.69	43.19	45.67	....	....	....
−0.01	−0.04	−0.07	−0.11	−0.16	−0.22	−0.30	−0.40	....	....	....
24.31	26.51	28.67	30.80	32.89	34.96	36.99	39.00	....	....	....
0.05	0.04	0.01	−0.02	−0.08	−0.14	−0.23	−0.32	....	....	....
24.15	26.33	28.47	30.58	32.66	34.72	36.74	38.74	....	....	....
−0.01	−0.03	−0.07	−0.12	−0.17	−0.23	−0.32	−0.41	....	....	....
21.84	23.75	25.62	27.45	29.24	30.99	32.71	34.39	....	....	....
0.03	0.00	−0.03	−0.07	−0.13	−0.20	−0.29	−0.39	....	....	....
19.63	21.30	22.93	24.52	26.08	27.60	29.08	30.54	....	....	....
−0.07	−0.09	−0.11	−0.13	−0.16	−0.19	−0.24	−0.28	....	....	....
0.062	0.083	0.078	0.111	0.156	0.171	0.235	0.303	....	....	....
0.143	0.162	0.182	0.251	0.343	0.455	0.595	0.729	....	....	....

Total average deviation (atm.) 0.081
Total average % deviation 0.209

TABLE XII.—Argon.

Pressure, atmospheres.

5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
....	....	....	....	....	....	....	....	....	....	....
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....	....	....	....	....	....	....	....	....	....	....
103.50	....	....	....	....	....	....	....	....	....	....
−0.11	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
77.81	84.49	91.06	....	....	....	....	....	....	....	....
−0.27	−0.31	−0.34	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....

TABLE XII.—Continued.

Density, moles/liter			0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
Temp. °*C*	Ref.		Pressure, atmospheres.								
−87.05	L	obs	7.42	14.48	21.15	27.44	33.36	39.02	44.34	49.35	54.11
		obs–calc	−0.01	0.02	0.04	0.03	0.00	0.04	0.04	0.02	0.02
−100	R	obs	6.88	13.31	19.32	24.92	30.14	35.01	39.55	43.79	47.74
		obs–calc	0.00	−0.03	−0.05	−0.10	−0.15	−0.18	−0.21	−0.21	−0.19
−102.51	L	obs	6.76	13.13	19.06	24.57	29.70	34.47	38.88	42.96	46.75
		obs–calc	−0.02	0.01	0.02	0.02	0.01	0.02	0.01	0.00	0.02
−109.88	L	obs	6.45	12.48	18.07	23.22	27.94	32.27	36.25	39.89	43.22
		obs–calc	−0.01	0.01	0.03	0.04	0.02	0.00	0.00	0.02	0.07
−113.80	L	obs	6.27	12.12	17.50	22.43	26.93	31.03	34.77	38.16	41.23
		obs–calc	−0.03	−0.01	0.00	−0.02	−0.04	−0.07	−0.07	−0.05	0.00
−115.86	L	obs	6.18	11.95	17.23	22.04	26.43	30.42	34.04	37.30	40.21
		obs–calc	−0.03	0.00	0.01	−0.02	−0.04	−0,06	−0.05	−0.03	0.00
−116.62	L	obs	6.15	11.88	17.11	21.89	26.24	30.19	33.77	36.96	39.80
		obs–calc	−0.03	0.00	−0.01	−0.03	−0.05	−0.06	−0.05	−0.05	−0.04
−119.20	L	obs	6.04	11.67	16.78	21.41	25.60	29.35	32.69	35.67	38.32
		obs–calc	−0.03	0.02	0.01	−0.02	−0.06	−0.12	−0.19	−0.23	−0.23
−120.24	L	obs	6.00	11.58	16.65	21.24	25.39	29.14	32.50	35.49	38.13
		obs–calc	−0.02	0.02	0.02	0.00	−0.02	−0.02	0.00	0.04	0.09
−121.21	L	obs	5.96	11.49	16.53	21.09	25.19	28.85	32.11	35.01	37.58
		obs–calc	−0.02	0.01	0.04	0.04	0.02	−0.01	−0.04	−0.03	0.03
−130.38	L	obs	5.58	10.67	15.24	19.31	22.91	26.07	28.80	31.13	33.06
		obs–calc	−0.01	0.01	0.01	0.00	0.00	0.01	0.04	0.10	0.17
−139.62	L	obs	5.17	9.83	13.91	....	....	....	....	....	....
		obs–calc	−0.03	−0.01	−0.03	....	....	....	....	....	....
−149.60	L	obs	4.74	8.93	12.48	....	....	....	....	....	....
		obs–calc	−0.03	−0.01	−0.05	....	....	....	....	....	....
Average deviation (atm.)			0.013	0.020	0.039	0.057	0.065	0.081	0.084	0.091	0.088
Average % deviation			0.207	0.107	0.147	0.142	0.158	0.174	0.185	0.198	0.195

TABLE XIII.—HYDROGEN.

Density, moles/liter			0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
Temp. °*C*	Ref.		Pressure, atmospheres.								
200	R	obs	19.56	39.44	59.63	80.15	101.02	....	....	....	....
		obs–calc	−0.01	−0.04	−0.11	−0.24	−0.43	....	....	....	....
100	R	obs	15.43	31.10	47.02	63.21	79.66	96.37	....	....	....
		obs–calc	0.01	0.01	0.00	−0.02	−0.08	−0.21	....	....	....
100	L	obs	15.42	31.06	46.93	....	....	....	....	....	....
		obs–calc	0.00	−0.03	−0.09	....	....	....	....	....	....
100	W	obs	15.43	31.09	47.01	....	....	....	....	....	....
		obs–calc	0.01	0.00	−0.01	....	....	....	....	....	....
50	R	obs	13.36	26.92	40.70	54.69	68.90	83.34	98.01	....	....
		obs–calc	0.01	0.02	0.04	0.04	0.01	−0.06	−0.18	....	....

TABLE XII.—Continued.

5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
				Pressure, atmospheres.						
58.69	63.14	....	....	....	....	....	....	....	....	....
0.09	0.27	....	....	....	....	....	....	....	....	....
51.41	54.82	58.00	....	....	....	....	....	....	....	....
−0.17	−0.14	−0.08	....	....	....	....	....	....	....	....
50.28	53.56	56.58	....	....	....	....	....	....	....	....
0.08	0.16	0.25	....	....	....	....	....	....	....	....
46.26	49.02	51.50	....	....	....	....	....	....	....	....
0.14	0.24	0.34	....	....	....	....	....	....	....	....
43.99	46.46	48.66	....	....	....	....	....	....	....	....
0.07	0.17	0.30	....	....	....	....	....	....	....	....
42.80	45.11	47.15	....	....	....	....	....	....	....	....
0.05	0.14	0.27	....	....	....	....	....	....	....	....
42.34	44.60	46.60	....	....	....	....	....	....	....	....
0.02	0.12	0.27	....	....	....	....	....	....	....	....
40.67	42.73	44.51	....	....	....	....	....	....	....	....
−0.18	−0.09	0.05	....	....	....	....	....	....	....	....
40.45	42.47	44.19	....	....	....	....	....	....	....	....
0.19	0.33	0.49	....	....	....	....	....	....	....	....
39.85	41.83	43.53	....	....	....	....	....	....	....	....
0.15	0.32	0.54	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
0.127	0.208	0.293	....	....	....	....	....	....	....	....
0.252	0.415	0.584	....	....	....	....	....	....	....	....

Total average deviation (atm.)	0.078
Total average % deviation	0.203

TABLE XIII.—HYDROGEN.

5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
				Pressure, atmospheres.						
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....

TABLE XIII.—Continued.

Density, moles/liter Temp. °C	Ref.		0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
			Pressure, atmospheres.								
20	L	obs	12.11	24.40	36.87	49.54	62.42	75.52	88.86	....	....
		obs–calc	0.00	0.02	0.03	0.04	0.04	0.03	0.02	....	....
0	R	obs	11.28	22.73	34.35	46.13	58.09	70.25	82.60	95.15	....
		obs–calc	0.00	0.03	0.05	0.06	0.05	0.04	0.01	−0.10	....
0	L	obs	11.28	22.71	34.30	46.05	....	....	....	....	....
		obs–calc	0.00	0.01	0.00	−0.02	....	....	....	....	....
0	W	obs	11.28	22.73	34.34	46.12	58.09	....	....	....	....
		obs–calc	0.00	0.03	0.04	0.05	0.05	....	....	....	....
−50	R	obs	9.21	18.54	27.99	37.56	47.26	57.11	67.10	77.24	87.55
		obs–calc	0.01	0.03	0.05	0.07	0.08	0.09	0.07	0.03	−0.03
−77	W	obs	8.09	16.28	24.58	32.98	41.48	50.09	58.82	....	....
		obs–calc	0.01	0.03	0.08	0.13	0.16	0.19	0.20	....	....
−100	R	obs	7.14	14.34	21.62	28.98	36.42	43.94	51.56	59.28	67.10
		obs–calc	0.01	0.02	0.05	0.07	0.10	0.10	0.11	0.10	0.08
−103.55	L	obs	6.99	14.04	21.17	28.37	35.65	43.01	50.46	58.02	....
		obs–calc	0.01	0.02	0.05	0.07	0.10	0.11	0.12	0.13	....
−104	W	obs	6.97	14.02	21.14	28.34	35.62	42.99	50.46	58.02	....
		obs–calc	0.00	0.04	0.07	0.12	0.17	0.21	0.26	0.29	....
−139.85	L	obs	5.48	11.00	16.55	22.14	27.76	33.43	39.16	44.94	....
		obs–calc	0.00	0.02	0.05	0.08	0.09	0.11	0.14	0.15	....
−147	W	obs	5.19	10.40	15.64	20.91	26.22	31.58	36.98	42.43	47.94
		obs–calc	0.01	0.02	0.05	0.07	0.11	0.15	0.19	0.22	0.26
−150	R	obs	5.06	10.14	15.24	20.37	25.53	30.72	35.95	41.22	46.53
		obs–calc	0.00	0.02	0.03	0.05	0.07	0.08	0.09	0.10	0.08
−164.10	L	obs	4.47	8.95	13.44	17.94	22.46	26.99	31.55	36.14	....
		obs–calc	0.00	0.01	0.03	0.04	0.07	0.08	0.10	0.11	....
−182.75	L	obs	3.70	7.38	11.05	14.70	18.35	22.00	25.65	29.30	32.96
		obs–calc	0.00	0.00	0.01	0.01	0.01	0.02	0.03	0.03	0.02
−183	R	obs	3.69	7.37	11.03	14.68	18.32	21.96	25.60	29.24	32.89
		obs–calc	0.00	0.01	0.02	0.03	0.04	0.05	0.05	0.06	0.06
−183	W	obs	3.69	7.37	11.04	14.70	18.35	22.01	25.67	29.33	32.99
		obs–calc	0.00	0.01	0.03	0.05	0.07	0.10	0.12	0.15	0.16
−190	W	obs	3.40	6.78	10.14	13.48	16.80	20.12	23.43	26.74	30.05
		obs–calc	0.00	0.01	0.03	0.04	0.04	0.06	0.08	0.10	0.11
−195.21	L	obs	3.18	6.33	9.46	12.57	15.65	18.72	21.77	24.80	27.83
		obs–calc	0.00	0.00	0.01	0.02	0.03	0.04	0.05	0.05	0.05
−204.63	L	obs	2.79	5.54	8.25	10.93	13.57	16.19	18.78	21.34	23.89
		obs–calc	0.00	0.00	0.00	0.01	0.01	0.02	0.02	0.02	0.03
−205	W	obs	2.78	5.51	8.21	10.87	13.50	16.10	18.67	21.22	23.74
		obs–calc	0.01	0.00	0.01	0.01	0.02	0.02	0.03	0.04	0.03
−207.9	R	obs	2.65	5.26	7.82	10.34	12.83	15.28	17.69	20.08	22.44
		obs–calc	0.00	0.00	−0.01	−0.01	−0.02	−0.02	−0.04	−0.04	−0.06
−212	W	obs	2.48	4.91	7.29	9.63	11.94	14.20	16.42	18.61	20.77
		obs–calc	0.00	−0.01	−0.01	−0.02	−0.01	−0.01	−0.01	−0.01	−0.01
−212.74	L	obs	2.45	4.85	7.20	9.50	11.76	13.98	16.16	18.31	20.43
		obs–calc	0.00	0.00	−0.01	−0.02	−0.02	−0.03	−0.04	−0.04	−0.04

TABLE XIII.—Continued.

Pressure, tamospheres.

5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
98.03 -0.12	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
75.03 0.03	83.09 -0.03	91.27 -0.11	99.57 -0.24	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
53.52 0.30	59.17 0.34	....	....	....	....	....	....	....	....	....
51.89 0.06	57.32 0.04	62.81 0.01	68.37 -0.04	74.00 -0.09	79.70 -0.17	85.49 -0.26	91.39 -0.34	97.39 -0.43	103.49 -0.53	....
....	....	....	....	....	....	....	....	....	....	....
36.63 0.02	40.32 0.01	44.02 0.00	47.74 -0.03	51.49 -0.05	55.27 -0.08	59.09 -0.11	62.96 -0.14	....	....	....
36.55 0.05	40.23 0.05	43.92 0.04	47.64 0.03	51.38 0.01	55.15 -0.01	58.96 -0.04	62.81 -0.07	66.69 -0.11	70.62 -0.16	74.62 -0.19
36.66 0.16	40.33 0.15	44.02 0.14	47.74 0.13	51.49 0.12	55.27 0.11	59.09 0.09	....	....	....	....
33.35 0.12	36.65 0.12	39.96 0.11	43.28 0.10	46.62 0.09	49.98 0.08	53.37 0.08	56.78 0.06	60.23 0.05	....	....
30.85 0.05	33.86 0.04	36.88 0.04	39.91 0.04	42.96 0.05	46.01 0.04	49.07 0.04	52.14 0.01	....	....	....
26.42 0.04	28.93 0.04	31.42 0.04	33.91 0.04	36.41 0.06	38.91 0.09	41.41 0.11	43.91 0.14	46.41 0.16	48.93 0.19	51.47 0.24
26.23 0.02	28.70 0.00	31.16 -0.01	33.61 -0.02	36.07 -0.02	38.53 -0.01	41.01 0.02	43.50 0.06	45.99 0.09	48.50 0.14	51.04 0.21
24.78 -0.06	27.10 -0.07	29.40 -0.08	31.70 -0.08	33.99 -0.07	36.27 -0.06	38.55 -0.04	40.83 -0.02	43.11 0.00	45.41 0.04	47.72 0.10
22.91 0.00	25.02 0.01	27.11 0.02	29.18 0.04	31.23 0.05	33.26 0.07	35.27 0.08	37.29 0.12	39.32 0.17	41.36 0.25	43.42 0.35
22.51 -0.05	24.56 -0.06	26.59 -0.06	28.61 -0.05	30.61 -0.04	32.60 -0.02	34.59 -0.02	36.57 0.06	38.55 0.12	40.53 0.19	42.51 0.26

TABLE XIII.—Continued.

Density, moles/liter Temp. °*C*	Ref.		0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
			Pressure, atmospheres.								
−217.33	L	obs	2.26	4.46	6.61	8.71	10.76	12.75	14.70	16.62	18.50
		obs–calc	0.00	−0.01	−0.01	−0.01	−0.02	−0.03	−0.04	−0.04	−0.04
−225.36	L	obs	....	....	....	....	....	....	12.13	13.64	....
		obs–calc	....	....	....	....	....	....	−0.05	−0.04	....
−231.39	L	obs	....	....	....	....	....	....	10.19	11.38	....
		obs–calc	....	....	....	....	....	....	−0.04	−0.03	....
−236.30	L	obs	....	....	....	....	....	....	8.59	9.53	....
		obs–calc	....	....	....	....	....	....	−0.02	0.01	....
−238.29	L	obs	....	....	....	....	....	....	7.95	8.77	....
		obs–calc	....	....	....	....	....	....	0.01	0.03	....
−239.91	L	obs	....	....	....	....	....	....	7.41	8.16	....
		obs–calc	....	....	....	....	....	....	0.02	0.06	....
−241.87	L	obs	....	....	....	....	....	....	6.76	7.39	....
		obs–calc	....	....	....	....	....	....	0.05	0.09	....
−243.88	L	obs	....	....	....	....	....	....	6.08	6.59	....
		obs–calc	....	....	....	....	....	....	0.07	0.12	....
Average deviation (atm.)			0.003	0.016	0.034	0.052	0.073	0.078	0.075	0.081	0.071
Average % deviation			0.045	0.108	0.166	0.196	0.215	0.227	0.294	0.355	0.209

TABLE XIV.—NITROGEN.

Density, moles/liter Temp. °*C*	Ref.		0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
			Pressure, atmospheres.								
400	R	obs	27.94	56.59	85.99	....	....	....	....	....	....
		obs–calc	−0.04	−0.14	−0.28	....	....	....	....	....	....
300	R	obs	23.76	48.03	72.86	98.29	....	....	....	....	....
		obs–calc	−0.02	−0.07	−0.15	−0.25	....	....	....	....	....
200	R	obs	19.56	39.45	59.71	80.37	101.46	....	....	....	....
		obs–calc	−0.01	−0.02	−0.04	−0.06	−0.10	....	....	....	....
150	R	obs	17.46	35.16	53.11	71.36	89.94	....	....	....	....
		obs–calc	0.00	0.00	0.00	−0.01	−0.02	....	....	....	....
100	R	obs	15.36	30.84	46.48	62.31	78.36	94.68	....	....	....
		obs–calc	0.00	0.00	0.00	0.01	0.02	0.04	....	....	....
50	R	obs	13.26	26.54	39.87	53.28	66.79	80.43	94.23	....	....
		obs–calc	0.01	0.02	0.04	0.06	0.08	0.08	0.08	....	....
20	L	obs	12.00	23.96	35.88	47.81	59.75	....	....	....	....
		obs–calc	0.01	0.03	0.04	0.05	0.03	....	....	....	....
20	V	obs	11.99	23.93	35.86	47.80	59.78	71.81	83.91	96.13	108.50
		obs–calc	0.00	0.00	0.02	0.04	0.06	0.06	0.03	−0.01	−0.07
0	R	obs	11.15	22.22	33.22	44.18	55.13	66.10	77.11	88.18	99.33
		obs–calc	0.00	0.03	0.05	0.07	0.09	0.10	0.10	0.08	0.03
0	L	obs	11.16	22.24	33.23	44.16	55.07	....	....	....	....
		obs–calc	0.01	0.05	0.06	0.05	0.03	....	....	....	....

TABLE XIII.—Continued.

Pressure, atmospheres.

5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
20.34	22.15	23.92	25.67	27.40	29.12	30.82	32.51	34.19	35.86	37.53
−0.04	−0.03	−0.03	−0.02	0.00	0.05	0.10	0.16	0.24	0.33	0.44
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0.075	0.071	0.053	0.066	0.054	0.066	0.083	0.107	0.152	0.229	0.256
0.192	0.176	0.138	0.142	0.125	0.141	0.164	0.210	0.296	0.438	0.562

Total average deviation (atm.)	0.068
Total average % deviation	0.203

TABLE XIV.—NITROGEN.

Pressure, atmospheres.

5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
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....	....	....	....	....	....	....	....	....	....	....
121.08	133.91	....	....	....	....	....	....	....	....	....
−0.11	−0.12	....	....	....	....	....	....	....	....	....
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TABLE XIV.—Continued.

Density, moles/liter			0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
Temp. °C	Ref.		Pressure, atmospheres.								
0	V	obs	11.15	22.20	33.19	44.14	55.08	66.03	77.02	88.08	99.24
		obs–calc	0.00	0.01	0.02	0.03	0.04	0.03	0.01	−0.02	−0.06
0	Br	obs	11.16	22.23	33.22	44.16	55.09	66.04	77.02	88.07	99.23
		obs–calc	0.01	0.04	0.05	0.05	0.05	0.04	0.01	−0.03	−0.07
−23.62	L	obs	10.15	20.17	30.05	39.83	49.54	59.21	68.87	....	....
		obs–calc	0.00	0.02	0.03	0.03	0.03	0.02	0.01	....	....
−50	R	obs	9.04	17.87	26.50	34.96	43.28	51.48	59.58	67.61	75.59
		obs–calc	0.00	0.01	0.01	−0.01	−0.03	−0.07	−0.13	−0.23	−0.35
−50.26	L	obs	9.04	17.88	26.51	34.96	43.26	51.45	59.57	....	....
		obs–calc	0.01	0.04	0.05	0.04	0.01	−0.02	−0.05	....	....
−81.10	L	obs	7.74	15.19	22.37	29.29	35.98	42.47	48.80	54.97	61.00
		obs–calc	0.02	0.04	0.06	0.06	0.04	0.01	−0.02	−0.07	−0.16
−100	R	obs	6.92	13.50	19.76	25.71	31.39	36.82	42.01	47.01	51.82
		obs–calc	0.00	0.00	0.00	−0.01	−0.02	−0.04	−0.08	−0.13	−0.19
−102.25	L	obs	6.83	13.31	19.47	25.33	30.91	36.22	41.29	46.16	50.85
		obs–calc	0.00	0.01	0.02	0.03	0.04	0.03	0.00	−0.02	−0.06
−121.19	L	obs	6.03	11.65	16.89	21.75	26.26	30.46	34.38	38.05	41.48
		obs–calc	0.01	0.01	0.02	0.02	−0.01	−0.02	−0.03	−0.02	−0.01
−130	R	obs	5.65	10.85	15.63	20.01	24.02	27.69	31.04	34.09	36.88
		obs–calc	0.00	−0.01	−0.02	−0.05	−0.07	−0.09	−0.11	−0.13	−0.12
−131.27	L	obs	5.59	10.70	15.41	19.74	23.73	27.38	30.70	33.71	36.45
		obs–calc	0.00	−0.04	−0.07	−0.07	−0.05	−0.01	−0.03	0.06	0.10
−141.53	L	obs	5.16	9.84	14.05	17.81	21.16	24.14	26.78	29.12	31.16
		obs–calc	0.01	0.01	0.00	−0.02	−0.05	−0.05	−0.01	0.07	0.18
−144.46	L	obs	5.03	9.56	13.62	17.23	20.42	23.22	25.66	27.78	29.60
		obs–calc	0.00	−0.01	−0.02	−0.03	−0.04	−0.04	−0.01	0.07	0.19
−146.32	L	obs	4.94	9.38	13.34	16.86	19.95	22.64	24.96	26.94	28.62
		obs–calc	−0.01	−0.02	−0.04	−0.04	−0.04	−0.03	0.01	0.08	0.20
−148.58	L	obs	4.85	9.20	13.07	16.47	19.43	21.98	24.14	25.93	27.41
		obs–calc	0.00	0.00	0.01	0.01	0.02	0.04	0.07	0.12	0.22
Average deviation (atm.)			0.006	0.026	0.044	0.046	0.042	0.043	0.044	0.076	0.134
Average % deviation			0.062	0.118	0.134	0.113	0.110	0.106	0.098	0.187	0.318

TABLE XIV.—A

Data of Smith and Taylor Compared with Special Equation Based on Their Measurements

Density, moles/liter			0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
Temp. °C	Ref.		Pressure, atmospheres.							
200	S + T	obs	19.55	39.40	59.58	80.10	101.00	122.32	144.10	166.39
		obs–calc	0.00	0.02	0.04	0.03	0.02	0.00	−0.03	−0.05
152.34	S + T	obs	17.55	35.31	53.30	71.55	90.09	108.95	128.18	147.83
		obs–calc	0.00	0.02	0.02	0.03	0.02	0.00	−0.01	0.00
100	S + T	obs	15.36	30.83	46.43	62.19	78.13	94.29	110.70	127.40
		obs–calc	0.01	0.03	0.04	0.05	0.05	0.05	0.05	0.05

TABLE XIV.—Continued.

Pressure, atmospheres.

5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
110.54	122.01	....	....	....	....	....	....	....	....	....
−0.11	−0.16	....	....	....	....	....	....	....	....	....
110.57	122.13	....	....	....	....	....	....	....	....	....
−0.08	−0.04	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
83.55	91.54	....	....	....	....	....	....	....	....	....
−0.50	−0.66	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
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....	....	....	....	....	....	....	....	....	....	....
56.47	61.00	....	....	....	....	....	....	....	....	....
−0.27	−0.35	....	....	....	....	....	....	....	....	....
55.38	....	....	....	....	....	....	....	....	....	....
−0.10	....	....	....	....	....	....	....	....	....	....
44.66	47.60	....	....	....	....	....	....	....	....	....
−0.03	−0.10	....	....	....	....	....	....	....	....	....
39.44	41.83	....	....	....	....	....	....	....	....	....
−0.10	−0.10	....	....	....	....	....	....	....	....	....
38.95	41.22	....	....	....	....	....	....	....	....	....
0.16	0.24	....	....	....	....	....	....	....	....	....
32.91	34.41	....	....	....	....	....	....	....	....	....
0.31	0.47	....	....	....	....	....	....	....	....	....
31.15	32.42	....	....	....	....	....	....	....	....	....
0.36	0.55	....	....	....	....	....	....	....	....	....
30.00	31.11	....	....	....	....	....	....	....	....	....
0.36	0.56	....	....	....	....	....	....	....	....	....
28.61	29.56	....	....	....	....	....	....	....	....	....
0.39	0.63	....	....	....	....	....	....	....	....	....
0.222	0.324	....	....	....	....	....	....	....	....	....
0.532	0.779	....	....	....	....	....	....	....	....	....

Total average deviation (atm.)	0.072
Total average % deviation	0.190

TABLE XIV.—A

Pressure, atmospheres.

4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
189.24	212.70	236.84	261.74	287.45	314.04	....	....	....	....	....	....
−0.04	0.01	0.13	0.38	0.77	1.33	....	....	....	....	....	....
167.95	188.58	209.77	231.57	254.03	277.18	....	....	....	....	....	....
0.05	0.15	0.31	0.56	0.91	1.36	....	....	....	....	....	....
144.44	161.85	179.67	197.96	216.75	236.07	....	....	....	....	....	....
0.08	0.13	0.23	0.39	0.62	0.92	....	....	....	....	....	....

TABLE XIV.—A—Continued.

Density, moles/liter			0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
Temp. °*C*	Ref.		Pressure, atmospheres.							
49.98	S + T	obs	13.25	26.51	39.80	53.14	66.55	80.08	93.76	107.65
		obs–calc	0.00	0.00	0.00	−0.01	−0.04	−0.06	−0.08	−0.06
0	S + T	obs	11.16	22.22	33.21	44.15	55.06	65.96	76.90	87.91
		obs–calc	0.01	0.01	0.01	0.01	0.00	−0.03	−0.05	−0.05
Average deviation (atm.)			0.004	0.016	0.022	0.026	0.026	0.028	0.044	0.042
Average % deviation			0.032	0.052	0.046	0.040	0.032	0.034	0.044	0.038

TABLE XV.—OXYGEN.

Density, moles/liter			0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
Temp. °*C*	Ref.								Pressure, atmospheres.		
100 .	R	obs	15.29	30.54	45.78	61.03	76.31	91.64	....	....	....
		obs–calc	0.00	−0.01	−0.02	−0.04	−0.07	−0.12	....	....	....
50	R	obs	13.19	26.26	39.22	52.10	64.91	77.67	90.41	103.14	....
		obs–calc	0.00	0.01	0.02	0.03	0.02	0.00	−0.03	−0.09	....
20	L	obs	11.93	23.67	35.25	46.68	57.97	....	....	....	....
		obs–calc	0.00	0.01	0.02	0.02	0.00	....	....	....	....
0	R	obs	11.09	21.95	32.61	43.09	53.42	63.59	73.64	83.58	93.43
		obs–calc	0.01	0.01	0.03	0.04	0.07	0.07	0.06	0.04	−0.02
0	L	obs	11.09	21.95	32.61	43.09	53.39	....	....	....	....
		obs–calc	0.01	0.01	0.03	0.04	0.04	....	....	....	....
−40.05	L	obs	9.40	18.48	27.27	35.79	44.06	52.10	59.97	....	....
		obs–calc	0.00	0.00	0.00	0.00	0.00	−0.01	0.01	....	....
−80.03	L	obs	7.71	15.01	21.92	28.47	34.69	40.60	46.21	51.54	56.60
		obs–calc	0.00	0.00	−0.01	−0.01	0.00	0.02	0.03	0.04	0.03
−102.46	L	obs	6.77	13.07	18.93	24.35	29.36	33.98	38.25	42.21	45.86
		obs–calc	0.01	0.01	0.02	0.02	0.01	−0.02	−0.04	−0.03	−0.02
−109.97	L	obs	6.45	12.41	17.90	22.94	27.54	31.74	35.58	39.07	42.23
		obs–calc	0.01	0.01	0.01	0.01	−0.01	−0.03	−0.02	−0.01	0.00
−113.98	L	obs	6.28	12.06	17.35	22.17	26.55	30.51	34.10	37.33	40.21
		obs–calc	0.01	0.01	0.01	−0.01	−0.03	−0.06	−0.06	−0.06	−0.05
−116.01	L	obs	6.19	11.87	17.05	21.77	26.04	29.89	33.37	36.49	39.25
		obs–calc	0.00	0.00	−0.01	−0.03	−0.05	−0.07	−0.06	−0.03	−0.01
−116.99	L	obs	6.15	11.79	16.93	21.60	25.81	29.60	33.00	36.03	38.72
		obs–calc	0.00	0.01	0.00	−0.01	−0.04	−0.06	−0.07	−0.07	−0.05
Average deviation (atm.)			0.004	0.008	0.015	0.022	0.028	0.046	0.042	0.046	0.026
Average % deviation			0.054	0.044	0.056	0.063	0.070	0.109	0.102	0.094	0.056

TABLE XIV.—A—Continued.

Pressure, atmospheres.

4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
121.77	136.15	150.82	165.82	181.18	196.93	....	....	....	....	....	....
−0.01	0.08	0.20	0.38	0.60	0.89	....	....	....	....	....	....
99.04	110.31	121.77	133.46	145.42	157.70	....	....	....	....	....	....
−0.01	0.07	0.21	0.42	0.73	1.16	....	....	....	....	....	....
0.038	0.088	0.216	0.426	0.726	1.132	....	....	....	....	....	....
0.026	0.056	0.146	0.226	0.350	0.498	....	....	....	....	....	....

Density range	0–5.0	0–7.0
Total average deviation (atm.)	0.033	0.202
Total average % deviation	0.040	0.116

TABLE XV.—OXYGEN.

Pressure, atmospheres.

5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
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....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
103.18	....	....	....	....	....	....	....	....	....	....
−0.13	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
61.42	....	....	....	....	....	....	....	....	....	....
0.00	....	....	....	....	....	....	....	....	....	....
49.22	52.30	55.12	57.71	60.10	....	....	....	....	....	....
−0.01	−0.01	−0.02	−0.05	−0.07	....	....	....	....	....	....
45.07	47.60	49.86	51.87	53.66	55.28	56.75	58.07	59.24	....	....
0.01	0.00	0.00	−0.01	−0.02	0.01	0.07	0.15	0.20	....	....
42.76	45.01	47.00	48.74	50.25	51.53	52.60	53.47	54.15	....	....
−0.05	−0.04	−0.01	0.03	0.09	0.13	0.15	0.16	0.12	....	....
41.67	43.78	45.62	47.19	48.52	49.63	50.55	51.31	51.95	....	....
0.01	0.03	0.06	0.10	0.15	0.20	0.27	0.35	0.48	....	....
41.08	43.14	44.91	46.40	47.65	48.68	49.50	50.14	50.62	....	....
−0.03	0.01	0.06	0.10	0.15	0.21	0.26	0.32	0.39	....	....
0.034	0.018	0.030	0.058	0.096	0.138	0.187	0.220	0.298	....	....
0.054	0.040	0.064	0.120	0.194	0.275	0.368	0.470	0.563	....	....

Total average deviation (atm.)	0.051
Total average % deviation	0.115

TABLE XVI.—AIR.

Density, moles/liter			0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
Temp. °C	Ref.		Pressure, atmospheres.								
200	R	obs	19.53	39.33	59.44	79.90	100.73	....	....	....	....
		obs–calc	−0.01	−0.01	−0.01	0.01	0.02	....	....	....	....
150	R	obs	17.44	35.05	52.88	70.95	89.30	107.97	....	....	....
		obs–calc	0.00	0.01	0.03	0.05	0.09	0.16	....	....	....
100	R	obs	15.34	30.77	46.30	61.98	77.84	93.92	110.25	....	....
		obs–calc	0.00	0.03	0.05	0.09	0.14	0.23	0.35	....	....
100	W	obs	15.33	30.70	46.17	61.78	77.57	93.56	109.78	126.26	....
		obs–calc	−0.01	−0.04	−0.08	−0.11	−0.13	−0.13	−0.12	−0.10	....
50	R	obs	13.24	26.47	39.70	52.97	66.30	79.72	93.25	106.94	....
		obs–calc	0.01	0.03	0.05	0.09	0.14	0.19	0.24	0.31	....
20	L	obs	11.99	23.89	35.73	47.51	59.27	....	....	....	....
		obs–calc	0.02	0.03	0.05	0.05	0.04	....	....	....	....
16	W,A	obs	11.80	23.50	35.11	46.66	58.20	69.77	81.37	93.03	104.80
		obs–calc	−0.01	−0.01	0.06	−0.07	−0.10	−0.11	−0.12	−0.13	−0.11
0	R	obs	11.14	22.15	33.06	43.90	54.69	65.46	76.21	86.98	97.80
		obs–calc	0.01	0.02	0.03	0.06	0.09	0.13	0.16	0.19	0.21
0	W	obs	11.14	22.14	33.02	43.81	54.54	65.26	75.99	86.75	97.58
		obs–calc	0.01	0.01	−0.01	−0.03	−0.06	−0.07	−0.06	−0.04	−0.01
0	K	obs	11.12	22.09	32.94	43.71	54.42	65.11	75.81	86.54	97.33
		obs–calc	−0.01	−0.04	−0.09	−0.13	−0.18	−0.22	−0.24	−0.25	−0.26
−35	W	obs	9.67	19.15	28.44	37.57	46.56	55.44	64.23	72.95	81.61
		obs–calc	0.01	0.04	0.06	0.08	0.10	0.12	0.13	0.14	0.13
−70.09	L	obs	....	....	....	....	....	....	51.92	58.58	....
		obs–calc	....	....	....	....	....	....	−0.07	−0.04	....
−78.5	W	obs	7.84	15.37	22.62	29.60	36.33	42.84	49.15	55.27	61.21
		obs–calc	0.01	0.03	0.05	0.06	0.07	0.08	0.09	0.09	0.06
−79.1	K	obs	7.79	15.26	22.43	29.34	36.02	42.47	48.72	54.78	60.69
		obs–calc	−0.01	−0.03	−0.06	−0.09	−0.10	−0.12	−0.13	−0.16	−0.17
−84.08	L	obs	....	....	....	....	....	....	47.04	52.77	58.33
		obs–calc	....	....	....	....	....	....	−0.07	−0.12	−0.17
−103.48	L	obs	....	....	....	....	....	....	40.13	44.68	48.99
		obs–calc	....	....	....	....	....	....	−0.13	−0.16	−0.19
−103.5	W	obs	6.77	13.17	19.22	24.92	30.29	35.36	40.16	44.72	49.06
		obs–calc	0.00	0.01	0.01	0.00	−0.03	−0.06	−0.09	−0.11	−0.11
−122.03	L	obs	....	....	....	....	....	....	33.45	36.81	39.91
		obs–calc	....	....	....	....	....	....	−0.13	−0.15	−0.13
−129.97	L	obs	....	....	....	....	....	....	30.50	33.35	35.93
		obs–calc	....	....	....	....	....	....	−0.16	−0.16	−0.10
−134.99	L	obs	....	....	....	....	....	....	28.61	31.13	33.35
		obs–calc	....	....	....	....	....	....	−0.18	−0.17	−0.11
−140.00	L	obs	....	....	....	....	....	....	26.70	28.88	30.75
		obs–calc	....	....	....	....	....	....	−0.21	−0.19	−0.10
−145.05	L	obs	....	....	....	....	....	....	24.74	26.56	28.05
		obs–calc	....	....	....	....	....	....	−0.24	−0.22	−0.13
Average deviation (atm.)			0.009	0.024	0.046	0.066	0.092	0.135	0.154	0.152	0.133
Average % deviation			0.078	0.113	0.139	0.148	0.164	0.204	0.327	0.308	0.251

TABLE XVI.—AIR.

Pressure, atmospheres.

5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
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....	....	....	....	....	....	....	....	....	....	....
116.74	128.89	....	....	....	....	....	....	....	....	....
−0.03	0.11	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
108.51	119.61	130.92	....	....	....	....	....	....	....	....
0.06	0.20	0.42	....	....	....	....	....	....	....	....
108.20	119.19	130.34	141.69	153.28	165.12	177.21	....	....	....	....
−0.25	−0.22	−0.16	−0.04	0.14	0.37	0.63	....	....	....	....
90.25	98.90	107.64	116.55	....	....	....	....	....	....	....
0.12	0.11	0.16	0.32	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
67.00	72.66	78.23	83.73	89.20	94.76	100.32	....	....	....	....
0.03	−0.02	−0.06	−0.10	−0.12	−0.01	0.11	....	....	....	....
66.47	72.17	77.80	83.40	88.95	94.44	99.87	....	....	....	....
−0.18	−0.14	−0.08	0.02	0.14	0.22	0.26	....	....	....	....
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....	....	....	....	....	....	....	....	....	....	....
53.22	57.21	61.03	64.72	68.34	71.82	75.26	....	....	....	....
−0.07	−0.01	0.06	0.17	0.34	0.50	0.72	....	....	....	....
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....	....	....	....	....	....	....	....	....	....	....
0.106	0.116	0.157	0.130	0.185	0.275	0.430	....	....	....	....
0.127	0.113	0.145	0.140	0.220	0.290	0.423	....	....	....	....

Total average deviation (atm.) 0.113
Total average % deviation 0.198

TABLE XVII.—CARBON DIOXIDE.

Density, moles/liter			0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
Temp. °C	Ref.		Pressure, atmospheres.								
100	A	obs	14.78	28.53	41.32	53.22	64.32	74.72	84.50	93.72	102.43
		obs–calc	0.01	0.01	0.02	0.01	0.01	0.04	0.08	0.10	0.06
100	An	obs	14.75	28.47	41.28	53.30	64.57	75.06	84.86	94.45	103.19
		obs–calc	−0.02	−0.05	−0.02	0.09	0.26	0.38	0.44	0.83	0.82
90	A	obs	14.34	27.61	39.86	51.16	61.61	71.32	80.39	88.86	96.80
		obs–calc	0.00	0.02	0.02	−0.01	−0.04	−0.05	−0.03	−0.05	−0.12
80	A	obs	13.89	26.64	38.33	49.06	58.93	68.04	76.46	84.23	91.41
		obs–calc	−0.01	−0.03	−0.05	−0.06	−0.05	0.00	0.06	0.07	−0.02
70	A	obs	13.45	25.69	36.86	47.04	56.32	64.77	72.47	79.50	85.92
		obs–calc	−0.01	−0.05	−0.05	−0.03	0.03	0.09	0.13	0.13	0.05
63.75	An	obs	13.18	25.17	36.10	46.04	55.10	63.33	70.80	77.53	83.66
		obs–calc	−0.01	0.01	0.11	0.26	0.50	0.76	1.02	1.18	1.29
60	A	obs	13.01	24.78	35.41	45.01	53.66	61.44	68.42	74.67	80.27
		obs–calc	−0.01	−0.03	−0.03	0.01	0.07	0.14	0.18	0.15	0.01
50	A	obs	12.58	23.86	33.94	42.93	50.93	58.03	64.28	69.77	74.59
		obs–calc	0.00	−0.01	−0.01	0.01	0.07	0.14	0.18	0.14	0.02
48.1	An	obs	....	....	....	....	....	....	64.11	69.62	74.42
		obs–calc	....	....	....	....	....	....	0.80	0.93	0.94
40	A	obs	12.15	22.94	32.48	40.86	48.19	54.58	60.09	64.79	68.78
		obs–calc	0.01	0.00	0.01	0.03	0.07	0.14	0.17	0.12	−0.03
35.5	An	obs	....	....	....	....	....	....	58.12	62.52	66.29
		obs–calc	....	....	....	....	....	....	0.10	0.11	0.10
35	A	obs	11.92	22.45	31.71	39.80	46.81	52.85	59.00	62.32	65.91
		obs–calc	0.00	−0.02	−0.01	0.03	0.08	0.14	0.19	0.16	0.02
32.5	An	obs	....	....	....	....	....	....	56.93	61.07	64.48
		obs–calc	....	....	....	....	....	....	0.18	0.17	0.06
32	A	obs	11.79	22.21	31.32	39.22	46.02	51.83	56.75	60.83	64.15
		obs–calc	0.00	0.03	0.05	0.08	0.12	0.17	0.21	0.18	0.02
31.1	An	obs	....	....	....	....	45.53	51.36	56.29	60.37	63.72
		obs–calc	....	....	....	....	−0.12	0.02	0.14	0.17	0.12
30	A	obs	11.69	21.95	30.92	38.70	45.39	51.08	55.84	59.76	62.93
		obs–calc	−0.01	−0.05	−0.05	−0.02	0.05	0.12	0.16	0.12	−0.02
20	A	obs	11.26	21.03	29.43	36.56	42.54	47.49	51.49	54.57	....
		obs–calc	0.00	−0.02	−0.03	−0.03	0.00	0.06	0.10	0.04	....
10	A	obs	10.82	20.10	27.89	34.33	39.58	43.80	....	....	....
		obs–calc	0.00	0.00	−0.05	−0.11	−0.13	−0.06	....	....	....
6.5	An	obs	10.63	19.76	27.48	33.86	....	....	....	....	....
		obs–calc	−0.03	−0.01	0.08	0.18	....	....	....	....	....
0	A	obs	10.38	19.15	26.39	32.19	....	....	....	....	....
		obs–calc	0.01	0.01	−0.02	−0.08	....	....	....	....	....
Average deviation (atm.)			0.007	0.021	0.034	0.050	0.065	0.090	0.136	0.122	0.050
Average % deviation			0.060	0.088	0.108	0.131	0.136	0.163	0.224	0.184	0.066

Note: The 100°, 63.75°, and 48.1° isotherms of Andrews show large variations from Amagat's data, and hence are excluded in the averages.

TABLE XVII.—Carbon Dioxide.

Pressure, atmospheres.

5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
110.71	....	....	....	....	....	....	....	....	....	....
−0.06	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
104.26	111.26	....	....	....	....	....	....	....	....	....
−0.31	−0.71	....	....	....	....	....	....	....	....	....
98.07	104.26	....	....	....	....	....	....	....	....	....
−0.24	−0.67	....	....	....	....	....	....	....	....	....
91.79	97.17	....	....	....	....	....	....	....	....	....
−0.19	−0.64	....	....	....	....	....	....	....	....	....
89.26	94.28	....	....	....	....	....	....	....	....	....
1.28	0.97	....	....	....	....	....	....	....	....	....
85.33	89.93	....	....	....	....	....	....	....	....	....
−0.24	−0.66	....	....	....	....	....	....	....	....	....
78.83	82.55	....	....	....	....	....	....	....	....	....
−0.24	−0.71	....	....	....	....	....	....	....	....	....
78.65	82.39	....	....	....	....	....	....	....	....	....
0.82	0.54	....	....	....	....	....	....	....	....	....
72.16	75.03	....	....	....	....	....	....	....	....	....
−0.32	−0.78	....	....	....	....	....	....	....	....	....
69.41	71.94	....	....	....	....	....	....	....	....	....
−0.06	−0.47	....	....	....	....	....	....	....	....	....
68.87	71.30	....	....	....	....	....	....	....	....	....
−0.27	−0.73	....	....	....	....	....	....	....	....	....
67.33	69.61	....	....	....	....	....	....	....	....	....
−0.12	−0.52	....	....	....	....	....	....	....	....	....
66.79	68.87	....	....	....	....	....	....	....	....	....
−0.33	−0.88	....	....	....	....	....	....	....	....	....
66.36	68.46	....	....	....	....	....	....	....	....	....
−0.15	−0.60	....	....	....	....	....	....	....	....	....
65.43	67.35	....	....	....	....	....	....	....	....	....
−0.34	−0.87	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
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....	....	....	....	....	....	....	....	....	....	....
....	....	....	....	....	....	....	....	....	....	....
0.221	0.687	....	....	....	....	....	....	....	....	....
0.286	0.871	....	....	....	....	....	....	....	....	....

Total average deviation (atm.)	0.128
Total average % deviation	0.203

TABLE XVIII.—METHANE.

Density, moles/liter			0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
Temp. °*C*	Ref.		Pressure, atmospheres.							
200	K + B	obs	19.39	38.77	58.18	77.66	97.26	117.06	137.11	157.46
		obs–calc	0.01	0.03	0.04	0.04	0.01	–0.02	–0.05	–0.09
150	K + B	obs	17.28	34.42	51.46	68.44	85.42	102.47	119.63	136.94
		obs–calc	0.01	0.02	0.03	0.02	0.00	–0.01	–0.03	–0.07
100	K + B	obs	15.16	30.06	44.74	59.23	73.59	87.85	102.08	116.32
		obs–calc	0.00	0.02	0.04	0.04	0.04	0.02	–0.01	–0.04
50	K + B	obs	13.04	25.69	37.98	49.94	61.62	73.08	84.35	95.49
		obs–calc	0.00	0.01	0.02	0.01	0.00	–0.02	–0.04	–0.06
0	K + B	obs	10.92	21.29	31.16	40.56	49.53	58.13	66.41	74.40
		obs–calc	0.00	–0.01	–0.02	–0.03	–0.06	–0.07	–0.06	–0.04
Average deviation (atm.)			0.004	0.018	0.030	0.028	0.022	0.028	0.038	0.060
Average % deviation			0.022	0.060	0.066	0.048	0.036	0.040	0.044	0.050

TABLE XVIII.—METHANE.

Pressure, atmospheres.

4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
178.18	199.36	221.09	243.46	....	....	....	....	....	....	....	....
−0.13	−0.12	−0.05	0.14	....	....	....	....	....	....	....	....
154.46	172.27	190.45	209.08	....	....	....	....	....	....	....	....
−0.11	−0.13	−0.10	0.00	....	....	....	....	....	....	....	....
130.64	145.09	159.74	174.62	....	....	....	....	....	....	....	....
−0.06	−0.06	−0.03	0.02	....	....	....	....	....	....	....	....
106.56	117.60	128.64	139.71	....	....	....	....	....	....	....	....
−0.05	−0.03	−0.01	0.01	....	....	....	....	....	....	....	....
82.16	89.73	97.10	104.23	....	....	....	....	....	....	....	....
0.02	0.11	0.19	0.17	....	....	....	....	....	....	....	....
0.074	0.090	0.076	0.068	....	....	....	....	....	....	....	....
0.052	0.066	0.060	0.048	....	....	....	....	....	....	....	....

Total average deviation (atm.) 0.045
Total average % deviation 0.049

TABLE XIX.—Ethyl Ether.

Density, moles/liter			0.385699	0.449982	0.539979	0.674973	0.899964	1.079957	1.349947	1.799929	2.699893
Temp. °C	Ref.		Pressure, atmospheres.								
325	B	obs	17.665	20.394	24.050	29.390	37.713	43.930	52.638	65.973	90.182
		obs–calc	0.012	0.032	–0.002	0.018	0.002	–0.023	–0.061	–0.014	0.072
300	B	obs	16.732	19.276	22.708	27.601	35.194	40.755	48.430	59.908	79.936
		obs–calc	0.004	0.017	0.020	0.008	0.011	–0.024	–0.056	0.021	0.058
275	B	obs	15.796	18.165	21.322	25.841	32.639	37.586	44.238	53.805	69.718
		obs–calc	–0.008	0.010	–0.001	0.027	–0.015	–0.017	–0.033	0.022	0.085
250	B	obs	14.890	17.076	19.958	24.038	30.120	34.400	40.028	47.730	59.378
		obs–calc	0.012	0.025	0.000	0.005	–0.004	–0.026	–0.024	0.058	0.006
225	B	obs	13.957	15.966	18.604	22.250	27.578	31.214	35.808	41.626	49.107
		obs–calc	0.004	0.019	0.012	–0.002	–0.015	–0.032	–0.021	0.072	0.014
200	B	obs	13.049	14.847	17.229	20.471	25.036	28.023	31.579	35.502	38.775
		obs–calc	0.022	0.005	0.003	0.001	–0.023	–0.041	–0.023	0.074	–0.017
175	B	obs	12.112	13.756	15.859	18.678	22.492	24.825	....	....	....
		obs–calc	0.011	0.019	0.001	–0.009	–0.032	–0.053	....	....	....
150	B	obs	11.190	12.667	14.479	16.880	....	....	....	....	....
		obs–calc	0.015	0.035	–0.011	–0.022	....	....	....	....	....
Average deviation (atm.)			0.011	0.020	0.006	0.012	0.015	0.031	0.036	0.044	0.042
Average % deviation			0.081	0.127	0.033	0.051	0.055	0.099	0.083	0.100	0.060

Total average deviation (atm.)	0.023
Total average % deviation	0.076

TABLE XX

SUMMARY OF THE AVERAGE DEVIATION OF THE PRESSURES CALCULATED BY THE EQUATION OF STATE FROM THE OBSERVED PRESSURES

See Tables IX to XIX

Gas	Maximum Pressure (Atmospheres)	Maximum Density (Moles/liter)	Temperature range (Degrees C.)	Number of Points	Total Average Deviation	
					Atmospheres	%
He	102	10	400 to −252	183	0.071	0.133
Ne	106	8.5	400 to −217	229	0.081	0.209
A	114	6	400 to −150	209	0.078	0.203
H_2	103	10	200 to −244	362	0.068	0.203
N_2*	134	5.5	400 to −149	214	0.072	0.190
O_2	103	9	100 to −117	137	0.051	0.115
Air	177	8	200 to −145	171	0.113	0.198
CO_2	111	5.5	100 to 0	148	0.128	0.203
CH^4	243	6	200 to 0	60	0.045	0.049
$(C_2H_5)_2O$	90	2.7	325 to 150	64	0.023	0.076
			Grand Average		0.077	0.178

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* The comparisons of the data of Smith and Taylor with the special equation based on their measurements alone are not included. To do so would reduce the deviations given in this table.

12. BIBLIOGRAPHY OF THE COMPRESSIBILITY DATA ON TEN GASES

SOURCES OF THE PRESSURE-VOLUME-TEMPERATURE DATA CONSIDERED IN THE COMPUTATION OF TABLES X TO XIX.

Author	Reference	Isotherms measured. (Degrees C)	Pressure Range. (Atmospheres)
	Helium		
Ramsay and Travers	Zeit. für Phys. Chem., *38*, 641 (1901); Phil. Trans., *197*(*A*), 47 (1901).	237.3	54–102
		11.2	28–101
K. Onnes	Leiden Comm., 102a; Verslag Akad. Amsterdam, Dec. 1907, p. 495.	100.35	43–67
		20	28–54
		0	27–50
		−103.57	21–33
		−182.75	14–18
		−216.56	10–11
K. Onnes	Leiden Comm., 102c; Verslag Akad. Amsterdam, March, 1908, p. 741.	−252.72	54–66
		−258.82	40–53
Martinez and K. Onnes	Leiden Comm., 164; Archives Néerlandaises, III A, *6*, 253 (1923).	−252.6	0.1–0.8
Penning and K. Onnes	Leiden Comm., 165 c, 166; Archives Néerlandaises, III A, 7, 166, 172 (1923).	−205.31	44–50
		−212.06	40–45
		−217.41	36–41
		−252.65	12–14
		−258.34	9–10
Boks and K. Onnes	Leiden Comm., 170 a; Fourth Int. Cong. Refrig., London, June, 1924.	20	27–63
		0	27–59
		−37.40	24–47
		−70.32	23–56
		−103.64	21–50
		−142.01	21–53
		−183.32	21–37
		−201.51	20–30
		−225.01	43–55
		−235.91	37–55
		−249.87	25–33
		−252.63	23–28
		−256.04	19–22
		−258.78	15–18
K. Onnes and Boks	Leiden Comm., 170 b; Fourth Int. Cong. Refrig., London, June 1924.	−268.88	<1
		−269.37	<1
		−269.69	<1
		−270.52	<1
van Agt and K. Onnes	Leiden Comm., 176b; Verslag Akad. Amsterdam, June 1925, p. 625.	69.86° K	<1
		20.55° K	<1
		20.53° K	<1
		20.51° K	<1
		16.65° K	<1

Author	Reference	Isotherms measured. (Degrees C)	Pressure Range. (Atmospheres)
Holborn and Schultze	Ann. der Physik, *47*, 1089 (1915).	100	20–50
		50	25–47
		0	25–50
Holborn and Otto	Zeit. für Physik, *10*, 367 (1922).	100	20–100
		50	25–99
		0	25–98
Holborn and Otto	Zeit. für Physik, *30*, 320 (1924).	−50	23–98
		−100	19–98
		−150	25–98
		−183	20–72
Holborn and Otto	Zeit. für Physik, *23*, 77 (1924); Wiss. Abh. Phys. Tech. Reich., *8* (*1*), 41 (1924).	400	26–98
		300	75
		200	23–99
Holborn and Otto	Zeit. für Physik, *33*, 1 (1925); Wiss. Abh. Phys. Tech. Reich., *9* (*1*), 163 (1925).	400 to −183 Smoothed	0–100 values
Holborn and Otto	Zeit. für Physik, *38*, 359 (1926).	−183	51–98
		−208	26–98
		−252.8	12–98
		−258	12–98
Bridgman	Proc. Nat. Acad. Sci., *9*, 370 (1923); Proc. Amer. Acad. Arts and Sci., *59*, 173 (1924).	95	3900–20000
		65	3900–20000
		30	3900–20000
	Neon		
Ramsay and Travers	Zeit. für Phys. Chem., *38*, 641 (1901); Phil. Trans., *197* (*A*), 47 (1901).	237.3	44–94
		11.2	39–94
K. Onnes and Crommelin	Leiden Comm., 147 d; Verslag Akad. Amsterdam, June 1915, p. 392.	20	23–93
		0	22–85
		−182.6	67–79
		−200.1	62–80
		−208.1	58–79
		−213.1	54–80
		−217.5	50–79
Crommelin, Martinez and K. Onnes	Leiden Comm., 154 a; Verslag Akad. Amsterdam, March 1919, p. 1316.	20	23–93
		0	22–85
		−103.01	36–78
		−141.22	34–79
		−182.60	32–63
		−200.08	26–80
		−208.10	24–79
		−213.08	23–80
		−217.52	21–79
Holborn and Otto	Zeit. für Physik, *23*, 77 (1924); Wiss. Abh. Phys. Tech. Reich., *8* (*1*), 41 (1924).	400	26–99
		300	26–99
		200	23–98
		100	24–99
		0	25–99
Holborn and Otto	Zeit. für Physik, *33*, 1 (1925); Wiss. Abh. Phys. Tech. Reich., *9* (*1*), 163 (1925).	400	27–98
		300	26–99
		200	22–99

Author	Reference	Isotherms measured. (Degrees C)	Pressure Range. (Atmospheres)
		100	39–99
		0	25–99
		−50	22–99
		−100	25–99
		−150	24–98
		−182.5	20–73
		400 to −182.5 Smoothed	0–100 values
Holborn and Otto	Zeit. für Physik, *38*, 359 (1926).	−207.9	20–89
	Argon		
Ramsay and Travers	Zeit. für Phys. Chem., *38*, 641 (1901); Phil. Trans., *197* (*A*), 47 (1901).	237.3	43–103
		11.2	30–101
K. Onnes and Crommelin	Leiden Comm., 118 b; 120 a; 121 b; 128; Verslag. Akad. Amsterdam, Oct. 1910, p. 582; Feb. 1911, p. 1177; May 1911, p. 68; June 1912, p. 256.	20.39	22–62
		18.39	37–62
		0	21–62
		−57.72	18–62
		−87.05	16–62
		−102.51	15–62
		−109.88	14–60
		−113.80	31–59
		−115.86	31–62
		−116.62	14–61
		−119.20	14–54
		−120.24	31–53
		−121.21	14–51
		−130.38	13–33
		−139.62	12–15
		−149.60	11–13
Holborn and Schultze	Ann. der Physik, *47*, 1089 (1915).	200	24–99
		150	21–98
		100	20–98
		50	25–98
		0	25–98
Holborn and Otto	Zeit. für Physik, *23*, 77 (1924); Wiss. Abh. Phys. Tech. Reich., *8* (*1*), 41, 1924.	400	26–100
		300	26–99
Holborn and Otto	Zeit. für Physik, *30*, 320 (1924).	−50	20–99
		−100	23–74
Holborn and Otto	Zeit. für Physik, *33*, 1 (1925); Wiss. Abh. Phys. Tech. Reich., *9* (*1*), 163 (1925).	400 to −100 Smoothed	0–100 values
Bridgman	Proc. Nat. Acad. Sci., *9*, 370 (1923); Proc. Amer. Acad. Arts and Sci., *59*, 173 (1924).	55	2600–20000
Masson and Dolley	Proc. Roy. Soc., London, *103A*, 524 (1923)	24.95	5–125
	Hydrogen		
Amagat	Ann. Chim. Phys., (5), *19*, 345 (1880).	20±	32–400

Author	Reference	Isotherms measured. (Degrees C)	Pressure Range. (Atmospheres)
Amagat	Ann. Chim. Phys., (5), *22*, 353 (1881).	100.1	39–420
		81.1	39–420
		60.4	39–420
		40.4	39–420
		17.7	39–420
Amagat	Compt. Rend., *107*, 522 (1888).	15	1000–3000
		47	1000–3000
Amagat	Ann. Chim. Phys., (6), *29*, 68 (1893).	200.25	150–900
		99.25	150–1000
		15.5	100–1000
		0	100–1000
		47.3	1000–2900
		15.4	700–3000
		0	100–2800
Wroblewski	Monat. für Chem., *9*, 1067 (1888); Wien. Ber., *97 II A*, 1321 (1889).	99.14	30–71
		0	30–71
		−103.55	20–71
		−182.45	15–71
Schalkwijk	Leiden Comm., 70; Verslag Akad. Amsterdam, June 1901.	20	6–60
K. Onnes and Hyndman	Leiden Comm., 78 c; Verslag Akad. Amsterdam, March 1902.	20	48–56
		0	45–53
K. Onnes and Braak	Leiden Comm., 97 a; Verslag Akad. Amsterdam, Dec. 1906 p. 517.	−103.57	32–50
		−135.71	29–33
		−182.81	47–55
		−195.27	41–50
		−204.70	35–62
		−212.82	31–61
		−212.98	30–35
		−217.41	46–59
K. Onnes and Braak	Leiden Comm., 99 a; 100 a; Verslag Akad. Amsterdam, June 1907, p. 162; Nov. 1907, p. 441.	−103.57	28–58
		−139.88	25–48
		−164.14	23–40
		−182.81	20–32
		−195.17	19–28
		−204.70	17–24
		−212.82	15–21
		−217.41	15–19
K. Onnes and Braak	Leiden Comm., 100 b; Verslag Akad. Amsterdam, Nov. 1907, p. 420.	100	31–50
		0	27–51
K. Onnes and Braak	Leiden Comm., 101 b; Verslag Akad. Amsterdam, Nov. 1907, p. 420.	Correction to temperatures of preceding articles.	
de Haas	Leiden Comm., 127 a; Verslag Akad. Amsterdam, April 1912, p. 1422.	20	0.1–1.1
K. Onnes and de Haas	Leiden Comm., 127 c; Verslag Akad. Amsterdam, May 1912, p. 92; June 1912, p. 282.	−252.63	< 1
		−255.46	< 1
		−257.26	< 1

Author	Reference	Isotherms measured. (Degrees C)	Pressure Range. (Atmospheres)
K. Onnes, Dorsman and Holst	Leiden Comm., 146 a; Verslag Akad. Amsterdam, June 1915, p. 344.	20	67–90
K. Onnes, Crommelin and Smid	Leiden Comm., 146 b; Verslag Akad. Amsterdam, June 1915, p. 350.	20	65–100
Martinez and K. Onnes	Leiden Comm., 164; Archives Néerlandaises, III A, *6*, 253 (1923).	−252.6	< 1
K. Onnes and Penning	Leiden Comm., 165 b; 166; Archives Néerlandaises, III A, *7*, 157 (1913); ibid, III A, *7*, 172 (1923).	−103.57	38–51
		−139.89	30–40
		−182.74	20–30
		−203.96	17–22
		−212.73	15–18
		−217.32	13–17
		−225.37	10–14
		−231.38	9–11
		−236.28	8–9
		−238.29	8–9
		−239.90	7–8
		−241.87	6–7
		−243.88	6–7
Crommelin and Swallow	Leiden Comm., 172 a; Fourth Int. Cong. Refrig., London, June 1924	−217.33	34–57
		−225.36	28–56
		−231.40	23–49
		−236.31	18–40
		−238.29	15–46
		−239.92	13–51
van Agt and K. Onnes	Leiden Comm., 176 b; Verslag Akad. Amsterdam, June 1925, p. 625.	90.23° K	< 1
		69.86° K	< 1
		20.55° K	< 1
		20.53° K	< 1
		20.51° K	< 1
		20.49° K	< 1
		18.16° K	< 1
		16.65° K	< 1
		15.64° K	< 1
		14.50° K	< 1
Witkowski	Krakauer Anzeiger, 305 (1905); Landolt-Börnstein Tabellen, I, (1923), p. 105.	100	5–60
		0	5–60
		−77	5–60
		−104	5–60
		−147	5–60
		−183	5–60
		−190	5–60
		−205	5–60
		−212	5–60
Konstamm and Walstra	Proc. Roy. Acad. Amsterdam, *17*, 203 (1914).	20	124–2280
		15.5	136–2260
Holborn	Ann. der Phys., *63*, 674 (1920)	100	20–99
		50	25–96
		0	25–97

Author	Reference	Isotherms measured. (Degrees C)	Pressure Range. (Atmospheres)
		100–0 Smoothed	0–100 values
Holborn and Otto	Zeit. für Physik, *23*, 77 (1924); Wiss. Abh. Phys. Tech. Reich., *8* (*1*), 41 (1924).	200	22–100
Holborn and Otto	Zeit. für Physik, *33*, 1 (1925) Wiss. Abh. Phys. Tech. Reich., *9* (*1*), 163 (1925).	−50	21–98
		−100	27–99
		−150	25–99
		−183	25–75
		200 to −183 Smoothed	0–100 values
Holborn and Otto	Zeit. für Physik, *38*, 359 (1916).	−207.9	20–98
Verschoyle	Proc. Roy. Soc. London, *111 A*, 552 (1926).	20	46–205
		0	46–205
Bartlett	Jour. Am. Chem. Soc., *49*, 687 (1927); ibid., *49*, 1955 (1927)	0	50–1000
Bridgman	Proc. Nat. Acad. Sci., *9*, 370 (1923); Rec. trav. Chim., *42*, 568 (1923). Proc. Amer. Acad. Arts and Sci., *59*, 173 (1924)	65	3900–17000
		30	2600–17000
	Nitrogen		
Amagat	Ann. Chim. Phys., (5), *19*, 345 (1880); Compt. Rend., *88*, 336 (1879).	20±	27–431
Amagat	Ann. Chim. Phys., (5), *22*, 353 (1881).	100.1	39–420
		75.5	39–420
		50.4	39–420
		30.1	39–420
		17.7	39–420
Amagat	Compt. Rend., *99*, 1017, 1153 (1884).	16	26–85
Amagat	Compt. Rend., *107*, 522 (1888).	15	750–3000
Amagat	Ann. Chim. Phys., (6), *29*, 68 (1893).	199.50	150–950
		99.45	150–950
		16.03	100–1000
		0	100–1000
		43.6	900–3000
		16	800–3000
		0	100–3000
Ramsay and Travers	Zeit. für Phys. Chem. *38*, 641 (1901); Phil. Trans., *197* (*A*), 47 (1901).	237.3	51–101
		11.2	55–104
Holborn and Otto	Zeit. für Physik, *10*, 367 (1922).	100	19–99
		50	25–99
		0	25–99
Holborn and Otto	Zeit. für Physik, *23*, 77 (1924); Wiss. Abh. Phys. Tech. Reich., *8* (*1*), 41 (1924).	400	26–99
		300	26–99
		200	23–97
		150	21–98
		100	19–99

Author	Reference	Isotherms measured. (Degrees C)	Pressure Range. (Atmospheres)
		50	50–97
		0	25–98
Holborn and Otto	Zeit. für Physik., *30*, 320 (1924).	0	100
		−50	20–99
		−100	25–98
		−130	21–67
Holborn and Otto	Zeit. für Physik, *33*, 1 (1925); Wiss. Abh. Phys. Tech. Reich., *9* (*1*), 163 (1925).	400 to −130 Smoothed	0–100 values
Smith and Taylor	Jour. Am. Chem. Soc., *45*, 2107 (1923); ibid., *48*, 3122 (1926).	200	61–320
		152.34	55–282
		100	48–240
		49.98	41–200
		0	34–160
K. Onnes and van Urk	Leiden Comm., 169 d; 169 e; Fourth Int. Cong. Refrig. London, June 1924.	20	36–63
		0	33–59
		−23.62	33–61
		−50.26	32–57
		−81.10	30–57
		−102.25	28–55
		−121.19	27–54
		−131.27	25–58
		−141.53	24–50
		−144.46	23–42
		−146.32	23–36
		−148.58	28–31
Verschoyle	Proc. Roy. Soc. London, *111 A*, 552 (1926).	20	25–205
		0	25–205
Bartlett	Jour. Am. Chem. Soc., *49*, 687, (1927); ibid., *49*, 1955 (1927).	0	50–1000
Bridgman	Proc. Nat. Acad. Sci., *9*, 370 (1923); Proc. Amer. Acad. Arts and Sci., *59*, 173 (1924).	68	3300–20000
	Oxygen		
Amagat	Compt. Rend., *91*, 812 (1880).	100	113–418
		14.7	113–418
Amagat	Ann. Chim. Phys., (5), *19*, 345 (1880).	20±	32–399
Amagat	Compt. Rend., *107*, 522 (1888).	15	1000–3000
Amagat	Ann. Chim. Phys., (6), *29*, 68 (1893).	199.5	150–950
		99.5	100–1000
		15.65	100–1000
		0	100–1000
		15.6	600–3000
		0	100–2900
K. Onnes and Hyndman	Leiden Comm., 78 b; Verslag Akad. Amsterdam, March 1902.	20	23–65
		15.6	36–51
		0	22–66
Kuypers and K. Onnes	Leiden Comm., 165 a; Archives Néerlandaises, III A, *6*, 227 (1923).	20	22–61
		0	21–50

Author	Reference	Isotherms measured. (Degrees C)	Pressure Range. (Atmospheres)
K. Onnes and Kuypers	Leiden Comm., 169 a; 169 b; Fourth Int. Cong. Refrig. London, June 1924.	−40.05	21–61
		−80.03	21–62
		−102.46	20–60
		−109.97	20–61
		−113.98	20–59
		−116.01	22–55
		−116.99	20–53
van Urk and Nijhoff	Leiden Comm., 169 c; Fourth Int. Cong. Refrig. London, June 1924.	20	36–62
		15.6	34–57
		0	37–54
Nijhoff and Keesom	Leiden Comm., 179 b; Verslag Akad. Amsterdam, Oct. 1925, p. 905.	−40.01	6–9
		−80.00	6–8
		−102.49	5–7
		−109.99	6–7
		−113.94	5–7
		−116.01	5–6
		−116.03	4–5
		−117.01	4–6
		−118.58	4–6
		−124.95	3–6
		−135.29	4–6
		−145.39	3–5
		−152.56	3–5
Holborn and Otto	Zeit. für Physik, *10*, 367, (1922).	100	20–100
		50	25–97
		0	25–99
Holborn and Otto	Zeit. für Physik, *33*, 1, (1925); Wiss. Abh. Phys. Tech. Reich., *9* (*1*), 163 (1925).	{100 to 0 Smoothed	0–100 values
Masson and Dolley	Proc. Roy. Soc. London, *103A*, 524 (1923).	24.95	30–125
	Air		
Amagat	Ann. Chim. Phys., (5), *19*, 345 (1880).	20±	32–400
Amagat	Compt. Rend., *99*, 1017, 1153 (1884).	16	26–85
Amagat	Compt. Rend., *107*, 522 (1888).	15	750–3000
Amagat	Ann. Chim. Phys., (6), *29*, 68 (1893).	200.4	150–1000
		99.4	100–1000
		15.7	100–1000
		0	100–1000
		45.1	800–3000
		15.7	700–3000
		0	100–3000
Witkowski	Phil. Mag., (5) *41*, 288 (1896).	100	10–120
		16	10–130
		0	10–130
		−35	50–125
		−78.5	20–130
		−103.5	20–130
		−130	20–80
		−135	30–65

Author	Reference	Isotherms measured. (Degrees C)	Pressure Range. (Atmospheres)
		−140	15–40
		−145	15–30
Koch	Ann. der Physik, *27*, 311 (1908).	0	26–202
		−79.1	25–200
Holborn and Schultze	Ann. der Physik, *47*, 1089 (1915).	200	24–99
		150	21–98
		100	20–98
		50	25–97
		0	25–98
Holborn and Otto	Zeit. für Physik, *10*, 367, (1922). Zeit. für Physik. *23*, 77, (1924); Wiss. Abh. Phys. Tech. Reich., *8* (*1*), 41 (1924). Zeit. für Physik. *33*, 1 (1925); Wiss. Abh. Phys. Tech. Reich., *9* (*1*), 163 (1925).	200 to 0 Smoothed	0–100 Values
Penning	Leiden Comm., 166; Archives Néerlandaises, III A, *7*, 172 (1923).	20	29–61
		−70.09	52–59
		−84.08	48–58
		−103.48	41–48
		−122.03	34–39
		−129.97	31–36
		−134.99	29–33
		−140.00	27–31
		−145.05	25–28
	Carbon Dioxide		
Andrews	Phil. Trans., *159*, 575 (1869); ibid., *166*, 421 (1876).	6.5	12–34
		13.1	48–90
		21.5	47–63
		31.1	55–85
		32.5	57–85
		35.5	57–108
		48.1	63–109
		63.8	18–223
		100	17–224
Amagat	Ann. Chim. Phys. (5), *19*, 345 (1880).	20±	32–400
Amagat	Ann. Chim. Phys., (5), *22*, 353 (1881).	100	39–420
		90.2	39–420
		80	39–420
		70	39–420
		60	39–420
		50	39–420
		40.2	39–420
		35.1	39–420
		18.2	105–420
Amagat	Ann. Chim. Phys., (6), *29*, 68 (1893).	258	75–450
		198	75–950
		137	50–950
		100	50–1000
		90	50–1000
		80	50–1000

Author	Reference	Isotherms measured. (Degrees C)	Pressure Range, (Atmospheres)
		70	45–1000
		60	45–1000
		50	45–1000
		40	40–1000
		35	40–90
		32	40–78
		30	37–1000
		20	35–1000
		10	33–1000
		0	31–1000
Keesom	Leiden Comm., 88; Verslag Akad. Amsterdam, Sept., p. 391; Oct., p. 533; Nov., p. 616, (1903).	57.75	66–136
		48.10	65–136
		41.95	65–135
		37.09	65–136
		34.02	65–137
		31.89	64–137
		30.98	63–139
		28.15	63–137
		25.55	63–138
Maas and Mennie	Proc. Roy. Soc. London, *110 A*, 198 (1926).	99.9	< 1
		50.3	< 1
		19.7	< 1
		0.1	< 1
		−24.9	< 1
		−50.3	< 1
		−70.2	< 1
	Methane		
Amagat	Ann. Chim. Phys., (5), *19*, 345 (1880).	20 ±	32–400
Amagat	Ann. Chim. Phys., (5), *22*, 353 (1881).	100.1	53–300
		79.8	53–300
		60.1	39–300
		40.6	39–300
		29.5	39–300
		14.7	39–300
Keyes, Smith and Joubert	Jour. Math. and Phys., M.I.T., *1*, 191 (1922).	200	69–322
		150	61–275
		100	53–228
		50	45–179
		0	36–130
Keyes and Burks	Jour. Am. Chem. Soc., *49*, 1403 (1927).	200	60–254
		150	53–218
		100	46–182
		50	39–145
		0	32–108
	Ethyl Ether		
Ramsay and Young	Zeit. für Phys. Chem., *1*, 433 (1887).	280 to 50	1–69
Keyes and Felsing	Jour. Am. Chem. Soc., *41*, 589 (1919).	340 to 150	8–41

Author	Reference	Isotherms measured. (Degrees C)	Pressure Range. (Atmospheres)
Beattie	Jour. Am. Chem. Soc., *46*, 342 (1924).	325	18–206
		300	17–175
		275	16–143
		250	15–112
		225	14–80
		200	13–48
		175	12–25
		150	11–17

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THE ELECTRICAL RESISTANCE OF ALLOYS UNDER PRESSURE.

By C. W. Ufford.

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INTRODUCTION.

In this paper, are presented measurements of the effect of hydrostatic pressures from atmospheric to 12000 kg/cm^2 on the electrical resistance of three series of alloys: lithium tin, bismuth tin, calcium lead; and one alloy of calcium and magnesium with 10 A% Mg. These three series were chosen since each consists of one metal whose resistance increases with pressure: lithium, bismuth and calcium; and another whose resistance decreases with pressure; tin and lead. In this way, a large variation of the pressure coefficient of electrical resistance as a function of the composition of the alloys is obtained. The pressure coefficients were measured at two temperatures for each series of alloys.

There are seven metals which increase in resistance as the pressure increases, the normal effect being for the resistance to decrease as the pressure increases: lithium, calcium, strontium, antimony, cesium, barium, cerium and bismuth. By including the most easily manipulated of these in the alloys, it is hoped to give a basis for explaining this abnormal effect which must be accounted for by a complete theory of electric conduction in metals.

PREVIOUS EXPERIMENTS.

Bridgman[1] has measured the effect of pressures up to 12000 kg/cm^2 on the electrical resistance of a large number of elements and five commercial alloys, including the five elements bismuth, calcium, lead, lithium and tin used in preparing the alloys for these experiments. Bridgman's results are used here for the end points at zero and one hundred per cent in the curves plotted with concentrations as abscissae.

Lisell[2] and Beckman[3] have measured the pressure coefficients of the binary alloys gold-silver, cadmium lead, gold-nickel, and copper-manganese up to pressures of 2700 atmospheres. There have been no measurements as far as we know on the series of alloys of these experiments.

Experimental Method.

The composition of the alloys was determined by weighing the two metals before they were alloyed. It was necessary to weigh lithium and calcium under white petroleum oil to prevent their being oxidized. The metal, covered with an oil film, was dipped into petroleum ether. Just as this evaporated, the metal was dropped dry into the weighed crucible of oil. The oxidation of lithium after this process was slight, and that of calcium even less. The alloys weighed from one to five grams.

The alloys were made by melting the two metals together in vacuum in an electric furnace. Bismuth and tin were melted in a pyrex tube connected directly to the vacuum pump. Lithium and tin were put in a small steel tube inserted in a steel cylinder, into the top of which a plug was screwed. The interior of the plugged steel cylinder was connected with the vacuum outside by a hole drilled along the axis of the plug, starting from the inside, not extending straight through the plug, but intersecting in an acute angle a hole drilled in from the outside of the plug. This largely prevented the lithium vapor from escaping from the bomb by forcing it to turn an acute angle away from the pump in its effort to get out. The metals in this steel bomb, placed in a continuously evacuated quartz tube, were melted in an electric furnace.

Calcium and lead reacted so violently that another method was used to prepare them. The metals were compressed mechanically into a small Chromel "A" cylinder fitted with a plug, through which a small hole was drilled. This cylinder was then sealed into an evacuated quartz tube, together with a steel cap containing a small amount of metallic calcium. This tube, separate from the vacuum pump, was heated in an electric furnace. As calcium sublimed, the vapor pressure due to the calcium in the cap was equal to that inside the cylinder containing the alloy, provided that the entire quartz tube was at a uniform temperature, so that there was no tendency for the calcium to escape from the cylinder. In spite of this, the reaction of the calcium with lead was so violent that the alloy flowed freely from the containing cylinder.

Some calcium-lead alloys rich in calcium were made in an atmosphere of argon gas under a pressure of four atmospheres to prevent the alloy from escaping from the cylinder, but the argon had practically no effect. Finally, the metals were put in a cylinder with a metal plug screwed tightly in against them, without any hole in the plug, so that it was impossible for any of the alloy to escape. This was the most successful way of making lithium-tin alloys from 0 to 30 A% tin, and calcium-lead alloys from 50 to 100 A% calcium. No lithium-tin alloys from 10 to 30 A% tin, nor calcium-lead alloys from 50 to 80 A% calcium were measured, because, even when it was possible to make them by this final method, it was so difficult to extrude them into wires suitable for mounting in the pressure apparatus, and the wires obtained were so brittle that all attempts failed. After melting, the cylinder containing the alloy was turned off in a lathe, a considerable portion of the alloy being cut away in order to remove any contamination from the cylinder.

The alloys were made into wires by extruding them in a cylindrical die made from Bessemer steel by cutting a one-quarter-inch hole to hold the alloy, enlarging the top of this hole to half an inch, and drilling through the bottom of it a hole .052 inch in diameter, which determined the diameter of the wire. The die was set in an arbor press with the lower part of the die immersed in a test tube of white petroleum oil to prevent the alloy's being oxidized as it came out hot.

The pressure was exerted by a piston one-quarter of an inch in diameter at the bottom, one-half an inch at the top, corresponding with the hole in the die. The upper part of the piston was made larger than the lower to give strength to the threads holding a cross bar by which it was removed from the die. This also prevented the piston from bending under pressure.

After the alloy to be extruded was put in the die, copper and steel washers were driven down hard on top of it. A plug, made to fit the reamed hole in the bottom of the die, prevented the hole from becoming clogged with alloy oxidized when the die and alloy were heated in an electric furnace to the proper temperature for extruding. The alloys rich in lithium and calcium were so brittle that they had to be heated nearly to the melting point before they could be extruded. The most satisfactory extrusion temperature was found to be about eight-ninths of the melting temperature of the alloy.

The extruded wire was set in a pyrex test tube, mounted on top of the three-terminal plug which screwed into the lower cylinder of the pressure apparatus. The method of measuring the resistance of the alloy and its change with pressure on a potentiometer has been described by Bridgman.[1b]

ACCURACY OF THE RESULTS.

From the calibration of the manganin pressure gauge against a standard gauge,[4] the probable error in the pressure reading is shown to be less than .1%. The errors in measuring the resistance arose from reading the slide wire, thermoelectromotive forces, poor contacts, chemical action, change of state of the alloy, hysteresis, and lack of temperature control. These disturbances tend to prevent the points taken with increasing and decreasing pressure from lying on a smooth curve. Therefore, in order to estimate the probable error in the change of resistance produced by the pressure apparatus for all these different reasons, the curve in which the points were most widely scattered was chosen to calculate the average deviation of all the points from a smooth curve. This gave a probable error of .4%. It therefore may be assumed that the pressure coefficients of resistance of the actual specimens measured are correct to within .5%.

The accuracy of the temperature coefficients is not as great as that of the pressure coefficients, since the former were calculated from the measurements taken at atmospheric pressure in measuring the change of resistance with pressure at two different temperatures. The two measurements were often taken on different days, and represent a single observation and not an average value. The average temperature coefficients from 0 to 30°C of the alloys of the bismuth-tin series at 20.31 and 30.20 A% tin were found by extrapolating the average coefficients from 30 to 75° and 30 to 50°C respectively, since no measurements were made on these alloys at 0°C.

The greatest error is in the composition of the alloys themselves, especially those rich in lithium and calcium. The weight could be found to within .2 mg. on the analytical balance used, which gave a probable error in the composition of the alloys of the bismuth-tin series of ±.02%. The bismuth used in the alloys was Kahlbaum's grade "K," the following analysis for which is given by Bucher.[5] In 100 grams of bismuth:

silver,	scarcely a trace
lead,	trace
copper,	1.0 mg. and
iron,	.1 mg.

A sample of tin from the Bureau of Standards was used in both the bismuth-tin and lithium-tin series. The lithium was Kahlbaum's grade "K." The calcium was from a sample purified specially for Professor Bridgman in the Cornell University Chemical Laboratory. The lead was the Bureau of Standards melting-point lead, for which Bridgman[6] gives the following analysis:

Ag .0002 to .0003%	
Sb .0019 to .0028%	
Sn .0008 to .0011%	
Cu .0003 to .0004%	
Fe .0004 to .0006%	
As, Bi, Zn, Co, Ni	trace
Cd Mn	none
Lead (by difference)	99.9948+%

In the two lithium-tin alloys with 9 and 10 A% tin, 2 or 3 A% excess lithium was melted with the alloy to allow for vaporization. The vaporization and violence of the reaction was such in the alloys rich in lithium and calcium as to make their composition doubtful, so that the alloy with 55.7 A% lead, which vaporized and flowed most violently, was analyzed chemically, giving an error of 5.7 A% in its composition. This is a fair estimate of the error in composition of the alloys rich in lithium and calcium.

RESULTS.

The results are presented in the form of tables. The average pressure coefficient of resistance to 12000 kg/cm^2 $(1/R_0)$ $(\Delta R/\Delta p)$ and the average temperature coefficient between the two temperatures at which measurements were made for each series are given in Table I. The average pressure coefficient to 12000 kg/cm^2 is the slope of the straight line drawn from the origin to the point corresponding to 12000 kg/cm^2 on the curve of relative change of resistance with pressure. In order to reproduce this curve, the deviation of the points from this straight line at 2, 4, 6, 8 and 10 $\times$ 10^3 kg/cm^2 are given in Table II. The deviation at atmospheric pressure and at 12000 kg/cm^2 is then zero. Thus, in order to find the relative change of resistance at any pressure and the temperature of one of the sets of measurements, the average pressure coefficient to 12000 kg/cm^2 is multiplied by the given pressure, as found from the deviation curve drawn through the points given in Table II. The deviation curves are in general approximately parabolas.

As an example, find the relative change of resistance of an alloy of bismuth and tin with 10.01 A% tin at 0°C and 8000 kg/cm^2. From Table I the average pressure coefficient of a bismuth-tin alloy of 10.01 A% tin at 0°C is $-.0_41135$. The relative linear change of resistance is thus $-.0_41135 \times 8000 = -.0\,9080$. From Table II, the deviation from linearity for a bismuth-tin alloy of 10.01 A% tin at 0°C is found to be —.00313. The sum of these two is —.09393, which is the relative decrease in resistance due to a pressure of 8,000 kg/cm^2.

The curves showing the relative change of resistance under pressure of the bismuth-tin alloy of .53 A% tin were too widely separated for increasing and decreasing pressure to plot a curve of deviation from a straight line. This separation is probably due to a change of state of the alloy. This is the one alloy measured which showed a change of sign of the pressure coefficient during the measurements, since the resistance first increases to a maximum and then falls as the pressure increases. The relative change of resistance is given for the pressures up to 12000 kg/cm^2 in Table III. The pressure coefficient and deviations from linearity of the relative change of resistance for the single calcium-magnesium alloy, with 10 A% mg., are included in Tables I and II.

TABLE I.

PRESSURE AND TEMPERATURE COEFFICIENTS OF ELECTRICAL RESISTANCE.

LITHIUM-TIN SERIES.

Composition	*Average coefficient*		*Temperature coefficient*
A% Sn	*0–12000 Kg/cm²*		*30–75° C*
	30° C	*75° C*	
0	$.0_5772$	$.0_5772$	.00398
9.0	$.0_59047$	$.0_59374$	.00989
10.0	$.0_58634$	$.0_59046$	.00284
30.1	$-.0_41062$	$-.0_41063$	.00385
40.0	$-.0_56595$	$-.0_56864$	.00271
71.4	$-.0_58419$	$-.0_58381$	.00173
95.1	$-.0_58886$	$-.0_59017$	.00415
100.0	$-.0_59295$	$-.0_59434$	.00394

CALCIUM-LEAD SERIES.

A% Pb			
0	$.0_41236$	$.0_41172$	.00304
9.5	$.0_41434$	$.0_41502$	.00390

TABLE I.—Continued.

CALCIUM-LEAD SERIES.

Composition *A% Pb*	*Average coefficient 0–12000 Kg/cm²* *30°C*	 *75°C°*	*Temperature coefficient 30–75°C*
17.5	$.0_41382$	$.0_41516$	.00312
18.8	$.0_41422$	$.0_41374$	.00294
29.6	$-.0_53931$	$-.0_54127$	.00474
55.7	$-.0_54524$	$-.0_54894$	.00260
63.8	$-.0_57478$	$-.0_57424$	.00278
74.8	$-.0_41065$	$-.0_59997$	.00303
90.0	$-.0_41131$	$-.0_41144$	.00333
100.0	$-.0_41224$	$-.0_41243$	.00375

PRESSURE AND TEMPERATURE COEFFICIENTS OF ELECTRICAL RESISTANCE.

BISMUTH-TIN SERIES.

Composition *A% Sn*	*Average coefficient 0–12000 Kg/cm²* *30°C*	 *0°C*	*Temperature coefficient 0–30°C*
0	$.0_42129$	$.0_4223$	.00392
Kahlbaum	$.0_42498$	$.0_4269$	.0028
.13	$.0_42331$	$.0_42220$	$-.000976$
.53	$.0_57205$	$-.0_6910$	$-.0_3444$
.89	$-.0_54178$	$-.0_58998$	$.0_3475$
5.02	$-.0_41034$	$-.0_41166$	.00270
10.01	$-.0_41053$	$-.0_41135$	.00323
20.32	$-.0_59729$		.00409
30.20	$-.0_59451$		.00332
39.87	$-.0_59216$	$-.0_59126$	.00355
50.39	$-.0_58998$	$-.0_58856$	.00350
68.51	$-.0_58185$	$-.0_57946$	.00367
90.00	$-.0_57621$	$-.0_57403$	.00375
97.99	$-.0_58131$	$-.0_58008$	.00455
100.00	$-.0_59295$	$-.0_59204$	.00433

CALCIUM-MAGNESIUM SERIES.

A%Mg	*0°C* *Average 0–6000 Kg/cm²*	*30°C*	
10.0	$.0_41636$	$.0_41496$	.00226

TABLE II.

Deviations from Linearity.

Lithium-Tin Series.
(30°C)

Composition	*Pressures*				
A% Sn	*2000*	*4000*	*6000*	*8000*	*10000*
0					
9.0	—.0$_3$396	—.00170	—.00267	—.00276	—.00202
10.0	—.00385	—.00524	—.00436	—.00331	—.00220
30.1	—.00916	—.0122	—.01189	—.00874	—.00458
40.4	—.00275	—.00368	—.00389	—.00338	—.00168
71.4	—.00851	—.00900	—.00741	—.00524	—.00288
95.12	—.00236	—.00378	—.00410	—.00380	—.00204

Lithium-Tin Series.
(75°C)

0					
9.0	No Deviation				
10.0	—.00197	—.00377	—.00451	—.00426	—.00286
30.1	—.00860	—.01152	—.01045	—.00796	—.00497
40.4	—.0$_3$993	—.00201	—.00288	—.00287	—.00199
71.4	—.00419	—.00753	—.00900	—.00802	—.00505
95.12	—.00215	—.00341	—.00391	—.00358	—.00223

Calcium-Magnesium Series.
(0°C)

A% Mg					
10.0	—.00306	—.00516	—.00590	—.00519	—.00324

Calcium-Magnesium Series.
(30°C)

10.0	No Deviation

Bismuth-Tin Series.
(0°C)

Composition	*Pressures*				
A% Sn	*2000*	*4000*	*6000*	*8000*	*10000*
.13	.0133	.01969	.0211	.02014	.0170
.530	Deviation not calculated				
.89	.00236	.00358	.00331	.00226	.00121
5.02	—.00130	—.00200	—.00230	—.00186	—.00108
10.01	—.00201	—.00299	—.00343	—.00313	—.00243
20.32					
30.20					

TABLE II.—(Continued).

BISMUTH-TIN SERIES.
(0°C)

Composition			*Pressures*		
A % Sn	*2000*	*4000*	*6000*	*8000*	*10000*
39.87	—.00264	—.00384	—.00418	—.00375	—.00260
50.39	—.00211	—.00335	—.00390	—.00340	—.00206
68.51	—.00180	—.00336	—.00373	—.00319	—.00188
90.00	—.0$_3$902	—.00141	—.00161	—.00156	—.00120
97.99	—.00221	—.00296	—.00321	—.00280	—.00188

BISMUTH-TIN SERIES.
(30°C)

A % Sn	*2000*	*4000*	*6000*	*8000*	*10000*
.13	.00346	.00616	.00812	.01012	.00947
.530	.01433	.02524	.02764	.02205	.0140
.89	.00926	.01321	.01348	.01151	.00667
5.02	.0$_3$183	.0$_3$496	.0$_3$522	.0$_3$417	.0$_3$248
10.01	—.0$_3$654	—.00107	—.00121	—.00119	—.0$_3$934
20.32	—.0$_3$379	—.00136	—.00170	—.00144	—.0$_3$705
30.20	—.00160	—.00258	—.00284	—.00241	—.00144
39.87	—.00197	—.00311	—.00342	—.00316	—.00197
50.39	—.00171	—.00309	—.00354	—.00334	—.00232
68.51	—.00213	—.00342	—.00376	—.00338	—.00194
90.00	—.00160	—.00274	—.00301	—.00262	—.00157
97.99	—.00224	—.00296	—.00321	—.00281	—.00185

CALCIUM-LEAD SERIES.
(30°C)

A % Pb	*2000*	*4000*	*6000*	*8000*	*10000*
9.52	—.00510	—.00880	—.00915	—.00769	—.00507
17.49	—.00405	—.00740	—.00830	—.00688	—.00463
18.80	—.00377	—.00659	—.00730	—.00615	—.00403
29.61	—.00141	—.00236	—.00259	—.00236	—.00159
55.7	—.00235	—.00317	—.00343	—.00300	—.00196
63.78	—.00229	—.00360	—.00386	—.00346	—.00230
74.76	—.00427	—.00620	—.00660	—.00480	—.00368
90.00	—.00353	—.00591	—.00631	—.00541	—.00339

CALCIUM-LEAD SERIES.
(75°C)

A % Pb	*2000*	*4000*	*6000*	*8000*	*10000*
9.52	—.00420	—.00756	—.00824	—.00672	—.00409
17.49	—.00403	—.00690	—.00785	—.00698	—.00461
18.80	—.00385	—.00696	—.00788	—.00636	—.00397
29.61	—.00147	—.00300	—.00322	—.00297	—.00202

TABLE II.—(Continued).

CALCIUM-LEAD SERIES.

(75°C)

Composition	*Pressures*				
A% Pb	*2000*	*4000*	*6000*	*8000*	*10000*
55.7	—.00197	—.00329	—.00339	—.00285	—.00178
63.78	—.00182	—.00308	—.00360	—.00314	—.00200
74.76	—.00343	—.00490	—.00508	—.00447	—.00278
90.00	—.00412	—.00622	—.00648	—.00594	—.00398

TABLE III.

RELATIVE CHANGE OF RESISTANCE.

BISMUTH-TIN ALLOY.

.53 A% Tin—0°C.

Pressure kg/cm^2	$\frac{\Delta R}{R_0}$
	Increasing Pressure
2000	.006910
4000	.010168
6000	.008453
8000	.005118
10000	—.001343
12000	—.011940
12446	—.01164
	Decreasing Pressure
12000	—.011352
11000	—.004764
9000	.005947
7000	.014367
5000	.019681
3000	.020707
1000	.013600

DEDUCTIONS FROM EXPERIMENT.

To illustrate the character of the results, the average pressure coefficient of electrical resistance from 0 to 12000 kg/cm^2 at 30°C and the average temperature coefficient of resistance between the two temperatures are plotted against the per cent composition of the alloy in figures 1 to 3. The curves of pressure coefficients at the other temperature are similar in all respects. The equilibrium diagrams of the alloys have been superposed on these diagrams for

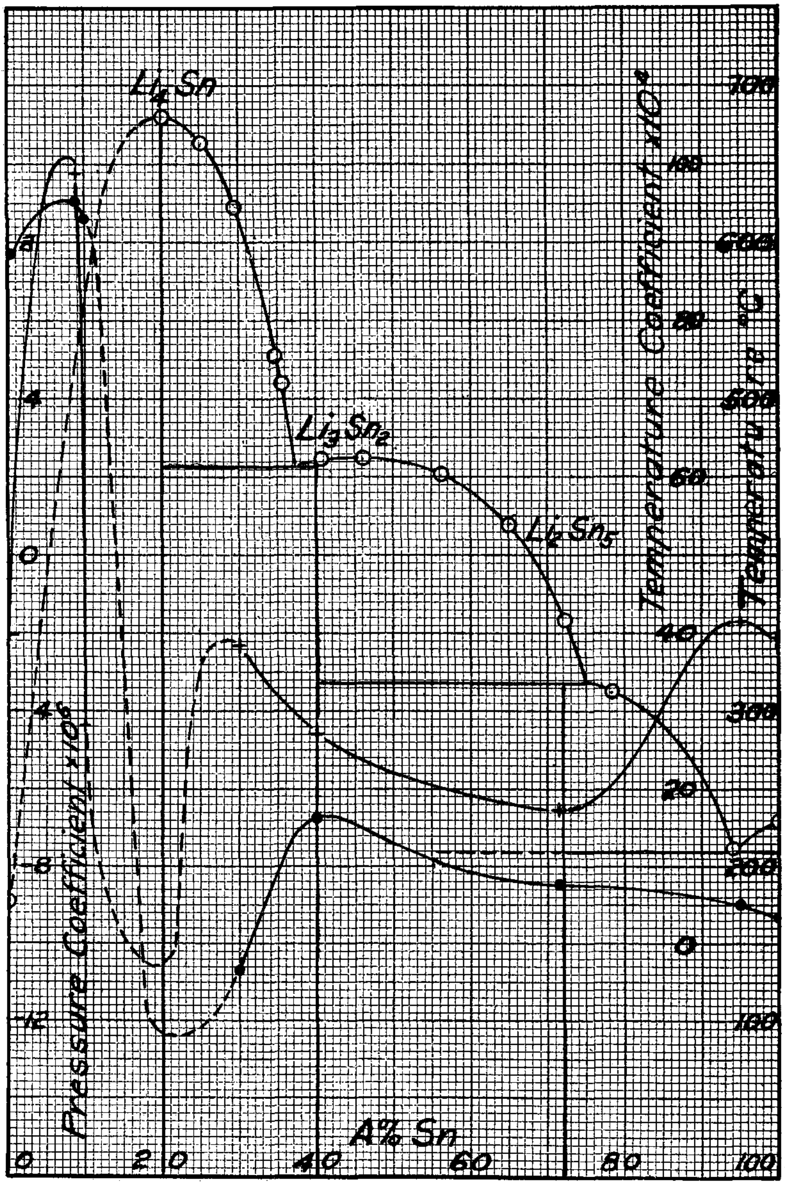

FIGURE 1. Lithium tin series at 30° C. The melting curve is drawn through the circles. The curve through the dots represents the average pressure coefficient of resistance 0–12000 kg/cm². That through the crosses represents the average temperature coefficient of resistance from 30 to 75° C. The abscissae of all three curves are concentrations in atomic per cent tin.

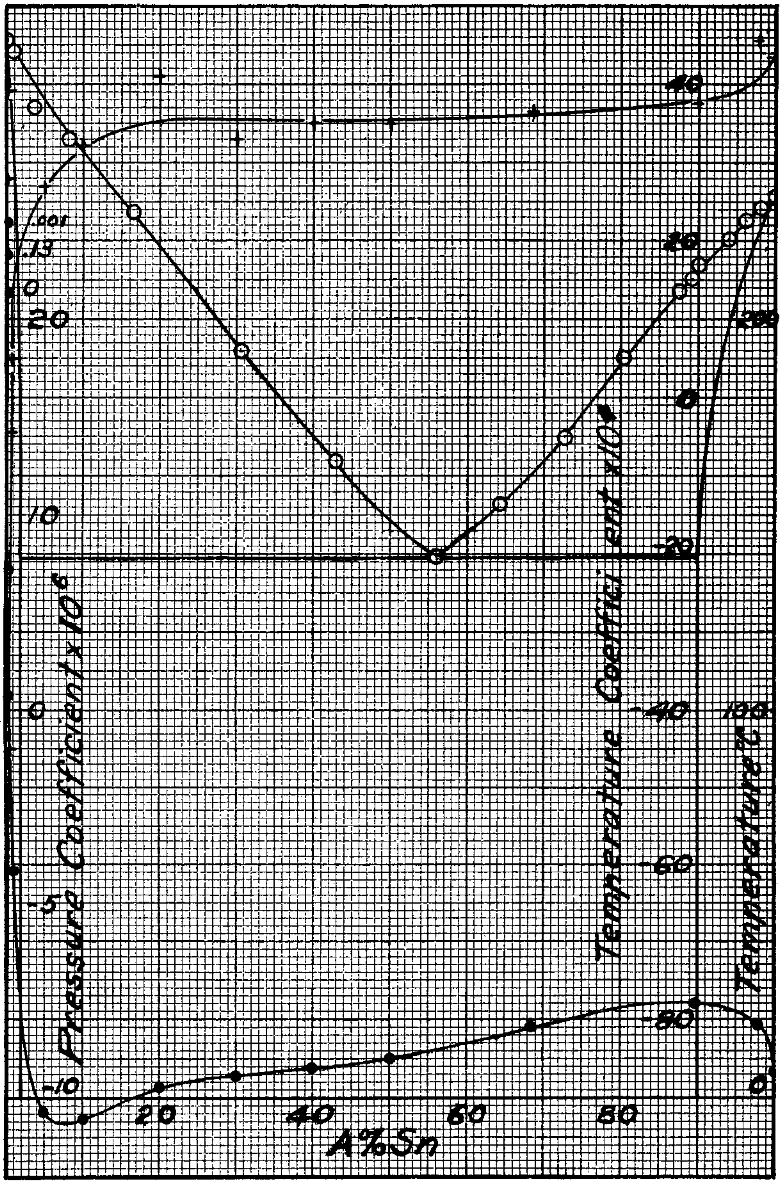

FIGURE 2. Bismuth tin series at 30° C. The melting curve is drawn through the circles. The curve through the dots represents the average pressure coefficient of resistance 0–12000 kg/cm^2. That through the crosses represents the average temperature coefficient of resistance from 30 to 75° C. The abscissae of all three curves are concentrations in atomic per cent tin.

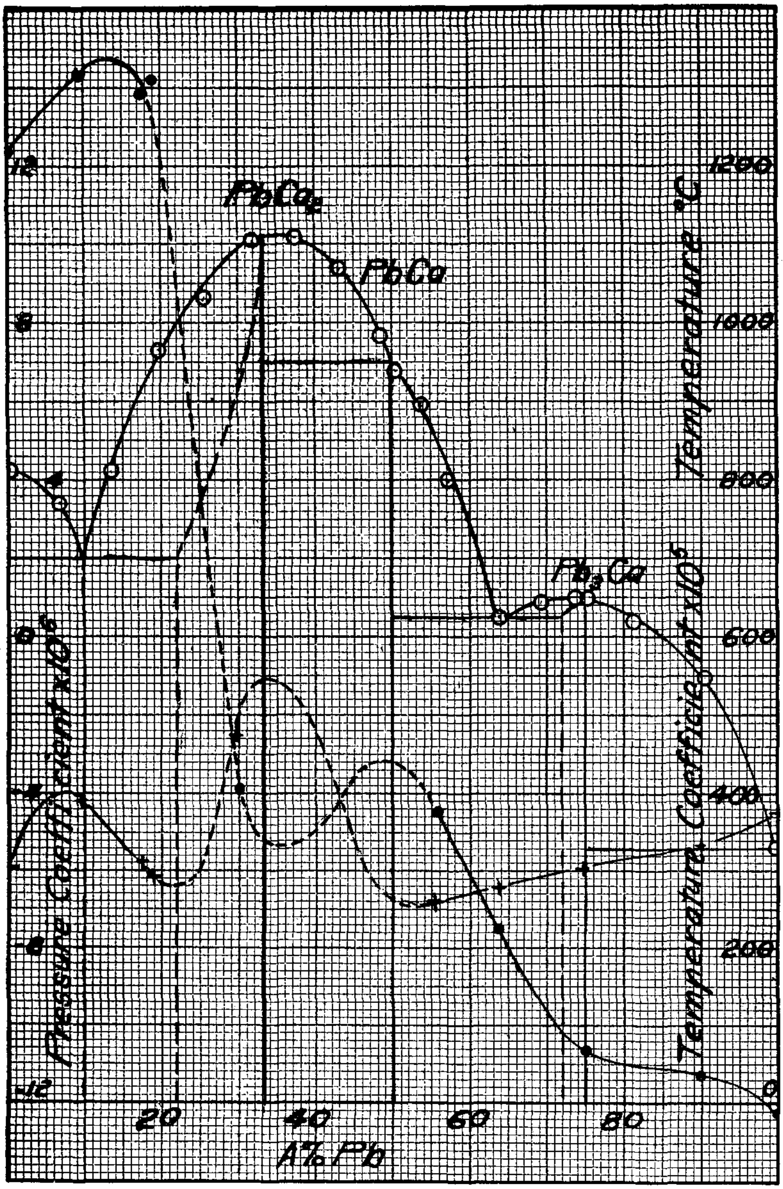

FIGURE 3. Calcium lead series at 30° C. The melting curve is drawn through the circles. The curves through the dots represents the average pressure coefficient of resistance 0–12000 kg/cm². That through the crosses represents the average temperature coefficient of resistance from 30 to 75° C. The abscissae of all three curves are concentrations in atomic per cent tin.

comparison. The equilibrium diagrams for the lithium-tin series have been investigated by Masing and Tammann,[7] the bismuth-tin series by Lepkowski,[8] Bucher,[9] Stoffel,[10] Kapp,[11] Matuyama,[12] and Endo,[13] and the calcium-lead series by Baar,[14] and Donski.[15]

The equilibrium diagrams of the lithium-tin and calcium-lead alloys were taken directly from Masing and Tammann[7] and Donski.[15] There is some confusion in the literature on the equilibrium diagram of the bismuth-tin series because of the three kinds of percent: atomic, weight, and volume. The consensus of opinion seems to be that the eutectic point occurs at 58 W% bismuth or 44 A% bismuth. Bucher's[9] statement that the eutectic point occurs at 58 W% bismuth, or 70.09 A% bismuth, is apparently reversed, since 70.9 W% bismuth is 58 A% bismuth. Stoffel,[10] whom he corrects, was not in need of correction. Again, the International Critical Tables, Vol. 2, p. 416, give an equilibrium diagram, with reference to Kapp,[11] which shows an eutectic point at 58 W% tin. This is apparently in error since Kapp gives 56 W% bismuth as the eutectic mixture. Matuyama[12] gives an equilibrium diagram for bismuth-tin alloys with the eutectic point at about 30 W% bismuth instead of 58 W% bismuth. Comparison of the melting curves found by Lepkowski,[8] Kapp,[11] and Endo[13] shows that Endo's results are a fair average of the other two, so that they were used here. The mean deviation of all the points including Endo's from a smooth curve through his points is 1.7 degrees.

The variations of several physical properties with composition of the alloys have been used to fix the limits of solubility of tin in bismuth and bismuth in tin. The properties so used are the duration time of eutectic crystallization, the electrical conductivity, the temperature coefficient of electrical resistance, the magnetic susceptibility, the thermo-electromotive force, the specific volume, and crystal structure as observed microscopically. These different methods fix the limit of solubility of tin in bismuth between 97.3 and 99.1 A% bismuth and of bismuth in tin between 5.4 and 14 A% bismuth. Endo's[13] values of 98.5 and 9.8 A% bismuth are a fair average. He used all the properties except the thermo-electromotive force.

The curves show a similarity between the lithium-tin and calcium-lead series, as contrasted with the bismuth-tin. The bends in the curves in the lithium-tin and calcium-lead series are apparently caused by the chemical compounds formed by these metals, although the course of the curves is doubtful in the region of the alloys rich in lithium and calcium. There has been some doubt as to whether calcium belonged to the lithium type or the bismuth type, so that the similarity of these curves for the lithium-tin and calcium-lead series offers evidence that calcium belongs to the lithium type. From figures 1 and 2 it seems plausible that lithium and bismuth should have different types of mechanism to account for their positive pressure coefficients, because of the sudden drop in the coefficient of bismuth with small concentrations of tin; and that the coefficient in bismuth should depend entirely on the crystalline structure of the metal and become normal in the liquid. The positive pressure coefficient in lithium is more stable in its alloys with tin which agrees with the fact that lithium retains its positive coefficient when melted.

Bismuth and tin do not form chemical compounds or mixed crystals, but are soluble in each other up to certain limits between which there is a mechanical mixture of crystals The bends in the curve of Fig. 2 coincide well with the values of these limits, which, as seen above, have been placed at 9.8 and 98.5 A% bismuth by Endo's measurements. The pressure coefficients of the bismuth-tin alloys in the region of mechanical mixture obey the additive law of mixtures, as expected, so that expressing the concentration in weight per cent, the pressure coefficients in this region from 11 to 70 W% tin may be calculated from the value at the end points.

The physical properties of dilute electrolytes are often similar to those of the pure solvent which has been subjected to pressure. This is approximately true also of the conductivity of a solid solution of bismuth in tin. There is an increase of 1.7% in the volume occupied by one atom of tin in the lattice due to putting 3.6 A% bismuth in tin. The conductivity decreases 7.6% as compared to an increase of 7.8% due to 9200 kg/cm^2, the pressure required to reduce the volume of pure tin 1.7%. The average pressure coefficient of tin is reduced 17.2% due to putting 3.6 A% bismuth in tin, and 19.7% due to 9200 kg/cm^2. The corresponding changes in pure bismuth do not agree. 1 A% tin and 3500 kg/cm^2 both decrease the volume of bismuth .9% 1 A% tin decreases the conductivity 35%, whereas 3500 kg/cm^2 decreases the conductivity 8%. 1 A% tin decreases the pressure coefficient 133%, whereas 3500 kg/cm^2 increases the pressure coefficient 9%. The discrepancy would probably be less if a smaller amount of tin were put into the bismuth lattice, but there is no data for the specific volumes of bismuth tin alloys with less than 1 A% tin.

The curvature of the lithium-tin alloys with the exception of 9

A% tin is greater than that of pure lithium, being about four times as great for the alloy of 30 A% tin, which has the maximum curvature. The curvature is a little less at 75° than at 30°C. The curvature of the calcium-lead alloys is without exception less than that of either calcium or lead, having a minimum of about one-third that of calcium at 29.6 A% lead. The curvature changes little with temperature, being less at 75° than at 30°C for the alloys with 9.5, 17.5, 63.8 and 74.8 A% lead.

The curvature of the bismuth-tin alloys with .13, .53, and .89 A% tin at 0 and 30°C and 5.02 A% tin at 30°C is abnormal, being concave toward the pressure axis. No example of this abnormal curvature has been observed before. Pure bismuth has normal curvature which changes to abnormal when a small amount of tin is added, being largest for the alloy with a .53 A% tin, which has a maximum in the curve of relative change of resistance against pressure. The abnormal curvature decreases again after this maximum is passed, becoming normal at about 5 A% tin, then increasing in magnitude until it passes through a maximum near the eutectic concentration, and finally through a minimum at the solubility limit of bismuth in tin. Heating the crystal slows up these progressive changes of curvature so that they occur at greater tin content at 30 than at 0°C. For example, the curvature has changed to normal in the alloy with 5.02 A% tin at 0, but not at 30°C.

The average temperature coefficients of electrical resistance from 30 to 75°C for the alloys of the lithium-tin and calcium-lead series in Table I are plotted against the concentration in figures 1 and 3. As in the case of the pressure coefficients, the bends in the curves may be correlated with the concentrations at which eutectic points and chemical compounds occur, except for alloys rich in lithium and calcium, where the curves are doubtful. The average temperature coefficients from 0 to 30°C for the alloys of the bismuth-tin series in Table I are plotted against the concentration in figure 2. Again as in the case of the pressure coefficients, bends occur in the curve at the limits of solubility, the curve being nearly straight in the region of mechanical mixtures from 20 to 80 A% tin. The curve does not have a maximum at 71 A% bismuth, as does the curve found by Bucher.[9] He attributed this maximum to the effect of the eutectic point, which he erroneously calculated to exist there. The temperature coefficient becomes negative for solid solutions of tin in bismuth.

The bismuth-tin alloys obey Matthiessen's[16] rule in the region of mechanical mixtures where the temperature coefficients of the alloys are less by approximately a constant amount than those of the pure metals. This rule does not hold for the lithium-tin nor for the calcium-lead series; nor would this be expected, because of the chemical compounds.

A more accurate account of the variation of the properties of the alloys with temperature is given by the change of the pressure coefficient of electrical resistance with temperature, since this could be more accurately calculated than the temperature coefficient. This is the same as the change of the temperature coefficient with pressure. The relative change of the pressure coefficient of the lithium-tin series with temperature has maxima at 20 and 66 A% tin, and a minimum at 35 A% tin, coinciding roughly with the composition of the chemical compounds Li_4Sn, Li_2Sn_5, and Li_3Sn_2. The magnitudes of the maxima at 20 A% tin and the minima at 35 A% tin are large compared to the small relative change over the rest of the range of composition. The relative change of the pressure coefficient of the bismuth-tin series with temperature is very small, except in the region of solid solutions of tin in bismuth where there is a sharp peak at .5 A% tin. The relative change of the pressure coefficient of the calcium-lead series with temperature has maxima at 15 and 70 A% lead, which might be correlated with the eutectic point at 11 A% lead and with Pb_3Ca. The curve bends sharply at 22.5 A% lead, where the eutectic mixture of 11 A% lead disappears from the mechanical mixture, and passes through a minimum at 50 A% lead corresponding to Pb Ca.

In general, therefore, the relative change of the electrical resistance of the alloys of the three series of lithium-tin, bismuth-tin and calcium-lead with both temperature and pressure may be correlated with the changes in structure of the alloys as given by the equilibrium diagrams.

The change of the solubility limits with pressure in the bismuth-tin series may be determined by comparing the initial and final curves for the instantaneous pressure coefficients at 0 and 12000 kg/cm^2 as a function of concentration. These curves shift .35 A% toward pure bismuth as the pressure increases to 12000 kg/cm^2, showing that tin becomes less soluble in bismuth as the pressure increases. The curves shift 8 A% away from pure tin toward bismuth as the pressure increases to 12000 kg/cm^2 at the solubility limit of bismuth in tin, showing that bismuth becomes more soluble in tin as the pressure increases.

From each curve of figures 1, 2, and 3, it may be seen that putting a foreign metal into a pure one increases the pressure coefficient of resistance of the pure metal algebraically. Since these curves for all pure metals have a normal curvature, hydrostatic pressure also increases the pressure coefficient. If this held in general, it would be justifiable to conclude that putting foreign atoms into the crystal lattice of any metal increases the algebraic value of the pressure coefficient of resistance, regardless of whether the pressure coefficient is positive or negative or what foreign metal is used. Further, hydrostatic pressure on the lattice has precisely the same effect. This is true not only of the metals in these experiments, but also of the alloys of iron with nickel and cobalt, measured by Bridgman but not yet published.

This relation also holds for the copper-manganese series measured by Lisell,[2] where a small amount of manganese increased the value of the pressure coefficient of copper from a negative to a positive one. The gold-silver, cadmium-lead, and gold-nickel series, whose pressure coefficients have been measured by Beckman[3], also follow this rule. The case of cadmium-lead alloys rich in cadmium is apparently an exception, but if Beckman's $a = 103 \times 10^{-7}$ is replaced by 115×10^{-7} as calculated from Bridgman's data for pure cadmium,[12] there will be a minimum in Beckman's curve and no exception. The points on the curve near pure cadmium show a tendency to rise, suggesting that Beckman's value for pure cadmium is too low. Beckman has plotted negative coefficients as though they were positive.

Platinum iridium and German silver were also found by Lisell[2] to have pressure coefficients which are greater than the coefficients of either of the pure metals of which they are composed, so that the pressure coefficient concentration curves have a maximum, indicating that the pressure coefficients of the pure metals are probably increased by impurities.

Putting foreign atoms into the crystal lattice of a pure metal, especially in a solid solution, as in the bismuth-tin series, causes the lattice to expand or contract as a whole. Similarly hydrostatic pressure causes the lattice to contract. Since the pressure coefficients of electrical resistance are similarly altered by these two processes, it seems probable that the conductivity of a metal depends on the arrangement of its atoms in space, or on the geometrical configuration of the crystal lattice. These experiments do not give a basis for explaining the positive pressure coefficients of lithium, calcium, and bismuth by an atomic mechanism, as it was hoped they would do.

SUMMARY.

The pressure coefficients of electrical resistance for the alloys of the three series lithium-tin, bismuth-tin, and calcium-lead for different concentrations, collected in Table I, constitute the principal experimental results.

These experiments show that distorting the crystal lattice by inserting foreign metals has the same effect on the pressure coefficient as does distorting the lattice by hydrostatic pressure. The change in the pressure coefficient is independent of the metal inserted in the crystal lattice.

Finally, it seems probable from these considerations that the mechanism of electric conduction in metals is connected with their geometrical configuration, the phenomena depending on the type of the crystal lattice and its uniformity or distortion.

I take pleasure in thanking Professor Bridgman for his interest and assistance. In particular, I appreciate his ideas which, suggested at the difficult points, have made this research possible.

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THE EFFECT OF PRESSURE ON THE RESISTANCE OF THREE SERIES OF ALLOYS.

By P. W. Bridgman.

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Introduction.

There are many phenomena connected with the electrical behavior of alloys which are difficult to understand. For several years one of the chief reasons for my dissatisfaction with the theory of metallic conduction on which I have been working has been the anomalous behavior of a number of alloys; the behavior of pure metals is more satisfactorily accounted for. It is therefore important to extend our knowledge of the electrical properties of alloys as much as possible; the measurements on the three series of alloys presented here are a contribution in this direction.

Materials and Experimental Methods.

For the series of Fe-Co alloys I am much indebted to Mr. W. C. Ellis,[1] who prepared the alloys and investigated many of their electrical properties as part of a thesis done at Rensselaer Polytechnic Institute under the direction of Dr. M. A. Hunter. The value of Ellis's work is much increased by an X-ray determination which he made of the crystal structure of all his alloys. The series consisted of eight different alloys. Six of these were in the form of drawn wires. Two of them, which could not be drawn, were in the form of swaged rods about 6 mm. in diameter; slender pieces suitable for the measurements under pressure were prepared from these rods by grinding. A special feature of the material prepared by Ellis is the high purity of his cobalt, which was electrolytic. Its high purity is evidenced by the low specific resistance and high temperature coefficient, as given by Ellis in his paper. A remarkable feature of the series is the fact, emphasized by Ellis, that the specific resistance of the alloys from 32 to 53 atomic per cent Fe have a lower specific resistance than either of the pure constituents.* So far as I know, this is the first example of such behavior in alloys; in fact one can frequently find the generalization made in the literature that the resistance of an alloy is greater than that of either component. As Ellis remarks, this effect may be expected to be of significance in theories of metallic conduction. Certainly such behavior lends great interest to as complete a knowledge as possible of the properties of this series of alloys.

The series of Fe-Ni alloys I owe to the kindness of Dr. L. W. McKeehan, at that time with the Bell Telephone Laboratories, Inc. He had a series of 15 alloys prepared and drawn to wire. The following description of the alloys is quoted from Dr. McKeehan's letter. "These wires are all hard drawn. In order to check the approximate composition and to get some idea of the variation of resistivity with composition, I have had the resistivity measured in the state furnished, and also on short samples that have been annealed at 1000° in vacuum for one hour and allowed to cool slowly, *i. e.*, requiring several hours to cool to room temperature. The results are given on the attached photostat (reproduced in Figure 1). In regard to the exact composition, I have not figures for all the alloys, but on those that have been analyzed the actual percentage of iron or nickel falls within less than ½% of the nominal composition."

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* I am not sure that this result can be regarded as established beyond all possibility of doubt. It is evident that the resistance of Co is extraordinarily sensitive to slight impurities; thus Kalmus[2] found for the specific resistance of pure Co reduced in hydrogen the value 8.96×10^{-6} at 20°, against 6.24 found by Ellis. The value of Ellis might possibly be materially reduced if there were a very slight amount of remaining impurity. The reduction need not be great in order to change Ellis's conclusion, because he finds for the resistance at 0° of his pure Co 5.60×10^{-6}, whereas the two alloys with lowest resistance, containing 33.3% and 50% Fe respectively, had resistances of 5.15 and 5.13. It is significant in this connection that Grüneisen[3] has indicated as the most probable value for the resistance of pure Co, on the basis of computations from the resistance at very low temperatures, made by means of Matthiesen's rule, on samples known to be impure, the figure 5×10^{-6}. Furthermore, it would appear that purer cobalt than that of Ellis has been prepared, for Holborn[4] gives for the temperature coefficient between 0° and 100° of pure Co 0.00658, against Ellis's 0.00604. Holborn does not give the specific resistance of his material.

In addition to the more or less complete results on these two series of alloys, I am taking this opportunity to publish some partial results obtained with my high pressure apparatus by Mr. J. R. Oppenheimer in 1925 on the Cu-Ni series. These results cover a range of composition up to about 50% Ni. The material was electrolytic copper from the Bureau of Standards, and commercially pure nickel of probably about 99% purity. The alloys were prepared by melting together in a vacuum in a quartz tube weighed amounts of the two constituents. Floating on the upper surface of the melted metal was a quartz capillary perhaps 8 cm. long and 0.5 mm. in diameter, closed at the upper end. When the melting was completed and the metals thoroughly mixed, air or nitrogen was suddenly admitted to the quartz tube, driving the molten metal up into the capillary, and thus producing a fine cast wire of the alloy. The wire so obtained was not subjected to any systematic heat treatment, but was used as it came from the tube, after cracking away

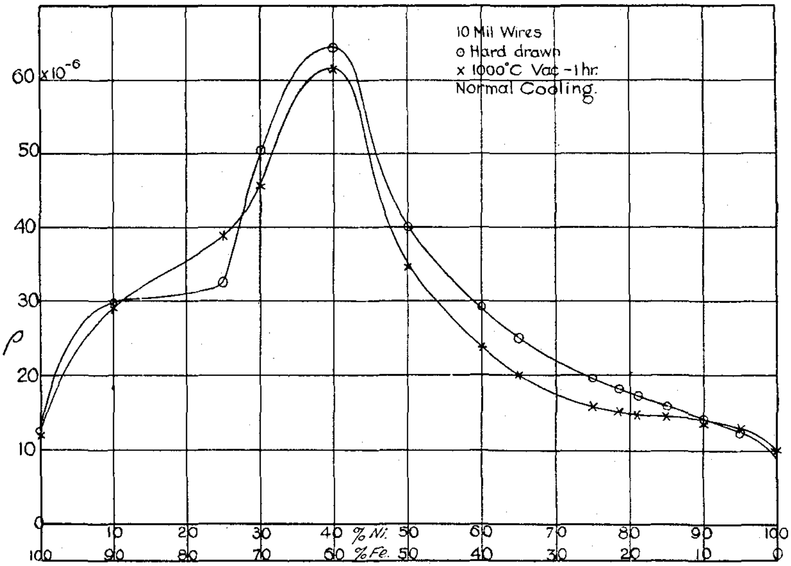

Figure 1. The specific resistance in the Fe-Ni series as a function of composition in the hard drawn condition, and after annealing.

the quartz. In general the capillary was always at a temperature lower than the melting point, so that the wire must have experienced a very rapid chilling from the melting temperature to several hundred degrees below.

The pressure measurements were made by the same method as that used in former work,[5] and no detailed description is necessary. The potentiometer method adapted to the measurement of small resistances was used. The description in the remainder of this paragraph applies to the Fe-Co and Fe-Ni series. The wire specimens were bent to the form of a hairpin, about 8 cm on either arm, and current and potential terminals were soft-soldered to either arm near the end. The changes of resistance listed in the following are measured changes; to convert to changes of specific resistance a correction equal to the linear compressibility must be applied. At a pressure of 10,000 kg. the change of resistance due to the change of dimensions is approximately 0.2%, or of the order of 1/10 of the measured change for the pure metals. The pressure range was 12000 kg./cm.^{2}; readings were made at 30° and 75°. At the lower temperature, pressure was transmitted with petroleum-ether to avoid distortion arising from the viscosity of kerosene under pressure. Before the pressure measurements, two seasoning applications of pressure were made, once to 2000 and once to 12000. The permanent changes of resistance produced by these preliminary applications were in almost all cases very small. This is evidence that no important change of internal composition was produced by pressure, such as an unmixing, and in fact inspection of the phase diagrams of these systems shows that no such effect is to be anticipated. In addition to the pressure coefficient of resistance, the mean temperature coefficient between 0° and 100° was obtained by linear extrapolation of the resistances at atmospheric pressure at 30° and 75°. Since the range 30°–75° is nearly in the middle of the range 0°–100°, linear extrapolation gives approximately the correct mean coefficient, although such an extrapolation would not in general be reliable because the relation between resistance and temperature is not linear. The dimensions of the specimens were also measured, and the specific resistance found. The chief error here is in the diameter of the wire, and should not be more than 1% or 2%. The diameter was measured with an ordinary micrometer, and varied from 0.025 cm. to 0.075 cm.

The measurements of the Cu-Ni series were less elaborate, and consisted of measurements by the potentiometer method of resistance as a function of pressure up to 12000 kg at 25°.

Experimental Results.

The numerical results of the measurements are given in the following tables. In all cases except one, the relation between pressure and resistance could be reproduced by a second degree formula in the pressure; the two constants of this formula are given in the tables. The one exception is the alloy 70% Fe, 30% Ni; for this the detailed dependence of resistance on pressure is shown in Table IV and in Table II only the mean coefficient to 12000 is given. The tables also contain the specific resistance at 30°, the mean temperature coefficient between 0° and 100°, and the mean deviation of a single pressure reading from a smooth curve in terms of the maximum pressure effect. This last gives an idea of the accuracy of the pressure measurements; of course the percentage deviation is much larger for those alloys with a small pressure coefficient.

Ellis's pure Co is materially purer than most of the Co whose electrical properties are listed in the literature, and much purer than the two samples for which I have previously measured the pressure coefficient.[6] The values given in the table should, therefore, supercede my previous values. Although the temperature coefficient of the new sample is much higher than that of my best previous sample, 0.0060 against 0.0044, the pressure coefficient is not materially different, the average coefficient between 0 and 12000 kg. of the new material being -0.950×10^{-6} against -0.934×10^{-6} found before. The difference is in the direction usually found to go with increasing purity.

The pressure effect on the resistance of Ellis's pure Fe was not measured, since I had previously measured[7] the effect on Fe of approximately the same grade of purity as that of Ellis. Ellis gives the impurity as 0.035%, and my American Ingot iron was stated to have about 0.03% impurity. Ellis's temperature coefficient is somewhat higher than mine, 0.00635 against 0.00621, so that on this ground Ellis's iron was probably somewhat purer than mine, but the difference is probably not sufficient to introduce an appreciable difference in the pressure coefficient. Neither the temperature coefficient of Ellis or myself is as high as the highest listed for iron, namely 0.00657 by Holborn.[8]

The values given in the table for the Fe-Ni series for pure Fe

TABLE I.

Results for Fe-Co Series.

Composition At. %		Effect of Pressure on Resistance $\frac{\Delta R}{R_0} = ap + bp^2$						Specific* Resistance at 30° $\times 10^6$	Mean* Temperature Coefficient 0–100°
		30°			75°				
Fe	Co	a $\times 10^6$	b $\times 10^{11}$	Deviation %	a $\times 10^6$	b $\times 10^{11}$	Deviation %		
0	100	− .958	+ 0.68	1.1	− .960	+1.04	2.7	6.62	.00604
10	90	− .401	+ 0.69	1.7	− .305	+ .81	2.2	8.29	534
20	80	+4.32	+14.	.5	+6.29	+8.0	.6	9.91	563
33.3	66.7	− .796	+ 0.6	.7	− .643	0	.7	5.84	446
50	50	− .473	+ 2.65	10.2	− .467	+1.63	4.0	5.78	425
66.7	33.3	+1.153	+ 3.10	.1	+ .898	+4.61	1.4	10.37	241
80	20	+1.408	+ .73	.3	+1.346	+ .39	.6	19.34	218
90.9	9.1	−1.413	+ .62	.3	−1.360	+ .49	.5	18.12	275
100.	0	−2.427	+ 1.14		−2.450	+1.00		10.52	635

* Values of Ellis.

TABLE II.

Results for Fe-Ni Series.

Composition Wt. %		Effect of Pressure on Resistance $\frac{\Delta R}{R_0} = ap + bp^2$						Specific Resistance at 30° × 10^6	Mean Temperature Coefficient 0–100°
		30°			75°				
Fe	Ni	a × 10^6	b × 10^{11}	Deviation %	a × 10^6	b × 10^{11}	Deviation %		
100	0	−2.427	+1.14		−2.450	+1.00		10.5	.00621
90	10	+0.036	−8.0	5.1	−0.091	−3.00	2.9	30.1	231
75	25	+2.822	− .92	.2	+2.315	− .92	1.9	32.4	262
70	30	+8.7	(See Table IV)	.1	7.7	(See Table IV)	.1	51.4	589
60	40	+9.100	−6.06	.1	+6.910	−7.08	.0	65.6	287
50	50	+5.904	+5.62	.2	+5.422	+3.68	.3	41.9	443
40	60	+3.002	+2.29	.9	+2.886	+1.70	....	30.7	474
35	65	+1.930	+1.09	.2	+1.776	+1.39	.4	25.7	473
25	75	+0.456		2.0	+0.394		3.1	18.9	628
21.5	78.5	−0.129		6.2	−0.181		12.2	18.2	462
19	81	−0.389		3.3	−0.461		4.2	17.1	446
0	100	−1.905	+0.50		−1.925	+0.56		(7.0)	634

TABLE III.

RESULTS FOR CU-NI SERIES.

Composition Wt. % Ni	Cu	Effect of Pressure on Resistance at 25° $\frac{\Delta R}{R_0} = ap + bp^2$ a	b
0	100	-2.03×10^{-6}	$+0.96 \times 10^{-11}$
0.145	99.855	-1.80	$+0.71$
1.50	98.50	-1.19	$+0.38$
4.58	95.42	-0.94	$+0.21$
8.23	91.77	-0.745	$+0.10$
12.9	87.1	-0.725	$+0.10$
24.3	75.7	-0.700	$+0.12$
50.8	49.2	-0.685	$+0.10$

TABLE IV.

RESISTANCE OF ALLOY 70% FE–30% NI AS FUNCTION OF PRESSURE.

Pressure kg/cm²	30° $\frac{\Delta R}{R(0.30°)}$	75° $\frac{\Delta R}{R(0.75°)}$
3000	.02478	.02114
6000	.05078	.04279
9000	.07751	.06473
12000	.10477	.08696

and pure Ni were taken from previous work.[7,9] It is probable that the end members of this series as supplied by Professor McKeehan had appreciable amounts of impurity. I measured the pressure and temperature coefficients of the nominally pure iron of this series, and found values considerably lower numerically than my best previous values for pure Fe: 0.00485 against 0.00621 for the temperature coefficient, and -2.09×10^{-6} against -2.43×10^{-6} for the pressure coefficient. It therefore seemed safer to use my previous values for both pure Fe and Ni. Slight impurities in the constituents produce by far the largest effects at the end of the series; there is no reason to expect any important error from impurities in the intermediate members of the series, between 10% and 90%, as given in the table.

Discussion of Results.

It is important to have before us the crystal structure of the various alloys. The data for the Fe-Ni and the Cu-Ni series will be found in International Critical Tables, Vol. I, p. 350–351, and for the series Fe-Co in the paper already quoted of Ellis.

In the Fe-Ni series, the two end terms have different structures, pure Fe being body centered cubic, and pure Ni face centered cubic (both at room temperature). The body centered structure of pure Fe persists up to between 25 and 30 At % Ni. In this range the effect of the addition of Ni is to increase slightly the side of the fundamental cube, that is to decrease the density slightly. The experimental error here is great; roughly the effect is linear with the amount of Ni, 30 At % Ni increasing the grating space from 2.84 to 2.88 Å, or 1.4%. Beyond 25 or 30% Ni the structure becomes face centered cubic, the same as that of pure Ni. The grating space decreases with increasing amount of Ni, roughly linearly from 3.61 at 25% to 3.53 Å at 100% Ni. In each case the effect of the addition of a slight amount of foreign metal is to distend the crystal structure, which is the opposite of the change produced by hydrostatic pressure. In the case of many dilute solutions of ordinary substances the effect of the addition of a small amount of solute is to produce the same change in physical properties as an extra hydrostatic pressure, that is, the effect of the dissolved substance may be likened to that of an increase of internal pressure. In the case of these Fe-Ni alloys, however, the effect is the exact opposite.

In the Fe-Co series the structure is body centered cubic from pure Fe up to about 70 A% Co. The ultimate effect of the addition of Co is to reduce the grating space, the value of which is 2.850 Å for pure Fe and 2.827 for 66.7% Co. The change of dimensions is not linear with composition, however, but up to about 33.3% Co the dimensions are independent of composition within an error of ± 0.001. The alloys with 80 and 90 A% Co are face centered cubic, with crystal parameters of 3.550 Å and 3.540 respectively. Finally, pure Co at room temperature is hexagonal close packed. As in the case of the Fe-Ni series, the effect of the addition of a small amount of foreign metal to pure Fe is not to produce the same change of dimensions as an external pressure.

The Cu-Ni alloys form an unbroken series of solid solutions, all of face centered cubic structure, the crystal unit varying from 3.605 for pure Cu to 3.527 Å for pure Ni.

The series of alloys measured here have in common the feature that at all compositions the crystal structure is that of a mixed crystal in which different kinds of atoms occur together, more or less haphazard, in the same crystal lattice. This is opposed to the structure of many binary alloys, in which over a range of composition there is a mechanical mixture in various proportions of mixed crystals of different fixed compositions, or of the pure components. In the range of mechanical mixture, the physical properties of such alloys are linear functions of the composition, the relation being that of the ordinary rule of mixtures. The alloys investigated by Ufford[10] had this structure throughout certain ranges, and within these ranges the pressure coefficient and other physical properties were linear functions of composition. Opposed to this, there is no range in which the pressure coefficient of these three series of alloys is a linear function.

The contents of the several tables are reproduced in Figures 2, 3, and 4 in order that the relations may be more easily grasped. The first and perhaps the most important result brought out by the figures is support of a generalization already made by Ufford,[10] namely that in all known cases the effect of adding a small amount of foreign metal to a pure metal is to increase algebraically its pressure coefficient of resistance. This is shown in the figures by the rise of the curve for pressure coefficient at either end on leaving the pure metal. This generalization is without known exception, and applies not only to these three alloys, but also to the three of very different type investigated by Ufford, and also to several series investigated by Lisell[11] and Beckman.[12] These new results are sufficient to show, however, that the explanation given by Ufford of this general effect cannot always be correct. Ufford pointed out that in the case of his alloys the addition of a small amount of foreign metal compresses the lattice; this is also the effect of hydrostatic pressure. This means, since in practically all cases hydrostatic pressure increases the pressure coefficient algebraically, that the effect of adding a foreign metal is equivalent, in the cases investigated by him, to an increase of external pressure. That this is not true in general is shown by the Fe-Ni series. Here the addition either of Ni to pure Fe or of Fe to pure Ni distends the structure, thus producing the same change of dimensions as that brought

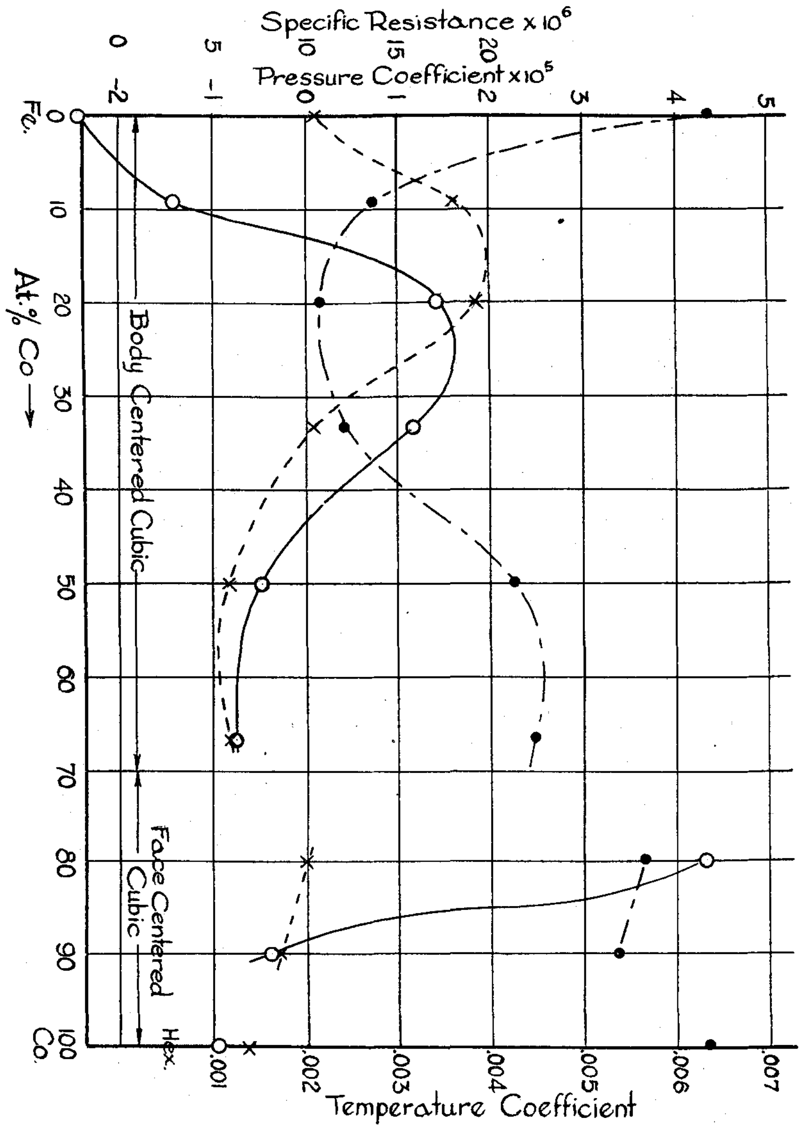

FIGURE 2. Specific resistance at 20° C (dotted line), pressure coefficient of resistance (full line), and temperature coefficient of resistance (dashed and dotted line), in the Fe-Co series as a function of composition.

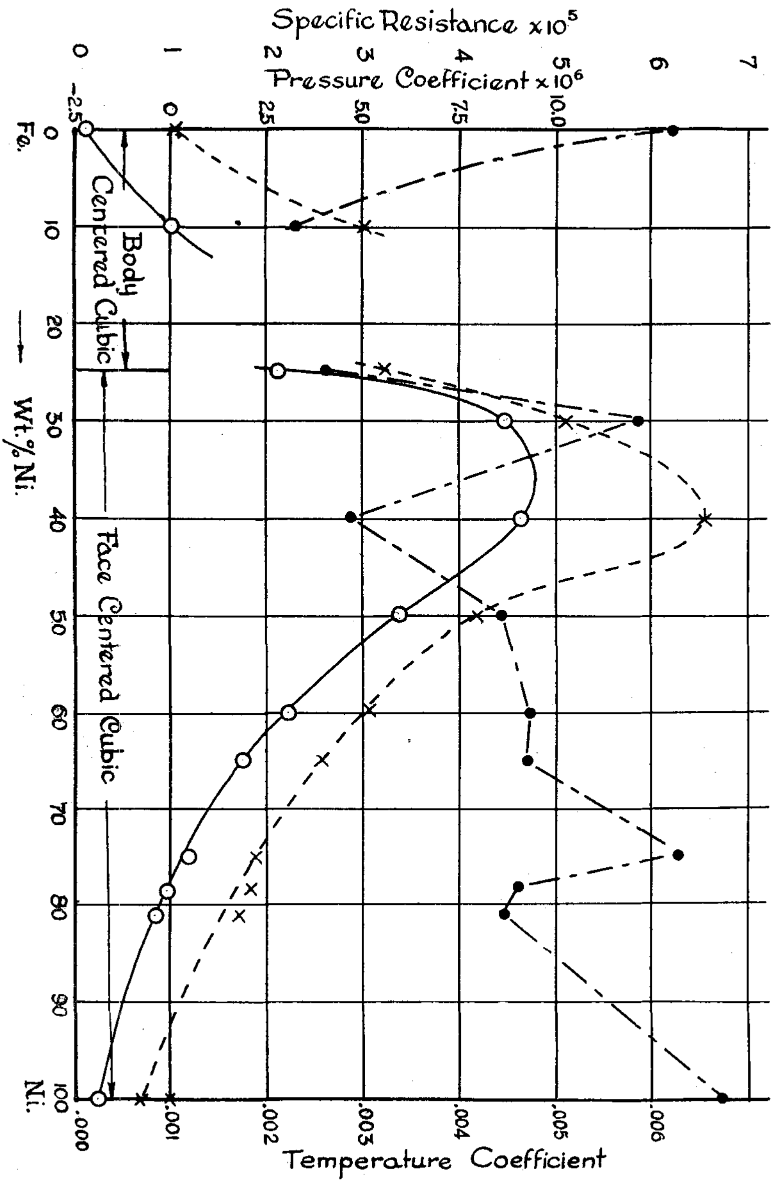

FIGURE 3. Specific resistance at 20° C (dotted line), pressure coefficient of resistance (full line), and temperature coefficient of resistance (dashed and dotted line), in the Fe-Ni series as a function of composition.

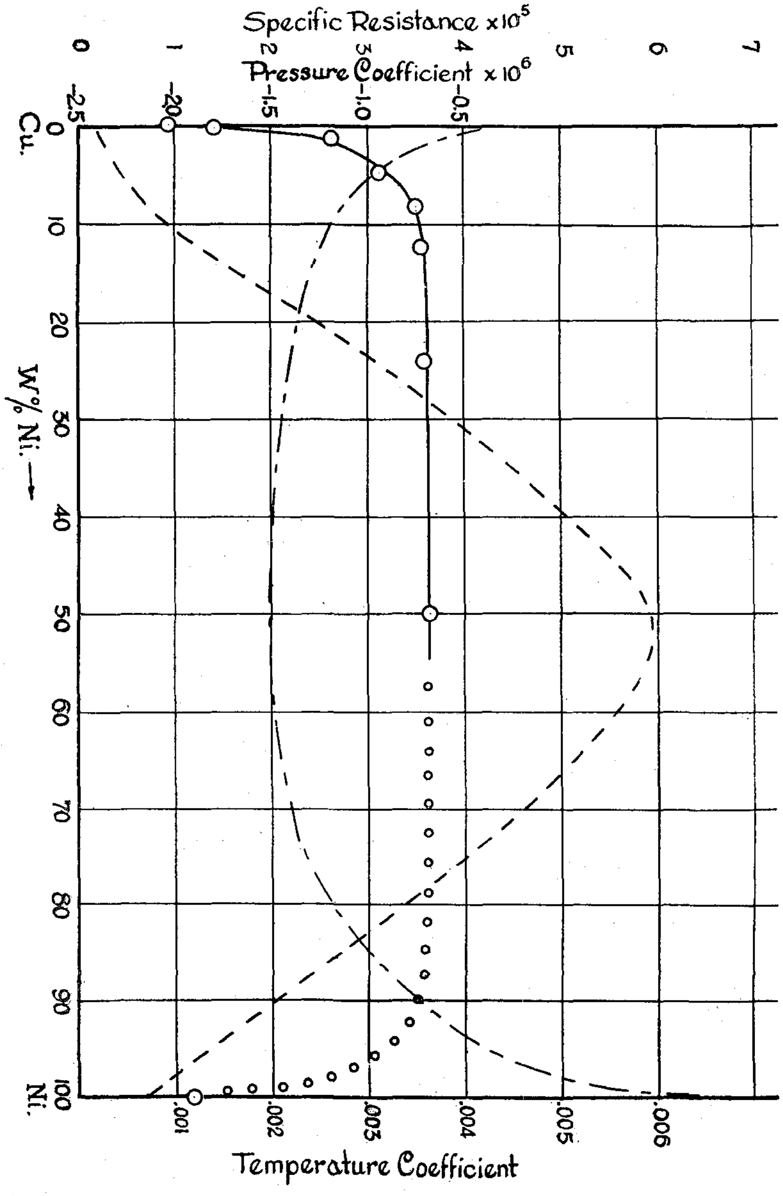

FIGURE 4. Specific resistance at 20° C (dotted line), pressure coefficient of resistance (full line), and temperature coefficient of resistance (dashed and dotted line), in the Cu-Ni series as a function of composition. A hypothetical extension of the curve for pressure coefficient beyond the range of experiment is shown by the small circles.

about by a decrease of external pressure, whereas the change of pressure coefficient is in the direction brought about by an increase of pressure. The same is also true with regard to the addition of Co to Fe, although the numerical discrepancy is not so great, and is probably true, although this is not entirely certain, for the addition of Cu to Ni.

An inspection of the figures shows much greater regularity in the specific resistance and pressure coefficient of resistance than in the temperature coefficient. For example, in the face centered range of the Fe-Ni series, the behavior of the temperature coefficient is highly irregular, but that of specific resistance and pressure coefficient is comparatively regular, both rising from the ends of the range to a maximum at some intermediate composition. This is also the behavior in the body centered range of the Fe-Co series; here the maximum of resistance is more displaced with respect to the maximum of the pressure coefficient than in the Fe-Ni series. The same behavior is shown qualitatively by the Ni-Cu series; the range of variation of specific resistance with composition is here, however, much larger compared to the variation of pressure coefficient than in either the Fe-Ni or Fe-Co series. It would be interesting to know whether the same parallelism holds in other parts of the range, but there are only two points in the body centered range of Fe-Ni, and also only two points in the face centered range of Fe-Co, and two points are not enough to answer the question.

Although there is undoubtedly a parallelism between specific resistance and pressure coefficient, the example of Cu-Ni shows that the parallelism cannot be so close that one can be said to be a function of the other. Between 10 and 50% Ni the specific resistance of Cu-Ni changes six fold, whereas the pressure coefficient changes hardly at all.

An interesting question that cannot be answered at present is as to the discontinuity of resistance and pressure coefficient when the lattice type changes. If the relations previously found for all known cases of melting and polymorphic change hold here,[13] one would expect the resistance to increase, when the lattice type changes, in the same direction in which the volume increases. To answer this, many more alloys must be investigated in the range between 10 and 25% Ni for the Fe-Ni series, and between 66.7 and 80% Co in the Fe-Co series, than were available for the present work.

One of the most interesting results which I hoped to get from these measurements was some suggestion as to the mechanism which is responsible for a positive pressure coefficient. The conclusion seems to be justified that in these three series of alloys the same thing that tends to cause a high specific resistance also tends to make the pressure coefficient positive. According to the view of conductivity in which I have been interested for some time[14] this would be interpreted in terms of a failure of perfect fit of adjacent atoms. If the atoms do not fit closely together, as they do not in a lattice composed of different sorts of atoms of different sizes, the electrons encounter difficulty in passing from atom to atom and the resistance is high. This is the explanation of the high resistance in the middle of a series of mixed crystals. Now if pressure tends to accentuate the lack of fit, it is to be expected that the pressure coefficient of resistance will be positive. It is natural to expect this to be the case. At pressures of the order of magnitude of those with which we are here dealing, part of the compressibility of a solid comes from the closing of the empty spaces between the atoms, the compressibility of the atoms themselves contributing only partly to the total effect. Hence as pressure increases the relative part of the total volume occupied by the atoms themselves increases, which means that the relative difference between the diameters of the different sorts of atoms is accentuated. But this is merely another way of saying that the lack of fit is accentuated, so that the result to be expected is an increase of resistance, which is the effect actually found.

The general features of this somewhat gross and materialistic point of view can be taken over into the recent picture of conduction on the basis of the wave mechanics given by Houston.[15] According to this picture electrical resistance arises from the scattering by the atoms of the waves which constitute the electrons. If the atoms are spaced with perfect regularity, as in a crystal at 0° Abs., there is little or no scattering, and the resistance is very low. When the arrangement becomes more irregular, as it does when temperature is increased, the scattering increases and resistance increases. If there is a permanent additional cause of irregularity, as when atoms of different sizes are forced to occupy the same lattice in an alloy, there is an additional reason for scattering, which explains the increase of resistance when a foreign metal is added to either pure component. If the irregularity in the space arrangement is accentuated by pressure, as we have just seen is to be expected on geometrical grounds, the scattering is also increased, and the pressure coefficient of resistance tends to become positive.

The reason for the positive pressure coefficient of alloys given in the last paragraph is different from the reason given by Houston for the positive pressure coefficient of pure metals. Houston connects the positive pressure coefficient of pure metals with an abnormally small decrease of compressibility with pressure, or even possibly an abnormal increase of compressibility. It would seem that such a mechanism is not necessary to account for the positive pressure coefficient of alloys. In order to check this point, I have measured by the conventional method, which has now been applied to many substances,[16] the compressibility as a function of pressure of invar, a Fe-Ni alloy of 37.5% Ni content, corresponding approximately to the alloy of maximum resistance and pressure coefficient found above. The following was found for the compressibility at 30° between atmospheric pressure and 12000 kg./cm.^{2}:

$$-\frac{\Delta V}{V^0} = 9.99 \times 10^{-7}p - 6.18 \times 10^{-12}p^2.$$

The second degree term has the normal sign, and numerically is larger, rather than smaller, than that characteristic of pure metals of about the same compressibility. The compressibility itself is unexpectedly large, being 75% greater than the value calculated by the rule of mixtures from the pure components.

It would seem, then, that Houston's explanation of the positive pressure coefficient of pure metals need have no application to alloys. Here we have an adequate picture of the situation in a lack of fit in the crystal lattice, which is accentuated by pressure. The facts disclosed by the compressibility measurement of invar, namely a high compressibility combined with an abnormally rapid decrease of compressibility with pressure, are in complete agreement with this picture.

Houston's explanation of the positive pressure coefficient of pure metals must be taken, I believe, with considerable reservation. His explanation finds a correlation between the positive coefficient and an abnormally small decrease of compressibility with pressure; if examination is made of the behavior of a number of metals, instead of the one or two which Houston considered, it will be found that there is in general no such correlation. It may well be that under the proper conditions we have the same mechanism in pure metals as in alloys, namely a lack of fit accentuated by pressure. This would especially be expected if the atom has not spherical symmetry. It is perhaps the present tendency, suggested by the wave mechanics, to emphasize the spherical symmetry of the atom, but that the atom cannot always be spherically symmetrical is shown by the mere existence of non-cubic crystals.

I am indebted to my assistant, Mr. Stephen Stark, for making most of the readings in the Fe-Ni and Fe-Co series. I am also much indebted to Mr. Oppenheimer for permission to publish the results which he obtained.

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63—8

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Vol. 63. No. 8.—December, 1928.

THE COMPRESSIBILITY AND PRESSURE COEFFICIENT OF RESISTANCE OF ZIRCONIUM AND HAFNIUM.

By P. W. Bridgman.

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By P. W. Bridgman.

Presented Oct. 10, 1928. Received Oct. 15, 1928.

The data of this paper were obtained in continuation of my program to extend the measurement of compressibility and resistance under pressure to all the pure elements available. I have previously published results[1] for the effect of pressure on the resistance of a very impure specimen of zirconium, made by depositing a covering of zirconium on a core of tungsten. The impurity was so great that the temperature coefficient of resistance was only 0.00004, and the specific resistance was 200×10^{-6}. The pressure coefficient of resistance, in kilogram units, of this impure zirconium, was -4.0×10^{-7}. So far as I know, no previous measurements have been made on the compressibility of zirconium, or on either the compressibility or resistance under pressure of hafnium.

For the material of this investigation I am greatly indebted to Dr. G. Holst, of the Naturkundig Laboratorium, Eindhoven, Netherlands, Philips Gloeilampen Fabrieken. In connection with an investigation of possible commercial uses, both these metals have been prepared in a state of high purity by the Philips Lamp Works.

Two pieces of zirconium were available. The resistance sample was a wire, about 0.050 cm. in diameter and 14 cm. long. The potentiometer method of measurement was used, which demands two current and two potential terminals. It is not possible to solder zirconium by any easy means, and therefore the terminals were attached by spring clips, in a way which has been used for a number of other metals. The compressibility was measured on a piece 2.7 cm. long, about 3 mm. in diameter. It was measured in the same apparatus as that used for many previous measurements,[2] which I have described as "the lever apparatus for short specimens."

Only a single piece of hafnium was available, about 2.6 cm. long and 2 mm. in diameter. The outside surface was not perfectly regular, the appearance being as if the piece were composed of several rather large crystal grains which were slightly staggered with respect to each other. This irregularity of figure is perhaps connected with the fact that the compressibility measurements were much more irregular than usual. Because of the very low resistance of this specimen (about 0.00112 ohms), the pressure coefficient could be determined only with much less accuracy than usual. I did not attempt to find the variation of pressure coefficient with temperature; neither was it possible to find any departure from linearity between pressure and resistance. It is no more possible to solder hafnium than zirconium, so again connections had to be made with springs. The potential terminals were placed only 1.2 cm. apart, to avoid as far as possible end effects. The current connections at the ends were specially constructed to touch the specimen as nearly as possible at all points around the circumference, so that the lines of current flow inside the specimen should be straight.

In addition to compressibility and pressure coefficient of resistance, the specific resistance, and temperature coefficient of resistance at atmospheric pressure were measured. The temperature coefficient was obtained from measurements at 30° and 75°; the values listed are the average coefficients between 0° and 100° (*i. e.*, $(R_{100} - R_0)/R_0$), obtained by drawing a straight line through the measurements at 30° and 75°.

The results follow. The pressure range was up to 12000 kg./cm.², as usual.

Zirconium.

Compressibility:

At 30°, $-\Delta V/V_0 = 10.97 \times 10^{-7}p - 7.44 \times 10^{-12}p^2$.

At 75°, $-\Delta V/V_0 = 11.06 \times 10^{-7}p - 7.80 \times 10^{-12}p^2$, pressure in kg./cm.²

At 30°, the average deviation of a single reading from a smooth curve was 0.6% of the maximum pressure effect, and at 75°, 0.7%. The decrease of compressibility with increasing pressure was well marked and far beyond experimental error.

Resistance:

$$\text{At } 30°,\ \Delta R/R\ (0\text{ kg}, 30°) = -4.31 \times 10^{-7}p + 6.5 \times 10^{-12}p^2,$$

$$\text{At } 75°,\ \Delta R/R\ (0\text{ kg}, 75°) = -6.01 \times 10^{-7}p + 5.8 \times 10^{-12}p^2.$$

The resistance decreases under pressure, which is the normal behavior. The coefficient is, however, unusually small, which accounts for the somewhat larger experimental error than usual. At 30° the average deviation of a single reading from a smooth curve was 1.4% of the maximum pressure effect, and at 75°, 1.6%. Expressed in terms of the total resistance, this means that at 30° the average deviation from a smooth curve was 0.006% of the total resistance, and at 75°, 0.009%. Not much better can be expected, considering the diameter of the specimen, and the fact that the contacts were made with springs.

The specific resistance at 30° was found to be 49.2×10^{-6}, and the mean temperature coefficient between 0° and 100° 0.00403. The latter figure is evidence of the high purity of this material. It is remarkable that the pressure coefficient of the sample originally measured should have been so close to that found above, in spite of the fact that its temperature coefficient and specific resistance showed very high impurity.

Hafnium.

Compressibility:

$$\text{At } 30°,\ -\Delta V/V_0 = 9.01 \times 10^{-7}p - 2.37 \times 10^{-12}p^2,$$

$$\text{At } 75°,\ -\Delta V/V_0 = 8.81 \times 10^{-7}p - 2.37 \times 10^{-12}p^2.$$

At 30° the average deviation of a single reading from a smooth curve was 1.7% of the maximum pressure effect, and at 75° 2.0%. The accuracy was not great enough to establish any deviation from linearity in the relative compressibility of hafnium and iron, which is the quantity directly measured. The second degree term in the formulas above comes from the compressibility of iron. It is probable that the second degree term is actually materially larger than given above. Neither is it certain that the compressibility at 75° should be less than that at 30°.

Resistance:

$$\text{At } 30°,\ \Delta R/R\ (0\text{ kg}, 30°) = -10.0 \times 10^{-7}p.$$

The average deviation of a single reading from a smooth curve was 6.0% of the total pressure effect, which means 0.09% in terms of the total resistance. Again I believe that this irregularity is as small as could be reasonably expected in view of the dimensions of the specimen, and the method of connection by spring clips.

The specific resistance at 30° was found to be 35.7×10^{-6}, and the mean temperature coefficient between 0° and 100°, 0.00398. Again the high value of temperature coefficient is presumptive evidence of high purity.

Discussion. The compressibilities listed above are cubic compressibilities, whereas the measured quantities were linear compressibilities. In converting the one into the other, the assumption was made that the compressibility is equal in all directions. Now this assumption is doubtless not strictly justified, because neither zirconium nor hafnium crystallizes in the cubic system, but instead in the close-packed hexagonal arrangement. However, the assumption of equal compressibility in all directions is probably approximately justified for these two metals, because the axial ratios are such (1.644 for hafnium and 1.593 for zirconium) as to correspond nearly to close packed *spheres,* for which the ratio is 1.63. It must nevertheless be recognized that the assumption of equal compressibility in all directions in a hexagonally close packed arrangement of spheres has not yet been justified by experiment, and should receive such a test.

In general comment on the results we may note in the first place that the compressibilities fit well into the vacant places in the periodic table, as is shown, for example, in Figure 1 of my paper in Proceedings American Academy 62, 207–226, 1927. It is perhaps not to be expected that the compressibility of hafnium should be less than that of zirconium, since in the majority of cases the heavier elements in a given column of the periodic table are more compressible. However, this observation is confined mostly to elements of lower atomic number than hafnium, and it is evident from the figure that there is some reverse tendency at higher atomic numbers.

The pressure effect on resistance has the normal sign in both these elements, but it is rather smaller for each than usual. It is to be noted that the pressure coefficient of resistance of hafnium is greater numerically than that of zirconium, although its compressibility is less. This emphasizes the fact frequently mentioned before that the pressure effect on resistance is not directly concerned with the volume changes produced by pressure.

I am indebted to my assistant, Mr. Stephen Stark, for making the readings of this paper.

THE JEFFERSON PHYSICAL LABORATORY,
Harvard University, Cambridge, Mass.

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THERMO-ELECTRIC PHENOMENA AND ELECTRICAL RESISTANCE IN SINGLE METAL CRYSTALS.

By P. W. Bridgman.

Investigations on Light and Heat made and published with aid from the Rumford Fund.

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INTRODUCTION.

In 1926 I published a paper[1] containing, in addition to some other matters, data for the thermal e.m.f. of single crystals of Zn, Bi, Cd, Sn, Sb, and Te as a function of the crystal orientation. The results thus obtained were not as complete or as accurate as could have been desired, the number of specimens being comparatively small, and in several cases the range of orientation being narrow. For example, the orientation of the specimens of Bi varied through only 22° instead of through the full 90°. A number of significant conclusions could be drawn from the data of that paper, but there were some important questions, as, for example, the precise way in which thermo-electric symmetry departs from Kelvin's relation (if indeed it departs at all), which could not receive a certain answer from the measurements. Furthermore, Linder[2] has also published thermo-electric data for single crystals, from which he draws conclusions not always the same as mine, so that a reëxamination was desirable.

In this paper improved results are obtained. In the first place, the method of producing single crystals has been much improved, so that it is now possible to cover the entire range of orientation, and in the second place, the apparatus by which thermo-electric force is measured has been improved so that now measurements can be made simultaneously with 16 different specimens, practically the only variable being the crystal orientation. This paper contains these new data, and a reëxamination, in their light, of the question of the symmetry relations. I have also remeasured, since the new specimens were better adapted to this purpose than the old ones, and since results disagreeing with my early ones have been since published by Schneider,[3] the specific resistance of most of these metals as a function of direction. I have further taken advantage of the new material to measure the effect of pressure on the resistance of Sb and Bi over the entire range of orientation.

THE METHOD OF PRODUCING SINGLE CRYSTALS.

The new method is a modification of that used before;[4] this consisted in lowering a mold containing the molten metal slowly through the bottom of a furnace maintained at a constant temperature above the melting point. If the rate of lowering is enough less than the rate of crystallization, the liquid solidifies in a continuation of the crystal lattice already laid down, and if solidification can be forced to start as a single crystal grain, as by making the lower end of the mold taper to a sharp point, the mold becomes filled, after solidification, with a single crystal grain. In the previous work the largest furnace used had an interior diameter of about 2.5 cm., so that this was the greatest diameter of any crystal that could be made. The control of the orientation of the crystal was the difficult feature of the previous work. The crystals show a striking inclination to grow with the planes of easiest cleavage parallel to the vertical axis of the furnace; there is, however, considerable haphazard variation from this preferred direction, and by making a large number of castings, it was possible to cover a considerable range of orientation. The range depends to a marked degree on the nature of the metal. The direction of growth was much less strongly preferential in rods of small diameter than in the larger rods.

The new method depends on the use of a furnace of 10 cm. internal diameter, against the former 2.5. This makes it possible to use a mold consisting of criss-cross arms of glass tubing (a photograph of such a mold is shown in Figure 1), arranged at a variety of angles. If the rate of lowering is slow enough, the mold, as before, becomes filled with a single crystal grain. By breaking the mold at the corners of the arms, one has effectively a number of short crystal rods of a variety of orientations out of a single crystal. Of course this method does not give control of the orientation of any individual rod, but it does ensure that the orientation of the aggregate of rods is spaced over the entire range, which is the essential thing. The molds may be made of any desired degree of complication; my assistant, Mr. Philip Dalton, developed much skill in making them, and produced molds with as many as 50 arms. I myself was usually satisfied with more modest molds of 12 arms that could be made simply from a single length of tubing, bent first into a hairpin at the middle, and then the two arms of the hairpin bent to the desired angles.

Various new devices of manipulation were used. One of the most important matters is the deposition of some kind of a coating on the inside of the glass, to prevent the metal from sticking to the glass, and to allow the easy removal of the glass by cracking. If the mold is not coated in some way, the sticking is so bad as to make quite impossible the removal of the glass without mechanical deformation of the metal inside; this is particularly serious when the diameter of the castings is small, as here. Formerly, the tube had been washed with a dilute solution of Nujol in petroleum ether, and then dried by heating in vacuum; this left enough deposit to accomplish the purpose. It was, however, difficult to control the operations exactly, and failures were numerous, either because the deposit had been entirely oxidized away, or because so thick a deposit was left that gas bubbles were generated in sufficient number to break the continuity of the castings. The new material used for coating the molds was an enamel made by the General Electric Co. for insulating copper wire. This was made very dilute with $CHCl_3$, the interior of the mold was washed with it, allowed to drain in a vertical position for several hours, and then the mold was heated progressively with a Bunsen burner, first driving off the volatile matter, and then carbonizing the residue, which was left on the inner walls as a black deposit of carbon. The thickness of this deposit was now appropriately reduced by heating the entire mold in an electric

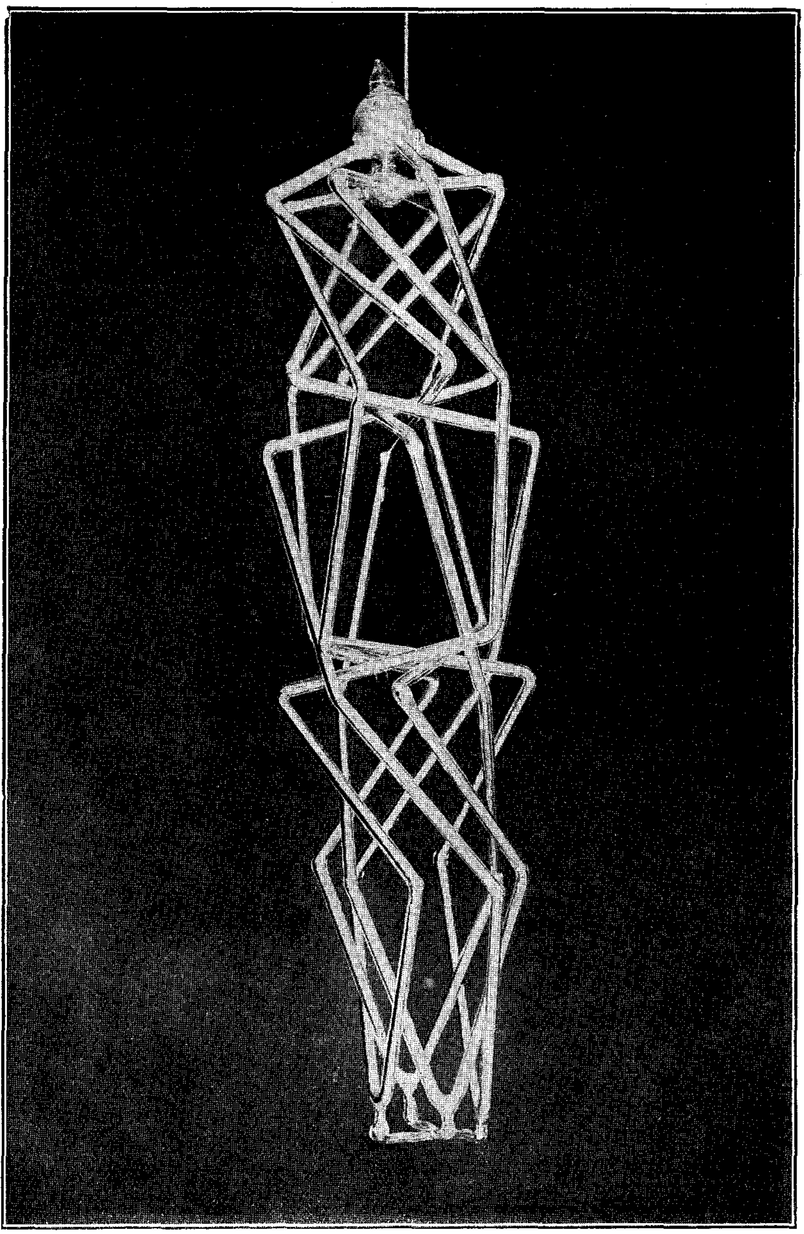

FIGURE 1. Photograph of the glass mold in which many single crystal rods with a wide range of orientation may be cast in one operation.

furnace to a temperature of 400° or 500°. If the mold is sealed to prevent oxidation, the deposit is unaffected by this treatment, but if air is allowed to enter, the deposit is oxidized and completely disappears in the course of 10 minutes or so. By controlling the amount of air and the time of heating, the deposit may be reduced to such a thickness that there is a just perceptible blackening of the walls, which is the ideal to strive for.

Filling the mold with molten metal could be accomplished in various ways; the diameter of the glass tubes used was so small that the metal mensicus would not break, even in an inverted position. It was therefore possible, when convenient, to fill the mold from an upper chamber, allowing the molten metal to run down one arm and up the other with no danger of gas bubbles breaking the continuity. In some cases I found it convenient to fill by sucking the molten metal up into the mold from an open lower reservoir. I had used this method some time ago in preliminary work, and it has been recently described in a publication by Sachs.[5] I found it necessary to suck the metal up very slowly, otherwise gas bubbles were apt to appear. The rate of rise may be made as slow as desired by connecting the upper end of the mold to a large vessel, which is exhausted very slowly through a fine capillary attached to the vacuum pump. To prevent the metal in the open reservoir from oxidizing during the filling, the furnace may be filled with an atmosphere of CO_2.

THE APPARATUS FOR MEASURING THERMAL E.M.F.

The previous apparatus consisted of two temperature baths at different temperatures, with the single crystal rods leading from one bath to the other, and with copper leads soldered to the ends of the crystals. The e.m.f. was measured as a function of the temperature difference. The apparatus could be used with only one crystal rod at a time, so that besides the disadvantage of slowness, slight temperature irregularities in the different runs might superpose vagaries on the differences due to differences of orientation in the different specimens. The new apparatus was so designed that 16 different samples could be measured with a single set-up, thus very much increasing the speed, and decreasing any error due to temperature irregularities. The 16 crystal rods, each about 3 mm. in diameter and about 7.5 cm. long, were passed through small stuffing boxes, arranged in a row in a block of bakelite 2.5 cm. thick. This block of bakelite was clamped, between asbestos packing, between two heavy cast brass cylinders about 7 cm. inside diameter, provided with openings with machined faces to press against the bakelite. A section is shown in Figure 2 and an elevation in Figure 3. The dimensions are such that the ends of the crystal rods project to about the middle of the brass cylinders. The cylindrical brass castings are connected at top and bottom to two separate reservoirs, each filled with kerosene. A rapid circulation is maintained through the cylindrical castings, past the projecting ends of the crystals, into the reservoirs at the top and into the castings at the bottom. The reservoirs are maintained at different constant temperatures by thermostatically controlled electric heating arrangements. In addition to the stirrers at the upper ends of the castings, the reservoirs also have independent stirrers, in order to keep the temperature as uniform as possible. One of the baths was maintained at a constant temperature of about 20°. Since sometimes the room rose above this temperature, artificial cooling was maintained by a current of tap water circulating through a coil of copper tube placed in the reservoir. The thermostatic control with electric heating was superposed on this constant cooling. The other bath was varied in temperature, the temperatures chosen being 37°, 54°,

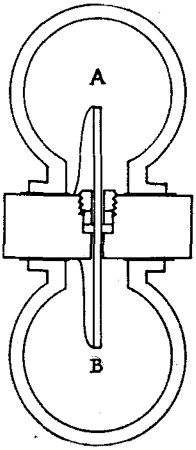

FIGURE 2. Section of part of the apparatus for measuring thermal e.m.f. of single crystal rods. The rod reaches from A to B. The two ends A and B are maintained at different temperatures by two streams of oil, circulated through two regulated sources.

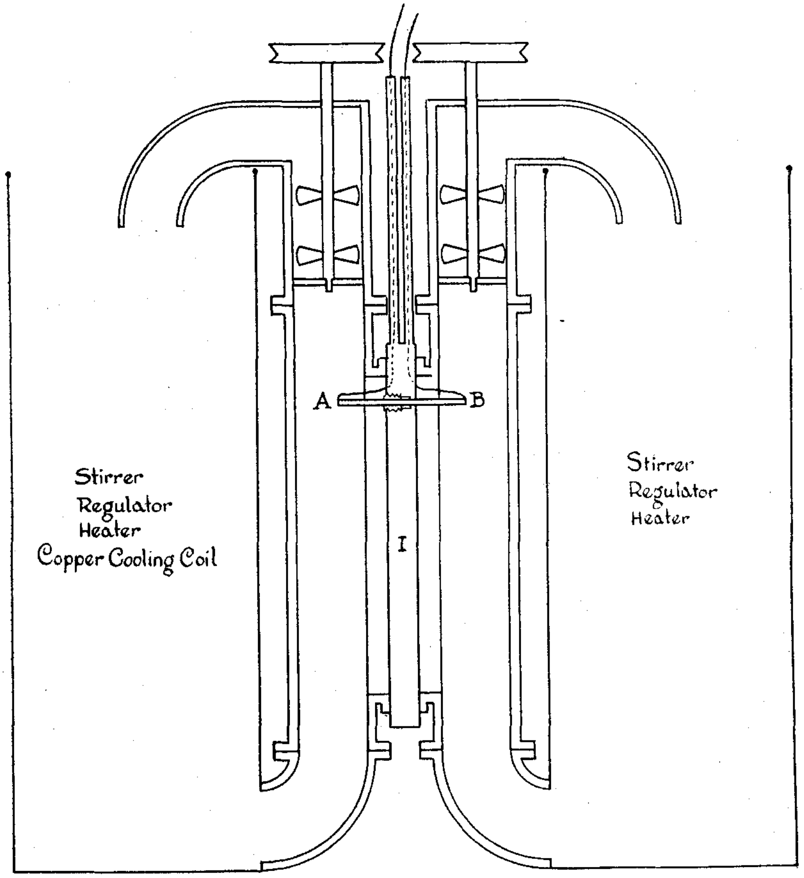
A
B
Stirrer
Regulator
Heater
Copper Cooling Coil
I
Stirrer
Regulator
Heater

FIGURE 3. Section in elevation of part of the apparatus for measuring thermal e.m.f. of single crystal rods. There is provision for measuring 16 rods at the same time. A single sample rod is shown at AB. The others are mounted in a similar way in the rest of the bakelite block I.

70°, and 88°. Temperatures were read with calibrated mercury-in-glass thermometers. The temperature regulator was a conventional mercury-in-glass affair; four of these regulators were used, each set permanently to one of the four temperatures. After readings at one temperature had been completed, the regulator for the next higher temperature was substituted for the former one, and temperature was run up rapidly to the desired point with a heavy heating current. The operation of changing temperature and coming to equilibrium again at the new point occupied from 10 to 15 minutes.

Copper leads, all from the same spool of wire, were soldered to the two ends of the crystal rods, brought out through a stand-pipe arrangement at the upper end of the bakelite block, and connected to the arrangement for measuring e.m.f. Any one of the 16 rods could be connected to the measuring arrangement through a bank of 32 all-copper switches, mounted in a closed box, and operated by long handles from outside to eliminate parasitic e.m.f.'s due to slight temperature inequalities. The circuit by which e.m.f. was measured is shown in Figure 4. The method is essentially a deflection method, this being much more rapid than the potentiometer method used before, and sufficiently accurate. A Pye galvanometer, G, which has a low period and a steady zero, was used, with mirror and scale. The circuit contained various resistances in series or in parallel, R_1 and R_2, with the galvanometer, by which its sensitivity could be appropriately varied, so that for each different sort of metal the throw for the maximum temperature difference was made about 40 cm. There were also arrangements by which an acid Weston cell, E_1, which permits a fairly large current to be drawn from it without deterioration, could be connected into the circuit in order to calibrate the galvanometer. This calibration was made at the beginning and the end of every run and varied only slightly from day to day. The e.m.f. of the acid cell was checked by comparison with a standard Weston cell, E_2, which could also be thrown into the same circuit in such a way that the comparison was made by a null method, thus drawing no current from the standard cell. R_3, which is a decade box reaching to 10,000 ohms in steps of 1 ohm, and R_4, which reaches to 100,000 in steps of 10,000, were used in making the various calibrations. Error from galvanometer drift and parasitic e.m.f.'s in parts of the apparatus beyond the thermocouple switch were limited by using the double throw of the galvanometer, instead of its deflection from rest. The period of the galvanometer was of the order of 5 seconds, so that it was possible to complete the measurements on a single rod in 20 seconds or less, and the entire series in 5 minutes. Constancy of the bath temperature was checked by repeating at the end of the series the measurement on the first couple; this also checked whether temperature equilibrium had been attained before beginning the run.

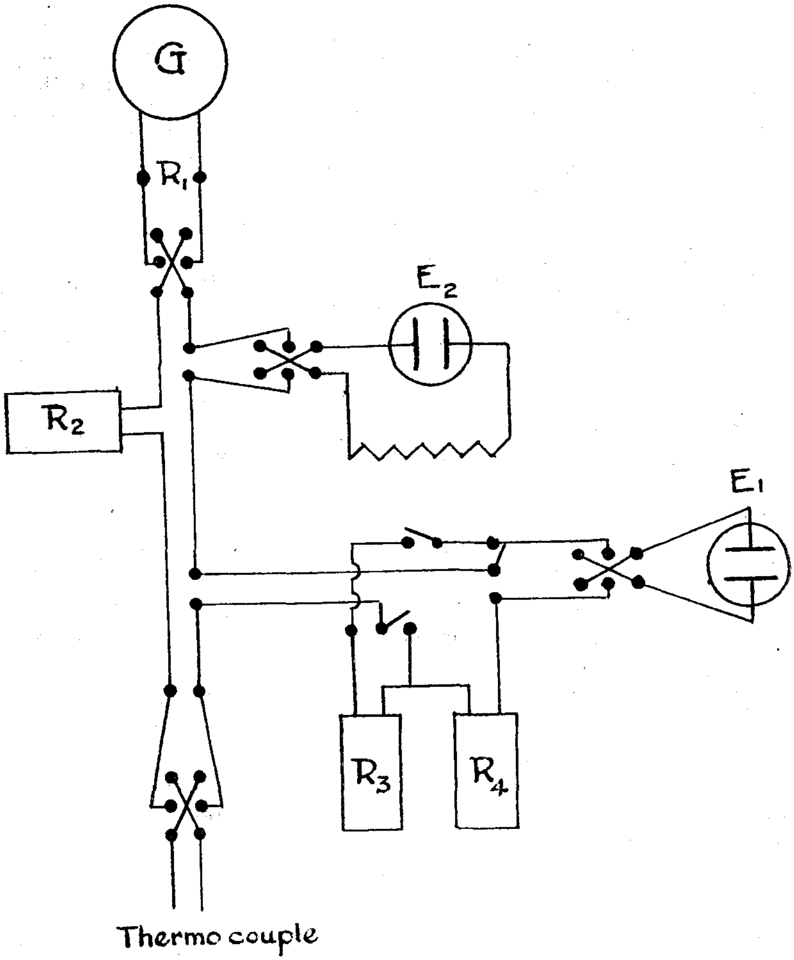

FIGURE 4. The circuits for measuring thermal e.m.f.

Before the adoption of the method just described, another piece of apparatus was built and tested, in which 16 crystal rods were led between two heavy copper bars, 5 cm. on a side, maintained at different temperatures. The crystal rods dipped into wells drilled in the copper bars, filled with stagnant oil, and of a diameter only slightly greater than that of the rods. The difficulty with this apparatus was that temperature equality between the crystal bars and the copper bars was not attained; this was tested by thermocouples directly soldered to the ends of the rods. Apparently the only way by which a positive temperature control can be attained is to immerse the crystal rods in a rapidly circulating stream of liquid.

METHODS OF CALCULATION.

The first step in the computation was to reduce the double throws of the galvanometer to volts by means of the calibration. Next a small correction, which could be determined by linear interpolation, was applied to these voltages to bring the temperature of the cold junction to exactly 20°; the actual temperature was usually a fraction of a degree below this. Then, by linear interpolation between 20° and the maximum temperature, the voltages at the intermediate temperatures were calculated which would have been found if the relation between temperature and voltage had been linear. The difference between the linear and the actual voltage was then plotted against temperature. The difference curve was, in almost every case, parabolic within the error of measurement; the condition for this is that the curve should be symmetrical about the mid point and that the ordinates at the one-quarter and the three-quarter points should be three-quarters of the maximum ordinate. A parabolic difference curve means that within experimental error the relation between thermal e.m.f. and temperature is of the second degree. The constants in this second degree relation could then be found at once in terms of the total voltage, e_0, between 20° and the maximum temperature, and the deviation from linearity at the mid point, Δ. In fact:

$$(\text{thermal e.m.f.}) = \left(\frac{e_0 + 4\Delta}{t_0}\right)(\tau - 293.1) - \frac{4\Delta}{t_0^2}(\tau - 293.1)^2,$$

where t_0 is the temperature range from 20° to the maximum, and τ is Abs. temperature.

If Kelvin's relation holds, that is, if at every temperature the thermal e.m.f. of a crystal rod against any homogeneous substance is a linear function of $\cos^2\theta$, θ being the angle between the length of the rod and the crystal axis, it is evident that e_0 and Δ must also be linear functions of $\cos^2\theta$. e_0 and Δ were accordingly plotted against $\cos^2\theta$, and the best smooth curve drawn through the points. If the relation turned out to be linear, then the values of e_0 and Δ at the extreme points, that is, for $\cos\theta = 1$ and $\cos\theta = 0$, were read from the curve. From the e_0 and Δ found in this way the values of the constants in the two-power series in temperature for thermal e.m.f. against copper could be calculated at the two extreme orientations. From the series formula the Peltier heat against copper and the Thomson heat can be found by differentiation. If e_0 or Δ was not linear against $\cos^2\theta$, as was the case for two metals, then the procedure just described must be repeated at enough orientations between 0° and 90° to define the behavior.

DETAILED DATA.

Zinc. The material was Kahlbaum's best zinc. A considerable number of castings were made: of these there were two sets in two many-arm molds with from 30 to 50 orientations, and at least five castings in the simpler mold with 12 orientations.

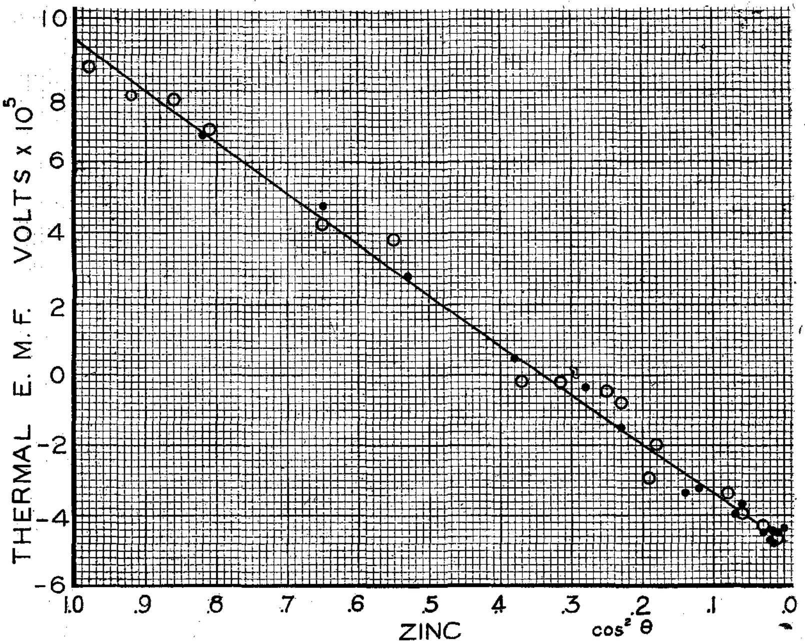

FIGURE 5. Thermal e.m.f. of single crystal zinc against copper between 20° and 88° C. The abscissa is the square of the cosine of the angle between the length of the rod and the principal crystal axis.

Measurements of thermal e.m.f. were made on two sets of 16 rods, one set from each of the larger molds, selected to cover as uniformly as possible the entire orientation range. The angle of the basal plane with the length was determined by cleavage, but instead of cleaving at room temperature, the rod was cooled in liquid air and cleaved while still cold by pressing into it the point of a knife. This trick of cleaving in liquid air is due to Boydston;[6] it is well worth while, the cleavage being much easier to produce and much sharper, and there being less danger of bending the rod.

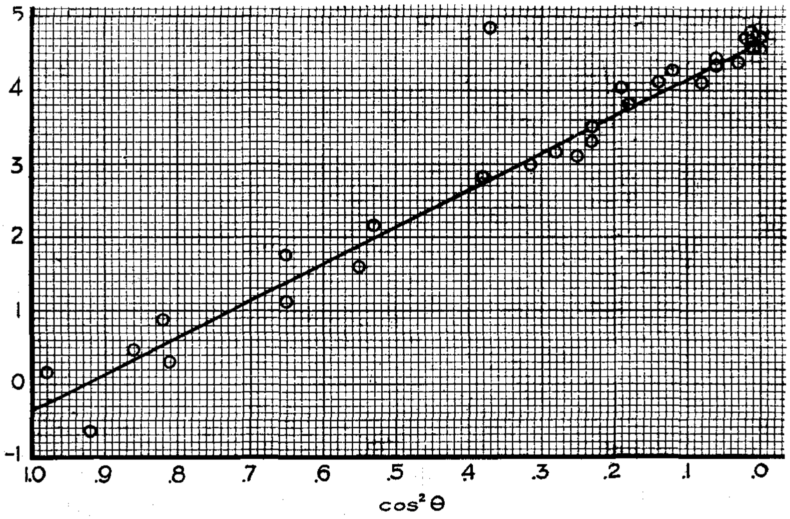

FIGURE 6. Shows the deviation from linearity in the relation between temperature and thermal e.m.f. of single crystal zinc against copper. The deviation is shown in 10^{-6} volts at the mean temperature 54°, the temperature range being from 20° to 88° C. The abscissa is the square of the cosine of the angle between the length of the rod and the crystal axis.

The results of the measurements of thermal e.m.f. are plotted in Figure 5 and Figure 6; in Figure 5 the total e.m.f. of the couple Zn-Cu between 20° and 88° is plotted against $\cos^2\theta$, and in Figure 6 the deviations from linearity at the mean temperature also against $\cos^2\theta$. All the e_0's of one set of rods have been corrected by the constant amount, 1×10^{-5} volts, to bring the absolute values into agreement with those of the other set. The difference in absolute values is doubtless due to a parasitic e.m.f. in parts of the circuit beyond the reversing switch, due to different temperature conditions in the room during the two sets of readings, and is evidently without significance, since the quantity in which we are interested here is the variation of e.m.f. with direction in the crystal. The deviations from linearity of the two sets of 16 lay on the same straight line without correction, as should be the case if the difference in e_0 is due to parasitic effects as suggested.

It will be seen from the figures that e_0 and Δ are both linear functions of $\cos^2\theta$; that is, the relation of Kelvin is satisfied for zinc. The results contained in the figures may be used to give the following formulas:

$$(\text{t.e.m.f.})_{\text{Zn}_{\shortparallel}-\text{Cu}} = 1.372 \times 10^{-6} \times (\tau - 293.1) + 3.03 \times 10^{-10} \times (\tau - 293.1)^2 \text{ Volts};$$

$$(\text{t.e.m.f.})_{\text{Zn}_{\perp}-\text{Cu}} = -0.428 \times 10^{-6} \times (\tau - 293.1) - 3.97 \times 10^{-9} \times (\tau - 293.1)^2.$$

Whence:

$$(\text{t.e.m.f.})_{\text{Zn}_{\shortparallel}-\text{Zn}_{\perp}} = 1.800 \times 10^{-6} \times (\tau - 293.1) + 4.27 \times 10^{-9} \times (\tau - 293.1)^2,$$

and since

$$P = \tau \frac{dE}{d\tau} \quad \text{and} \quad \sigma = \tau \frac{d^2E}{d\tau^2},$$

we have for the Peltier and Thomson heats:

$$P_{\text{Zn}_{\shortparallel}-\text{Zn}_{\perp}} = \tau \times [1.800 \times 10^{-6} + 8.54 \times 10^{-9} \times (\tau - 293.1)],$$

$$\sigma_{\text{Zn}_{\shortparallel}-\text{Zn}_{\perp}} = \tau \times 8.54 \times 10^{-9}.$$

The designation $\text{Zn}_{\shortparallel}$ means that the crystal axis (of six-fold symmetry) is parallel to the length of the rod. The sign convention is the usual one. If $(\text{t.e.m.f.})_{\text{Zn}_{\shortparallel}-\text{Zn}_{\perp}}$ is positive, current flows from $\text{Zn}_{\shortparallel}$ to $\text{Zn}_{\perp}$ at the hot junction.

It is probable that within the limits of error the relation between temperature and e.m.f. is not quite represented by a second degree expression in the temperature, but the curves of deviation from linearity are systematically unsymmetrical, in that the deviation at

¾ of the temperature range (71°) is slightly greater than at the ¼ temperature (37°). The difference is of the order of 4×10^{-7} volts. It is in such a direction as to decrease the curvature at the lower temperature and increase it at the higher temperature end of the range; this means that σ increases with rising temperature somewhat more rapidly than proportionally to the absolute temperature, as given by the formula. In order to make partial allowance for this slight departure from symmetry, the deviation, Δ, at the mean temperature was calculated by taking 0.4 of the sum of the deviations at the ¼, ½ and ¾ points, this being the fraction to be used if the deviation is accurately parabolic. An idea of the smoothness of the readings, and so of the accuracy, is given by the fact that the actual Δ at the ½ point usually differed from the calculated Δ by only 0.1 or 0.2 mm. deflection of the galvanometer, corresponding to from 4 to 8×10^{-8} Volts.

Compared with my previous measurements,[7] these new results show very important differences. The previous results were obtained with 8 rods, with a maximum angle between basal plane and length of 55°. The results indicated by the previous measurements were that P is not linear against $\cos^2 \theta$ (or against the specific resistance, which is the same thing), but that σ (or Δ) is linear. These new and greatly improved results do not substantiate this conclusion, but both P and σ are linear within satisfactory limits of error. The values given in the previous paper for Peltier heat and Thomson heat of Zn_{11} against Cu were obtained by an extrapolation to nearly double the range of measurement, and cannot, therefore, be expected to be accurate. It follows that the difference between the P and the σ for parallel and perpendicular directions in the former paper (p. 132) cannot be accurate. In fact, the former paper gives for $P_{11-\perp}$ at 0°, $1.001 \times 10^{-6} \times \tau$ against $1.629 \times 10^{-6} \times \tau$ found here; and for $\sigma_{11-\perp}$, $6.2 \times 10^{-9} \times \tau$ against $8.5 \times 10^{-9} \times \tau$ found now. The previous value for the basal plane parallel to the length avoids error from the long range extrapolation, but here there is another source of error in that the copper was not from the same source in the two sets of experiments. In fact the previous measurements give for $P_{Zn_\perp - Cu}$ at 0° the value $-.147 \times 10^{-6} \times \tau$, whereas the present value is $-.269 \times 10^{-6} \times \tau$. The difference, which amounts to about a tenth of a micro-volt per degree, does not seem too large for the thermal e.m.f. between two different varieties of commercial copper.

The conclusion that thermal e.m.f. satisfies Kelvin's law agrees with the conclusion reached by Linder[2] from his measurements up to 200°. Linder was of the opinion, however, that at 300° and 400° his departures from the relation were greater than experimental error. It seems to me, however, that this effect of Linder's might well be due to the very steep temperature gradient at the junction, which would make difficult the exact determination of the temperature, and which would have an increasing effect at high temperatures.

After the completion of the thermo-electric measurements, I made new measurements of specific resistance. There were several reasons for doing this. I desired to submit the $\cos^2$ law of Kelvin for resistance to a more accurate check than had yet been made; my previous measurements had been made on only eight samples scattered over a range of orientation of 60° instead of the whole 90°, and there were rather serious differences between the values of Grüneisen and Goens[3] and of myself for the ratio of the resistance parallel and perpendicular to the axis, Grüneisen and Goens finding the ratio 1.080 against my 1.036. At first, on repeating the measurements, disturbingly irregular results were obtained. After considerable work, the source of the irregularities was found in the very great sensitiveness of resistance to slight mechanical deformations. If the rod is slightly bent in removing from the glass mold, or if it sticks to the glass so that deformations are produced by unequal thermal expansion between the mold and metal on cooling, the resistance is materially increased. The reason that this source of error had not been more disturbing in the previous measurements was that the larger diameter of the rods, 6 mm. against 3mm. used here, made deformation incidental to cracking away the glass mold much less likely. It is perhaps surprising that thermal e.m.f. should be much less sensitive to deformation than resistance; this was proved not only by the much greater regularity of the results for thermal e.m.f., which were obtained on rods which must have suffered some deformation, but also by special tests, in which the rods were deliberately bent and straightened, with no measurable change in thermal e.m.f.

It finally proved impossible to obtain measurements of the resistance which were sufficiently regular on rods which had been removed from the molds, and accordingly the resistance was measured before removing the glass. This was done as follows. The potentiometer method of measuring resistance was used. Current terminals were attached by soldering copper leads to the two ends of the rod, the glass being broken away a sufficient distance to make this possible. The potential terminals were two needles, mounted on a frame at a fixed distance apart, and pressed by springs into contact with the metal through two V-shaped notches ground through the glass wall of the mold with a fine glass-cutting wheel dressed to a sharp edge with a diamond. The notches were ground to such a depth as to just break through the glass. In this way, the only appreciable deformation was confined to the ends near the current terminals, and so was without effect in the region between the potential terminals, which determines the resistance. The angle of the basal plane with the length was determined after the measurements by cooling in liquid air, and by cleavage, as before. The diameter was measured with a micrometer after the glass mold was broken away; the fluctuations from uniformity of diameter in the space between the potential terminals was of the order of 1%.

The specific resistances at 20° of two sets of rods, obtained with all these precautions, are shown in Figure 7 plotted against $\cos^2\theta$. Except for one bad point, the results are fairly regular. In selecting from these data the most probable values for the specific resistance, I assumed that almost all sources of error would give too high a resistance, and that therefore the lowest of the values found are most probably correct. Figure 7 shows that within experimental error specific resistance is linear against $\cos^2\theta$, so that the relation of Kelvin is satisfied. In fact there has never, as far as I know, been any reason to think that Kelvin's relation for resistance might not be correct.

In Figure 7 are also shown my previous values for resistance parallel and perpendicular to the axis. These both differ by a small and nearly constant amount from the value now found. The new value for the ratio of the two resistances is 1.039 (6.06/5.83) against 1.036 found before by extrapolation over a range of 30°. The absolute difference between my former and present results is perhaps to be explained by greater purity in my new supply of zinc; the fact that the difference is constant over the entire range of orientation is against the supposition that the previous high values were due to deformation. Plotted in the same diagram are also the values of Grüneisen and Goens. Their value for the resistance when the axis is perpendicular to the length is almost exactly the same as my new value, but their value for the parallel orientation is much higher.

It seems to be unquestionable that their large value is an effect of distortion, for it is of course true that a crystal is very deformable when the axis is parallel to the length, but comparatively rigid when perpendicular.

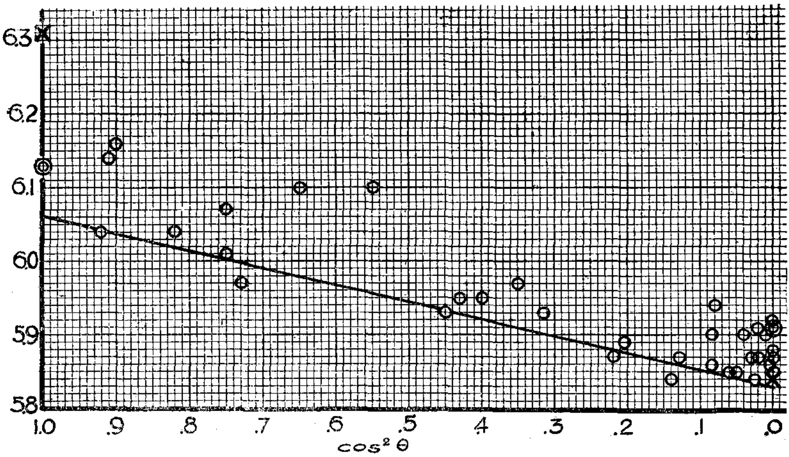

FIGURE 7. Specific resistance at 20° C in 10^{-6} ohms per cm. cube of single crystal zinc plotted against the square of the cosine of the angle between the crystal axis and the length of the rod. The double circles are my previous values and the crosses those of Grüneisen and Goens.

Cadmium. The material was Kahlbaum's best. At first the attempt was made to fill the molds by suction, drawing the melted Cd up from an open crucible, but this failed because of the unexpectedly rapid oxidation of the molten Cd, even at temperatures close to the melting point. The oxidation was sometimes so rapid that the coating of brown oxide spontaneously became red hot. The difficulty was not avoided by filling the furnace as completely as possible with a stream of CO_2, led in at the bottom and flowing out at the open upper end. The rapidity of oxidation seems to increase very markedly as the last traces of impurity are removed from the metal; the purest Cd of American make which I could obtain oxidized very much more slowly. The filling of the mold was finally accomplished by running the molten metal down from a sealed reservoir attached to the upper end of one of the branches. The two-branch mold with 12 sections was used. It was exhausted and flushed several times with CO_2 before filling.

Cd is perhaps the most difficult of the metals worked with here to compel to crystallize as one grain, the tendency to switch from one grain to another in the middle of a straight arm being particularly strong. The general behavior is consistent with the explanation that fresh nuclei of the solid are particularly likely to form from the liquid if it is subcooled by a small amount. The remedy is to make the temperature gradient unusually steep in the region where crystallization takes place, in this way reducing to a minimum the extent of the region in which there is any possible subcooling. A steep gradient in the desired locality was produced by lowering the mold into oil, and by various manipulations of the position of the surface of the oil and the temperature of the furnace, which can be learned only by trial. In general, the temperature of the furnace should be further above the melting point than is necessary for other metals.

The determination of the angle between the crystal axis and the length offered some difficulty. Cadmium does not cleave, even at liquid air temperature, and the small size of the rods made difficult the method previously used of marking the reflection pattern on a wooden ball slipped over the rod. At first I tried to determine the orientation by means of the specific resistance, but these results were very irregular, cadmium being even more deformable than zinc, and particularly difficult to prevent from sticking to the glass mold. After various unsuccessful attempts, including an attempt by locating the slip planes when stressed beyond the elastic limit, I returned to the optical method, not attempting, however, to locate the entire reflection pattern, but only the basal plane. A simple arrangement was made for this purpose, in which the crystal rod was mounted in a goniometer holder on an optical bench in a beam of parallel light from an electric arc, and the angle of reflection determined by sighting through two small peep holes. The error may easily be made less than 1° with very simply constructed apparatus. The difficulty with the method is in being sure that the plane of reflection is actually the basal plane. If the basal plane is approximately parallel to the length of the rod, the reflection from it is so much stronger than the reflection from any other plane that there is seldom any trouble. But if the basal plane is inclined at a high angle, then the reflection from it may be weaker than from one of the secondary planes more nearly parallel to the length, and in such a case, decision must be made with the help of other data. There are never more than a few discrete possibilities to choose between, and a knowledge of the specific resistance, or thermal e.m.f., or perhaps the slip pattern makes possible an unambiguous choice.

Measurements of thermal e.m.f. were made on 64 different samples, four sets of 16 each. This comparatively large number was used because the first measurements, in which the angle was determined from the specific resistance, had indicated a systematic departure from Kelvin's $\cos^2\theta$ law, and if this were really the case, I was anxious to settle it as certainly as possible. The suspected departure did not substantiate itself, however, on increasing the number of readings and determining the orientation in the better way.

The method by which the results were calculated for cadmium differed somewhat from that outlined above. The temperatures of the four sets of runs were so nearly alike that a very small correction sufficed to reduce the readings of any one set to a mean and standard set of temperatures: 20°, 37.20°, 54.03°, 72.00° and 88.14°. Four different plots of the thermal e.m.f.'s of all of the 64 rods between 20° and each of the other temperatures were then made against $\cos^2\theta$. In each case the points so obtained lay on a straight line, confirming Kelvin's law. The observed points for the interval 20°–88.14° are reproduced in Figure 8. Kelvin's law having thus been established, it remained only to get the best e.m.f.'s for the parallel and the perpendicular orientations; these were found from the intercepts of the best straight lines drawn through the experimental points. Next, the best second degree curve in the temperature was passed through these intercepts. In this way the following results were found:

$$(\text{t.e.m.f.})_{\text{Cd}_{\parallel}-\text{Cu}} = (1.357 \times 10^{-6})(\tau - 293.1) - 0.790 \times 10^{-8}(\tau - 293.1)^2 \text{ Volts},$$

$$(\text{t.e.m.f.})_{\text{Cd}_{\perp}-\text{Cu}} = -(1.616 \times 10^{-6})(\tau - 293.1) - 1.872 \times 10^{-8}(\tau - 293.1)^2.$$

Whence:

$$(\text{t.e.m.f.})_{\text{Cd}_{\parallel}-\text{Cd}_{\perp}} = (2.973 \times 10^{-6})(\tau - 293.1) + 1.082 \times 10^{-8}(\tau - 293.1)^2,$$

and for the Peltier and Thomson heats:

$$P_{\text{Cd}_{\parallel}-\text{Cd}_{\perp}} = \tau \times [2.973 \times 10^{-6} + 2.164 \times 10^{-8}(\tau - 293.1)],$$

$$\sigma_{\text{Cd}_{\parallel}-\text{Cd}_{\perp}} = \tau \times 2.164 \times 10^{-8}.$$

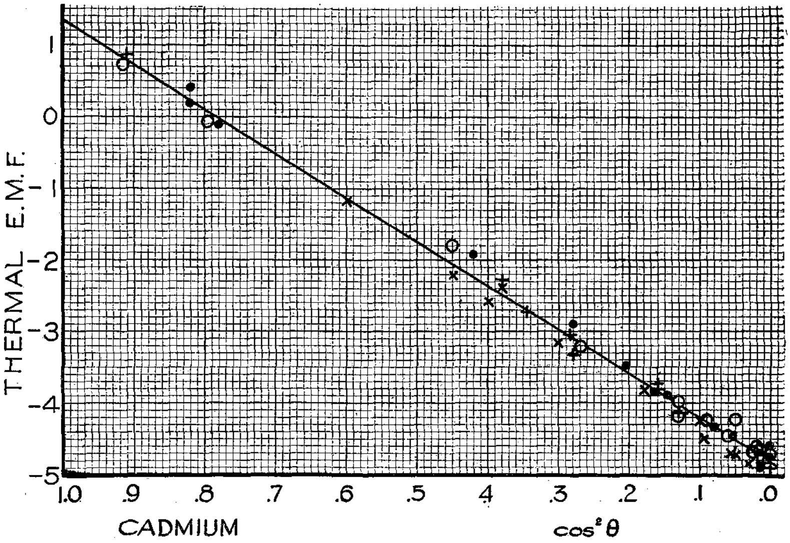

FIGURE 8. Thermal e.m.f. of single crystal cadmium against copper between 20° and 88.14° C. The ordinates are on an arbitrary scale, one large division corresponding to 40.15 × 10^{-6} volts. The abscissas are the squares of the cosine of the angle between the crystal axis and the length. The different designations of the observed points refer to different set-ups of the apparatus.

As in the case of zinc, the second degree curve does not quite reproduce the measurements, but there are small differences which are probably larger than experimental error. At the three temperatures 37.2°, 54.0°, and 72.0° the actual $(\text{t.e.m.f.})_{\text{Cd}_{\text{II}}-\text{Cu}}$ is to be found by adding to the values given by the second degree formula $- 0.2 \times 10^{-6}$, $+ 0.2 \times 10^{-6}$, and $+ 0.1 \times 10^{-6}$ Volts, respectively, and similarly, to obtain the actual $(\text{t.e.m.f.})_{\text{Cd}_{\perp}-\text{Cu}}$ the above formula is to be corrected by $- 0.4 \times 10^{-6}$, $- 0.6 \times 10^{-6}$, and $+ 0.9 \times 10^{-6}$ Volts, respectively. The correction term has the effect of decreasing the curvature, and therefore σ, at the low temperature end, and increasing it at the high temperature end.

We now compare these new results with those previously found. The previous values were obtained with 14 rods, distributed over practically the entire orientation range. Because the previous results were obtained without extrapolation, better agreement is to be expected than in the case of zinc. There is, however, the uncertainty in the previous results that the orientation was determined in terms of the specific resistance, which we have just seen does not give regular results.

The previous results agree with the new ones in that Kelvin's relation was found to hold. At 0° C the previous values for $P_{\mathrm{Cd}_{\parallel}-\mathrm{Cd}_{\perp}}$ was $+ 2.95 \times 10^{-6} \times \tau$ against $+ 2.54 \times 10^{-6} \times \tau$ now found, and for $\sigma_{\mathrm{Cd}_{\parallel}-\mathrm{Cd}_{\perp}}$, $+ 1.7 \times 10^{-8} \times \tau$ against $+ 2.2 \times 10^{-8} \times \tau$ now found.

Of the many sets of castings, only one was perfectly satisfactory for the determination of specific resistance, all the others sticking slightly to the glass or else having been partially deformed. Readings were made on this set before removing the glass mold, connections to the potential terminals being through notches ground in the glass, as in the case of zinc. The results are shown in Figure 9. It is evident that the $\cos^2$ law is satisfied within experimental error. My previous values are shown as double circles in the figure; these agree rather closely with the new values. I believe that a study of the data will suggest that the new value for the resistance when the basal plane is perpendicular to the length, 8.24, is somewhat preferable to the former value 8.30, in spite of the fact that the distribution of orientations in the previous work was somewhat wider. The values of Grüneisen and Goens[8] are also shown in the figure. These are higher than my values at both orientations by such an amount that their ratio of the resistances (8.45/7.11 = 1.188) does not differ by much from mine (8.24/6.82 = 1.207). The difference between my results and those of Grüneisen and Goens is in the direction to be explained by impurity or deformation in their material.

Antimony. The material was Kahlbaum's; I have not been able to obtain antimony from any other source of sufficient purity. The molds in which the single crystal rods were cast were made of quartz tubing; it would have been possible to use glass combustion tubing, but the softening point of this glass is so near the melting temperature, and other mechanical difficulties in handling the glass are so great, that the much greater convenience of the quartz much outweighed the greater expense. Fortunately antimony does not stick to quartz (or to glass either) so that it was not necessary to coat the mold, which would have been difficult at temperatures above a red heat. The first batch of castings was made in one of the complicated many-arm molds. The greatest difficulty with antimony

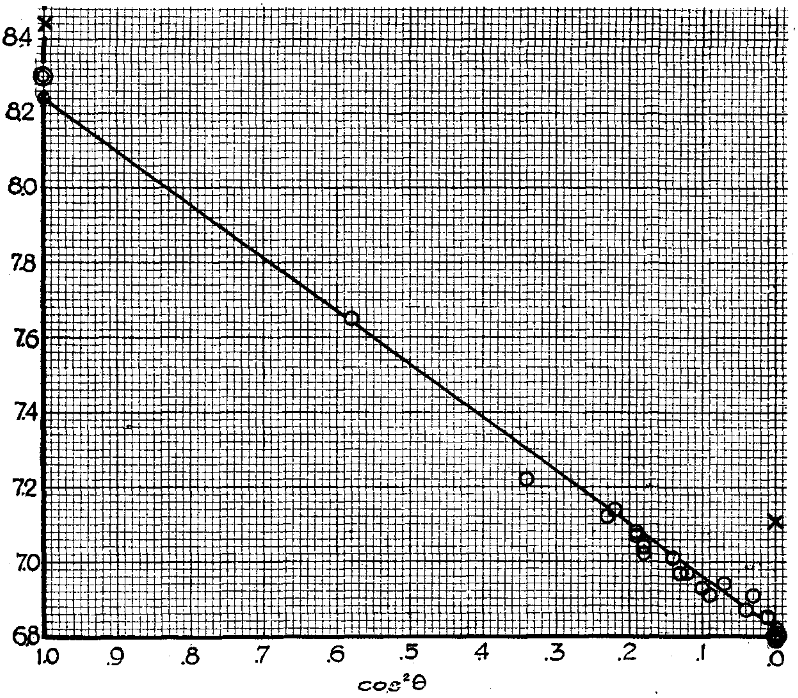

FIGURE 9. The specific resistance in 10^{-6} ohms per cm. cube at 20° of single crystal cadmium against the square of the cosine of the angle between the crystal axis and the length. The double circles show my previous values for the extreme orientations, and the crosses the values of Grüneisen and Goens.

is in the appearance of bubbles during filling; many of these appeared in the filling of the many-arm mold, and they could not be eliminated by any means that suggested themselves, so that many of the arms were of no use. I therefore returned to the much simpler mold with only six arms, made by bending a straight piece of quartz tubing at the desired angles. This was filled by slow suction from above, and the tube sealed off. The melted antimony was contained in an open porcelain crucible in the bottom of the furnace, and oxidation was sufficiently prevented by filling the furnace with a stream of CO_2. The advantage of this simple method is that if a bubble of gas appears during the filling, the mold can be allowed to empty itself and the process repeated until successful. Two of these six-arm molds could be filled at the same time from the same crucible, and thus 12 castings obtained by the same lowering operation. The higher orientations (cleavage perpendicular to the length) are more reluctant to appear with antimony than with most of the other metals, and even when once obtained, manipulation is difficult because of extreme brittleness. It is almost impossible to crack the quartz mold away from a rod with cleavage plane perpendicular to the length without breaking the rod. The method which I finally adopted was to grind the mold away from two sides, holding the quartz tube pressed lengthwise against a flat grinding surface, until the quartz was entirely ground through, as shown in section in Figure 10, when the remains of the mold will drop off.

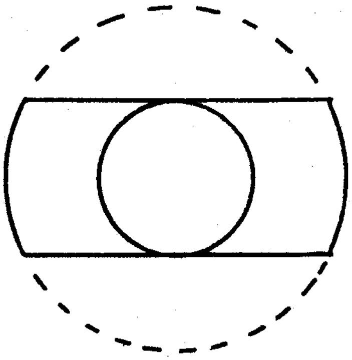

FIGURE 10. Shows the method of grinding away the quartz mold from the single crystals of antimony in order to avoid breaking the crystal when the cleavage plane is very nearly perpendicular to the length.

The orientation was determined with a simple goniometer from the cleavage, made with a knife at room temperature.

Three sets of runs were made to get the thermal e.m.f. The first set was on 12 rods obtained from the complicated many-arm mold. The second set was on 16 rods selected so as to give the widest range of orientation from four batches of castings made in the simple 6-arm molds. None of these rods was of very high orientation. The third set was on two pieces, both from the same original rod, with the cleavage plane at 83° to the length. These pieces were so short that it was not possible to use the regular arrangements; a new bakelite block was therefore made, machined so as to be only 3 mm. thick at the part where the rods were inserted, instead of 2.5 cm., and with special stuffing boxes of minimum depth. These two pieces from the same rod gave identical results, and they are shown in the following figures slightly separated.

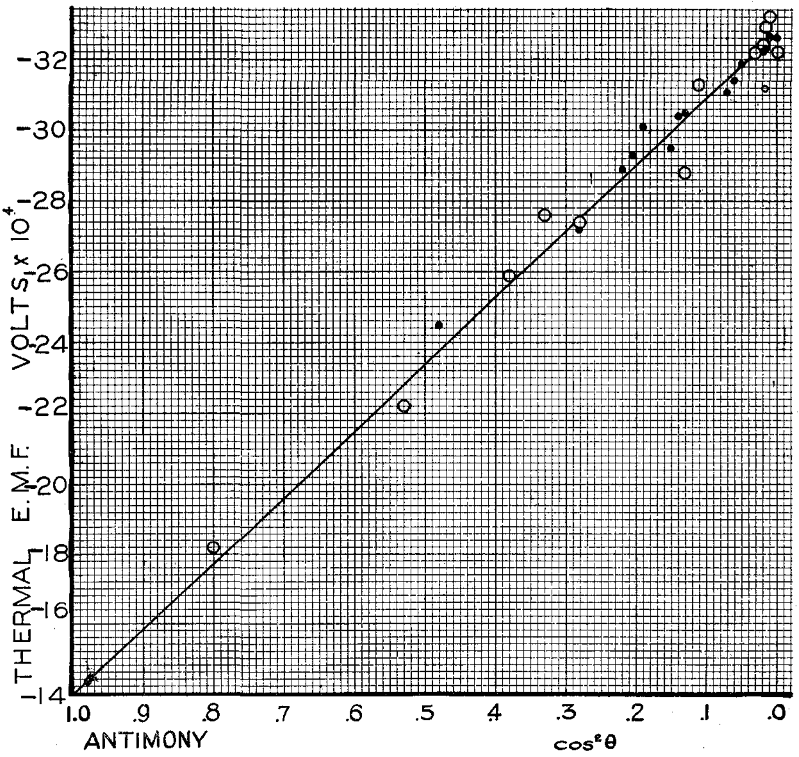

FIGURE 11. Thermal e.m.f. between 20° and 88° C in 10^{-4} volts of single crystal antimony against copper as a function of the square of the cosine of the angle between the crystal axis and the length.

The results for antimony were not as regular as for the other metals; doubtless the reason is connected with the short length of the specimens, so that irregular temperature fluctuations, due to greater proximity of the walls, were more important. The results for thermal e.m.f. between 20° and 88° are shown in Figure 11 for all specimens, and in Figure 12 for the deviation from linearity at the mean temperature. It is seen that within the limits of experimental error there is no reason to suspect a departure from Kelvin's $\cos^2$ law. The great scattering of the points for the deviation from linearity shows that there is considerable error in the curvature deduced from these results.

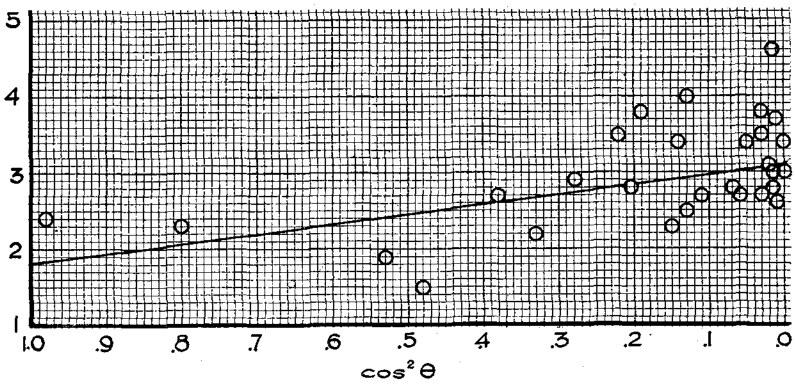

FIGURE 12. The deviation from linearity in the relation between temperature and thermal e.m.f. of single crystal antimony against copper. The deviation is shown in 10^{-5} volts at the mean temperature 54°, the temperature range being from 20° to 88° C. The abscissa is the square of the angle between the crystal axis and the length of the rod.

In spite of this comparatively large uncertainty, it is again probable that a second degree series in the temperature does not sufficiently reproduce the results. The departure from linearity was not quite symmetrical about the mean, the deviation being greater on the higher temperature side, so that the actual curvature is less at the lower temperature end and greater at the high temperature end than given by the second degree expression. The dissymmetry is of the order of 10^{-5} Volts, or ½% of the maximum t.e.m.f. I do not believe that the data justify an attempt to make a more precise statement about a possible departure from the second degree relation.

Expressing the results in the form of the best second degree series, the following values were found:

$$(\text{t.e.m.f.})_{\text{Sb}_{11}-\text{Cu}}$$
$$= -19.5 \times 10^{-6} \times (\tau - 293.1) - 1.55 \times 10^{-8} \times (\tau - 293.1)^2 \text{ Volts,}$$

$$(\text{t.e.m.f.})_{\mathrm{Sb}_{\perp}-\mathrm{Cu}}$$
$$= -46.2 \times 10^{-6} \times (\tau - 293.1) - 2.66 \times 10^{-8} \times (\tau - 293.1)^2.$$

Whence:

$$(\text{t.e.m.f.})_{\mathrm{Sb}_{\parallel}-\mathrm{Sb}_{\perp}}$$
$$= +26.7 \times 10^{-6} \times (\tau - 293.1) + 1.11 \times 10^{-8} \times (\tau - 293.1)^2,$$

and

$$P_{\mathrm{Sb}_{\parallel}-\mathrm{Sb}_{\perp}} = \tau \times [26.7 \times 10^{-6} + 2.22 \times 10^{-8} \times (\tau - 293.1)],$$
$$\sigma_{\mathrm{Sb}_{\parallel}-\mathrm{Sb}_{\perp}} = \tau \times 2.22 \times 10^{-8}.$$

In the previous paper I was able to measure the t.e.m.f. of only one sample of antimony, with the cleavage plane parallel to the length. For this I found:

$$(\text{t.e.m.f.})_{\mathrm{Sb}_{\perp}-\mathrm{Cu}}$$
$$= -61.6 \times 10^{-6} \times (\tau - 293.1) - 8.5 \times 10^{-8} \times (\tau - 293.1)^2.$$

This former value is without doubt much too high; the reason for it is not evident.

The value of the Peltier heat between $\mathrm{Sb}_{\parallel}$ and $\mathrm{Sb}_{\perp}$ is so large as to have a special significance that must be emphasized. At 20° C the formula above gives for $P_{\mathrm{Sb}_{\parallel}-\mathrm{Sb}_{\perp}}$ the value $293 \times 26.7 \times 10^{-6} = 0.0078$ Volts. This means that 1 coulomb absorbs 0.0078 joules ($= 7.8 \times 10^4$ ergs) in flowing from the parallel to the perpendicular direction. The amount absorbed by one electron is therefore $-1.59 \times 10^{-19} \times 7.8 \times 10^4 = -1.24 \times 10^{-14}$ ergs. Now the energy of a gas molecule at 20° C is 6.05×10^{-14} ergs, so that in antimony one electron, in flowing from a direction of motion perpendicular to one parallel to the axis of the crystal, absorbs *as heat* 0.2 of the classical equipartition energy. It will be seen later that the corresponding quantity for bismuth is about twice as great. This is evidently a most important point for any theory of conduction. It would seem to indicate that the thermal energy of the conduction electrons must be of the same order of magnitude as given by classical equipartition. It is not obvious what account would be given of this situation by such a theory as that of Sommerfeld,[8] in which the part of the energy of the electrons which varies with temperature is supposed to be much less than the classical amount.

The determination of the specific resistance of this new material was a comparatively straightforward matter, the danger of deformation being very small because of its brittleness. The results are shown in Figure 13. Again the cos² law is satisfied within experimental error. My previous values are also shown in the figure; these were obtained from only four specimens, with the cleavage plane at 0°, 0°, 47°, and 66° to the length respectively. It is not surprising that the agreement is not better. It is to be noted that the abnormal result previously found is sustained, namely that the resistance across the cleavage plane is less than parallel to the plane; in all other metals the behavior is the reverse. The discrepancy between my previous results and the new one for cleavage plane perpendicular to the length is in the direction to be accounted for by minute fissures on the cleavage plane in the former sample.

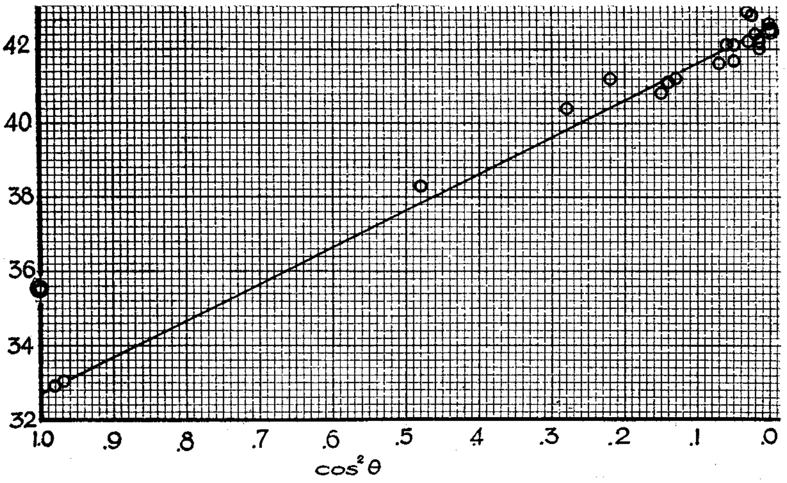

FIGURE 13. The specific resistance in 10^{-6} ohms per cm. cube at 20° C of single crystal antimony as a function of the square of the cosine of the angle between the crystal axis and the length. My previous values for the extreme orientations are shown by the double circles. The best values seem to be: 32.7×10^{-6} for the cleavage plane perpendicular to the length, and 42.6 for the parallel orientation.

The Pressure Coefficient of Resistance of Single Crystal Antimony. Advantage was taken of the greater range of orientation of the specimens now available to extend my previous measurements[9] of the effect of pressure on resistance. The previous measurements were made on two samples with the cleavage plane at 0° and 47.5° to the length. The new measurements were made on three specimens, with the cleavage plane at 7°, 42.5°, and 83° to the length. Measurements were made in the routine manner with the potentiometer. Current and potential leads were both soldered to the specimen, using very fine leads and a minimum amount of solder for the potential leads. Measurements were made at 30° and 75°, thus incidentally giving values for the temperature coefficient of resistance. The transmitting medium was petroleum-ether. In this way the irregular effects with the samples with high orientations previously found were completely avoided; these effects were ascribed to minute fissures produced by the viscosity under pressure of the transmitting medium. The regularity of the new results is shown by the following values for the average departure of a single reading from a smooth curve, in terms of percentage of the maximum pressure effect, at 30° and 75° respectively for the three orientations in the order given above: 0.50%, 4.0%; 0.18%, 0.22%; 0.08%, 0.17%. The larger error for the cleavage plane parallel to the length is explained by the much smaller value of the pressure coefficient in this direction.

The change of resistance produced by pressure cannot be represented within experimental error by any simple expression. For the 83° and the 42.5° orientations, the deviations from linearity are unsymmetrical about the mean pressure, the greatest deviations occurring at pressures less than the mean. The asymmetry is more pronounced for the 42.5° orientation. For the 7° orientation, the deviation from linearity reverses sign, being negative for low pressures and positive for high pressures; this effect is the more marked at the low temperature. In view of all these complications, the results are not indicated by a formula, but by a table (Table I), giving the resistance at 1000 kg intervals of pressure at 30° and 75°. In making the calculations, the relative resistance of each sample was first expressed in terms of its resistance at atmospheric pressure and 30° C as unity. These numbers were then multiplied by the specific resistance at 30° and atmospheric pressure to give the figures in the table. The specific resistance at 30° and atmospheric pressure was obtained from Figure 13, using in passing from 20° to 30° the values of the mean temperature coefficient shown in Table I. This mean temperature coefficient was obtained by linear extrapolation of the resistance measured at 30° and 75° during the pressure runs. The resistance of antimony is not linear with temperature, so that extrapolation is not quite correct, but it can only be a very little in error, because the range of temperature of the measurement, 30°–75°, is so nearly in the middle of the range 0°–100°.

The resistances listed in Table I are thus nearly specific resistances; they are, however, not quite this, because the resistances under pressure were obtained with terminals fixed to the specimen. To get specific resistances from these measurements, a correction must be applied for the distortion under pressure. From the elastic constants which I have previously determined it may be easily computed that when the basal plane is parallel to the length, the measured resistance at 10,000 kg must be corrected by subtracting 1.69% to convert to specific resistance, and when the basal plane is perpendicular to the length, the correction is an addition of 0.68%. These figures are also approximately the figures for the 7° and 83° directions respectively given in the table.

TABLE I.

Resistance of Antimony.

Pressure kg./cm.²	Resistance $\times 10^6$					
	Basal Plane at 7° to length		Basal Plane at 42°.5 to length		Basal Plane at 83° to length	
	30° C	75° C	30° C	75° C	30° C	75° C
0	44.41	53.37	40.01	48.81	34.60	42.47
1000	44.58	53.38	40.30	49.09	35.22	43.21
2000	44.76	53.40	40.66	49.43	35.95	43.94
3000	44.93	53.43	41.06	49.72	37.05	44.89
4000	45.11	53.51	41.52	50.09	37.78	45.94
5000	45.28	53.56	42.00	50.48	38.61	46.83
6000	45.45	53.65	42.49	50.89	39.60	47.86
7000	45.63	53.74	43.01	51.30	40.63	48.92
8000	45.80	53.78	43.53	51.73	41.70	50.02
9000	45.98	53.81	44.05	52.16	42.80	51.16
10000	46.16	53.82	44.58	52.59	43.93	52.32
11000	46.33	53.82	45.13	53.03	45.07	53.48
12000	46.50	53.78	45.66	53.38	46.20	54.60
Mean Temp. Coefficient, 0°–100°	.00495		.00551		.00572	

Comparing these new results with the previous values, the most important qualitative features found before are verified, but the numerical agreement is not very close. For the 7° direction, the minimum of resistance with increasing pressure is found again. As before, this minimum of resistance occurs first at higher temperatures. The pressure coefficient perpendicular to the basal plane is, as before, found to be very much greater than parallel to the plane. The difference is so great that at some pressure not very far beyond the maximum of 12,000 the initial abnormality of a resistance greater parallel than across the cleavage plane will be wiped out. Previously the disappearance of the abnormality was found actually to take place within the experimental range of pressure.

Tin. Kahlbaum's tin was used. Two sets of measurements of thermal e.m.f. were made, on two sets of 16 rods, selected from two different castings in the complicated mold with many branches. The orientation of these small rods was not easy to determine. Tin has no cleavage and the basal plane is absent from the reflection pattern, so that to determine the orientation from the reflection pattern would have demanded the location of several of the other planes, and not enough of these were usually present to make this possible. Accordingly I had to be satisfied with a measurement of specific resistance to give the orientation. Fortunately, tin is not as deformable as zinc or cadmium, and furthermore its resistance is comparatively insensitive to deformation, as was shown by direct trial, and also by the fact that the measurements of thermal e.m.f. were very much more regular when plotted against resistance than had been the case with zinc or cadmium.

In Figure 14 is plotted the total e.m.f. against copper between 20° and 87.8° as a function of the specific resistance at 20°. The readings of one set have been corrected by a small constant amount to make their mean agree with the other set; the difference between the two sets is doubtless due to different parasitic e.m.f.'s during the two runs, as has already been explained in connection with zinc. Within experimental error, thermal e.m.f. could be represented by a second degree series in the temperature. The deviations of the observed points from linearity at the mean temperature are shown in Figure 15. It is evident that within error the deviation is a linear function of the specific resistance, as is demanded by Kelvin's law. The total e.m.f., on the other hand, seems to be very definitely not a linear function. To a sufficient degree of approximation, the

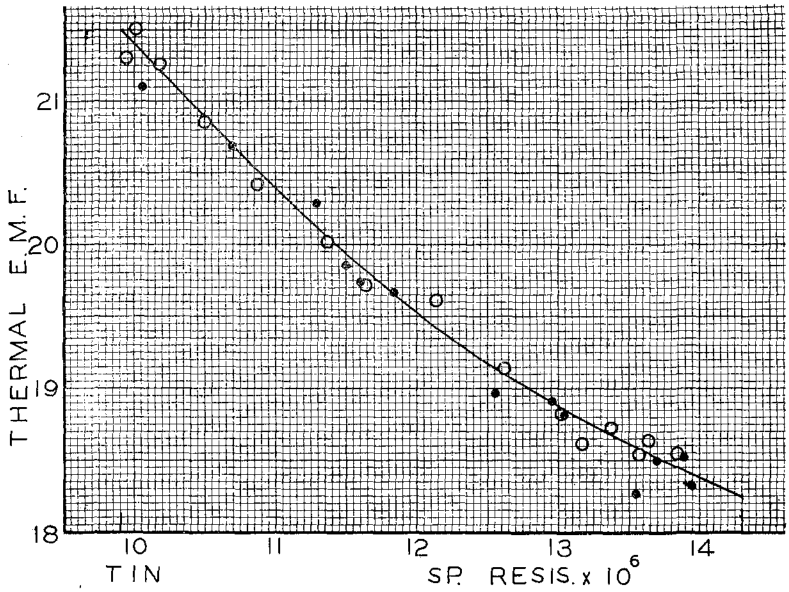

FIGURE 14. Thermal e.m.f. between 20° and 87.8° of single crystal tin against copper as a function of the specific resistance of the tin at 20°. The ordinates are centimeters deflection of the galvanometer; one centimeter is the equivalent of 1.096 × 10^{-5} volts.

e.m.f. is a second degree function in $\cos^2\theta$, so that the results can be reproduced by describing the behavior at the 0°, 45°, and 90° orientations. The results are as follows:

$(\text{t.e.m.f.})_{\text{Sn}_{\parallel}-\text{Cu}}$

$$= (2.650 \times 10^{-6}) \times (\tau - 293.1) + 4.29 \times 10^{-9} \times (\tau - 293.1)^2 \text{ Volts},$$

$(\text{t.e.m.f.})_{\text{Sn}_{45}-\text{Cu}}$

$$= (2.909 \times 10^{-6}) \times (\tau - 293.1) + 3.44 \times 10^{-9} \times (\tau - 293.1)^2,$$

$(\text{t.e.m.f.})_{\text{Sn}_{\perp}-\text{Cu}}$

$$= (3.307 \times 10^{-6}) \times (\tau - 293.1) + 2.47 \times 10^{-9} \times (\tau - 293.1)^2,$$

$(\text{t.e.m.f.})_{\text{Sn}_{\parallel}-\text{Sn}_{45}}$

$$= (-.259 \times 10^{-6}) \times (\tau - 293.1) + 0.85 \times 10^{-9} \times (\tau - 293.1)^2,$$

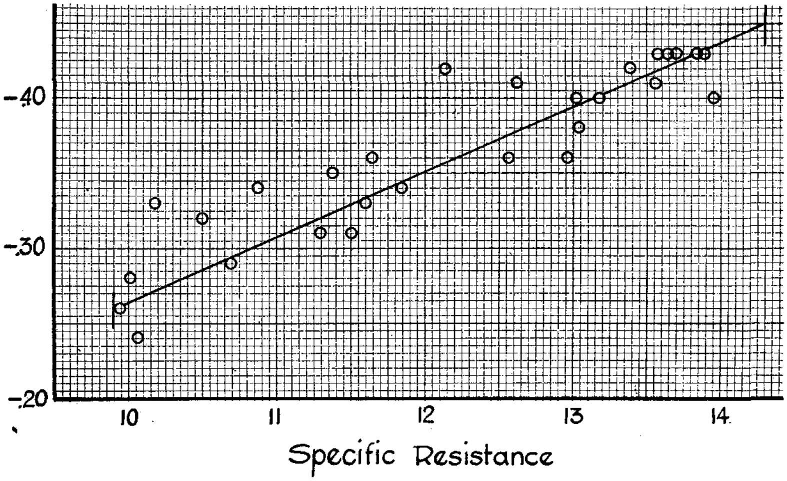

FIGURE 15. The deviation from linearity in the relation between temperature and thermal e.m.f. of single crystal tin against copper. The deviation is shown in terms of centimeters deflection of the galvanometer (1 cm. = 1.096×10^{-5} volts) at the mean temperature 53.9°, the temperature range being from 20° to 87.8° C. The abscissa is the specific resistance at 20° × 10^6.

$$(\text{t.e.m.f.})_{\text{Sn}_{11}-\text{Sn}_\perp}$$

$$= (-.657 \times 10^{-6}) \times (\tau - 293.1) + 1.82 \times 10^{-9} \times (\tau - 293.1)^2,$$

$$P_{\text{Sn}_{11}-\text{Sn}_{45}} = \tau \times [-.259 \times 10^{-6} + 1.70 \times 10^{-9} \times (\tau - 293.1)],$$

$$\sigma_{\text{Sn}_{11}-\text{Sn}_{45}} = \tau \times 1.70 \times 10^{-9},$$

$$P_{\text{Sn}_{11}-\text{Sn}_\perp} = \tau \times [-.657 \times 10^{-6} + 3.64 \times 10^{-9} \times (\tau - 293.1)],$$

$$\sigma_{\text{Sn}_{11}-\text{Sn}_\perp} = \tau \times 3.64 \times 10^{-9}.$$

The previous results* for tin were obtained from 14 specimens, of which only one had the basal plane approximately perpendicular to the length, the 13 others having the angle less than 40°; the results, furthermore, were much more scattering than the present ones, so that there is no doubt that the new results are much to be preferred. The value formerly found for the Peltier heat at 0° C from the parallel to the perpendicular direction was $\tau \times (-.75 \times 10^{-6})$ against $\tau \times (-.73 \times 10^{-6})$ found here. The goodness of the agreement must be in part fortuitous. The previous measurements showed no difference in the Thomson heat between the perpendicular and the parallel directions; the difference found here is small. The e.m.f.'s against copper previously found were about 0.20×10^{-6} Volts lower than the new values; the difference is doubtless due to difference in the quality of the copper.

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* In the previous paper,[1] the signs of the Peltier heat for tin given in Table II on page 132 should be negative instead of positive.

There are, of course, no measurements of the resistance as a function of the orientation, as there were for the other metals, since the method of determining the orientation did not permit this.

Bismuth. The material was Kahlbaum's. A large number of castings were made in the complicated many-arm molds. The orientation was determined from the cleavage plane; in a few cases the cleavage was made at liquid air temperatures, but cleavage at room temperature of rods as small as these was so easy that this refinement was not necessary. Four sets of measurements of thermal e.m.f. were made from four sets of 16 rods, selected to cover the orientation range.

Within experimental error the relation between e.m.f. and temperature is of the second degree in the temperature. In Figure 16 is shown the total e.m.f. between 20° and 88° against copper of the 64 specimens, plotted against $\cos^2\theta$, and in Figure 17 the deviations from linearity at the mid temperature. As in the case of tin, the total e.m.f. seems to definitely depart from Kelvin's $\cos^2$ law. The departure is not large, and the results can be sufficiently reproduced by giving the formulas for the 0°, 45°, and 90° orientations. The results follow:

$$(\text{t.e.m.f.})_{\text{Bi}_{\parallel}-\text{Cu}} = 1.084 \times 10^{-4} \times (\tau - 293.1) - 7.27 \times 10^{-8} \times (\tau - 293.1)^2 \text{ Volts},$$

$$(\text{t.e.m.f.})_{\text{Bi}_{45}-\text{Cu}} = .868 \times 10^{-4} \times (\tau - 293.1) - 3.46 \times 10^{-8} \times (\tau - 293.1)^2,$$

$$(\text{t.e.m.f.})_{\text{Bi}_{\perp}-\text{Cu}} = .572 \times 10^{-4} \times (\tau - 293.1) + 0.26 \times 10^{-8} \times (\tau - 293.1)^2.$$

Whence follows:

$$(\text{t.e.m.f.})_{\text{Bi}_{\parallel}-\text{Bi}_{45}} = .216 \times 10^{-4} \times (\tau - 293.1) - 3.81 \times 10^{-8} \times (\tau - 293.1)^2,$$

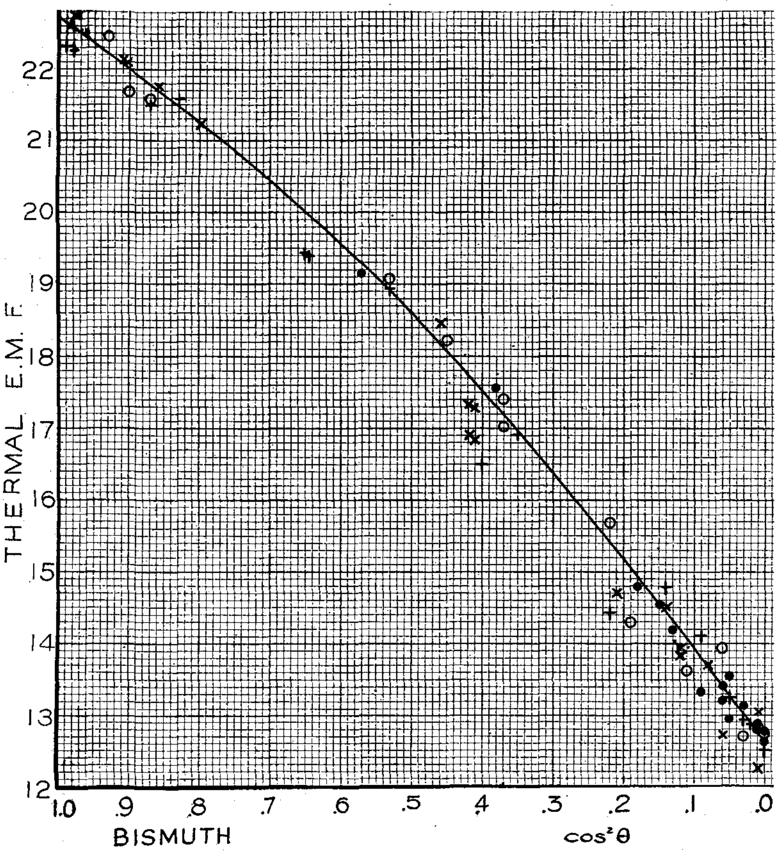

FIGURE 16. Thermal e.m.f. between 20° and 88° of single crystal bismuth against copper as a function of the square of the cosine of the angle between the crystal axis and the length. The ordinates are centimeters deflection of the galvanometer; one centimeter is the equivalent of 3.10 × 10^{-4} volts. The different symbols on the observed points refer to runs with different set-ups.

$$(\text{t.e.m.f.})_{\text{Bi}_{\parallel}-\text{Bi}_{\perp}}$$

$$= .512 \times 10^{-4} \times (\tau - 293.1) - 7.53 \times 10^{-8} \times (\tau - 293.1)^2,$$

and:

$$P_{\text{Bi}_{11}-\text{Bi}_{45}} = \tau \times [.216 \times 10^{-4} - 7.62 \times 10^{-8} \times (\tau - 293.1)],$$

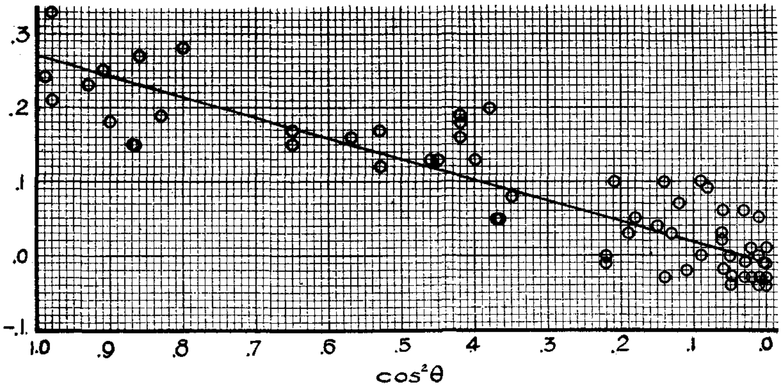

FIGURE 17. The deviation from linearity in the relation between temperature and thermal e.m.f. of single crystal bismuth against copper. The deviation is shown in terms of centimeters deflection of the galvanometer (1 cm. = 3.10 × 10^{-4} volts) at the mean temperature 54°, the temperature range being from 20° to 88°. The abscissa is the square of the cosine of the angle between the crystal axis and the length.

$$\sigma_{Bi_{\parallel}-Bi_{45}} = \tau \times [-\,7.62 \times 10^{-8}],$$

$$P_{Bi_{\parallel}-Bi_{\perp}} = \tau \times [.512 \times 10^{-4} - 15.06 \times 10^{-8} \times (\tau - 293.1)],$$

$$\sigma_{Bi_{\parallel}-Bi_{\perp}} = \tau \times [-\,15.06 \times 10^{-8}],$$

My previous results for bismuth covered a range of orientation of only 21°, so that a comparison with previous results is profitable only for the perpendicular orientation. The former value for the Peltier heat against copper was 0.578 × 10^{-6} × τ at 0° C, against 0.571 × 10^{-6} × τ found now. The agreement is good; better agreement is to be expected here than for the other metals because the absolute magnitude of the effects are so much larger for bismuth. Since my previous results were published, Boydston[10] has measured the thermal e.m.f. of single crystal bismuth. He used constantan as the metal of comparison, so that his results are not directly comparable with mine. It may be deduced from his Figure 1 on page 913 of his paper that the thermal e.m.f. between 0° and 100° C of a couple composed of $Bi_{\perp} - Bi_{\parallel}$ is about 5.4 × 10^{-3} Volts, against 4.7 × 10^{-3} to be deduced from my formulas above. Boydston's material covered a range of orientation up to within 23° of perpendicularity of basal plane to length, so that no large error is to be expected in his extrapolation to 90°. A more probable source of error I believe to be impurity in Boydston's bismuth. His metal was obtained from Mallinckrodt, and contained 0.04% of silver. This is not so stated in his paper, but I have learned from correspondence that the omission was due to an oversight. Now I have already found that silver[11] in amount even smaller than this may be responsible for important changes in the properties; I shall return to this question in discussing the resistance.*

In my previous work[9] the specific resistance had been found from a series of measurements on specimens covering an orientation range of only 45°. The points were furthermore rather scattering (Figure 6 on page 531 of my previous paper). In deducing from those measurements the most probable value of resistance, the guiding idea was that the lowest values were most likely to be correct, since high values can easily be produced by slight fissures on the cleavage plane due to deformation. The values deduced for specific resistance at 20° were: $\rho_{\perp} = 109 \times 10^{-6}$ and $\rho_{11} = 138 \times 10^{-6}$. The ratio of the two resistances was thus only 1.27, against nearly 1.6 reported by practically all early observers.† Adopting a suggestion endorsed by Borelius and Lindh, I attributed the difference to the effect of fissures. Since that work, a new determination has been made by Schneider,[6] who finds 1.62 for the ratio; Schneider is apparently inclined to accept my explanation of the discrepancy as due to the effect of fissures. It seemed therefore important to settle this question as definitely as possible. I was in a position to much improve on my former results because I could now cover the entire range

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* The thermo-electric properties of bismuth as a function of orientation have also been measured by Borelius and Lindh,[12] but it is not possible to compare their results with mine because of the very great variations which they find are produced by the temperature history of the specimen. About all that can be said is that their thermal e.m.f. between the two directions is of the same order of magnitude as that found here.

† Grüneisen (Handbuch der Physik, vol. XIII, p. 12) quotes Borelius and Lindh as giving the ratio 1.34, but it is not obvious from what part of their work this figure was obtained. Probably it was deduced from Figure 2 on page 613 of their paper in Ann. Phys. 51, 1916. But Borelius and Lindh were apparently not willing to commit themselves as to just what the ratio of the specific resistances really is, in view of the large effect which they attributed to fissures. Thus on page 614 of the paper just quoted they say: "This survey makes it very probable that the excess resistance (parallel to the axis over that perpendicular to the axis) becomes very small or entirely vanishes under sufficient pressure." In a later paper (Ann. Phys. 53, p. 124, 1917), in describing the symmetry relations of the resistance, they give for the resistance at 20° C, perpendicular, at 45°, and parallel to the axis the figures 1.24, 1.41, and 1.92 × 10^{-6}, which makes the ratio 1.55, but they remark that the figure 1.92 can be brought down to 1.51 by pressure. The general impression which the work of Borelius and Lindh gives is that in their opinion most, if not all, of the differences observed between different directions in bismuth are due to fissures or internal stresses.

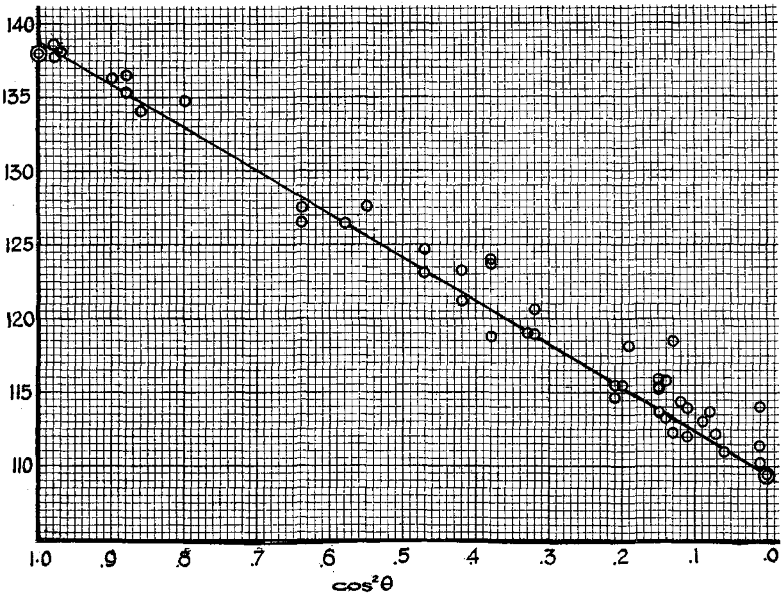

FIGURE 18. Specific resistance in 10^{-6} ohms per cm. cube at 20° of single crystal bismuth as a function of the square of the cosine of the angle between the crystal axis and the length.

of orientation. I did not make measurements on the crystals before they had been removed from the glass mold, as had been done for zinc and cadmium, but I selected from the large number of specimens those which had received little or no bending, and which sufficiently covered the entire range of orientation. All the results for the specific resistance of these selected specimens, *making no discards,* are shown in Figure 18 plotted against $\cos^2\theta$. It is evident in the first place, that Kelvin's $\cos^2$ law for specific resistance is satisfied.

In the figure are also shown as double circles my previous values for the parallel and perpendicular orientations; it is evident that these values were substantially correct.

Since the experimental work of this paper was done, a paper has appeared by Kapitza (Proc. Roy. Soc. 119, 358, 1928), in which a very careful investigation was made of the resistance as a function of direction. Kapitza grew his crystals by a modified method in which the crystal was not constrained by a mold. He found specimens with abnormally high resistance, but a study of the effect of compression on these led him to accept the explanation that the effect is due to minute cracks. He gives as the best values for resistance: 139×10^{-6} perpendicular to the cleavage plane, and 107 parallel to it. The ratio is 1.30, not far from my value 1.27. I believe that my value is somewhat to be preferred, because Kapitza's values for the parallel orientation showed considerable scattering, from 107 to 114, but in any event there can now be no question whatever that the true ratio cannot possibly be as high as found by the early observers.

I now believe that there are two possible explanations for the large value found for this ratio by other observers; one is the effect of fissures, as already suggested. This is substantiated by my own observations made on specimens in which the bending was known to be severe, which gave points lying very far above the curve of Figure 18 (these points are not shown in the diagram), by the observation of Borelius and Lindh[12] that the resistance of a rod with cleavage plane perpendicular to the length receives a large discontinuous decrease on applying a small longitudinal pressure in such a direction as to close the hypothetical fissures, and now by the recent work of Kapitza. But that this is not the only explanation became obvious on examining some of the specimens of Schneider, which had the high ratio, and which he was so kind as to send. On applying hydrostatic pressure, these showed no discontinuity whatever of resistance, as must have been the case if there had been cracks. I found that the temperature coefficient of resistance of these samples of Schneider was only 0.0032 instead of 0.0044 to be expected for pure bismuth. I have already found that a minute trace of silver has a large effect on temperature coefficient, and Schneider's bismuth is known to contain 0.04% silver. It seems possible that during the slow cooling incident to casting in unicrystalline form the silver may segregate on the basal planes in such a way as to increase the resistance to flow perpendicular to this plane, but not to affect it very much for parallel flow. Silver is one of the commonest impurities in bismuth, and it is also very difficult to remove, so that the same source of error may have been present in the material of some of the other observers.

ON THE ROTATIONAL SYMMETRY OF THERMAL E.M.F.

Altogether, I spent much more time in the investigation of bismuth than of the other metals, since the chance of finding new effects in bismuth is higher because of the larger numerical values to be expected. One of the questions investigated was that of the rotational symmetry of the thermo-electric properties. The symmetry is assumed to be of this sort in the argument of Kelvin, but if Kelvin's $\cos^2$ relation does not hold, as apparently it does not for bismuth, it becomes important to examine this assumption. I briefly touched on this question in my previous paper; I had not realized the importance of the question on gathering the data for that paper, and my argument rested on the general smoothness of the results and two fortuitous measurements for bismuth. I have now made a systematic examination of this question. In one set of 16 rods, not only was the orientation of the cleavage plane with respect to the length determined, but also the orientation of the secondary cleavage planes; these two together are sufficient to completely define the orientation of the rod with respect to the crystal. The secondary cleavage planes were determined from the triangular pattern which close examination will almost always disclose in the basal cleavage plane. Secondary cleavages may be started from the sides of this triangular pattern. The set of rods chosen all had the basal plane nearly parallel to the length, since for this orientation the effect would be expected to be a maximum. It is obvious that if the basal plane is perpendicular to the length, the orientation of the triangular pattern must be without effect on the longitudinal e.m.f. No correlation whatever was found between the thermal e.m.f. and the orientation of the secondary cleavage plane, thus establishing that within the limits of error indicated by the points in Figure 16 the thermo-electric properties have rotational symmetry about the trigonal axis, as assumed by Kelvin. Of course this proof applies only to bismuth, but it is probable that the other metals have rotational symmetry also. Direct proof would have been much more difficult for any of the others, except for antimony, but the regularity of the results when plotted against only a single parameter, the angle between axis and length, makes it probable that no other parameter enters.

Pressure Coefficient of Resistance. Previously I had measured the effect of pressure on the resistance of two samples of single crystal bismuth,[13] with the basal plane parallel and at 33° to the length. These measurements are now extended to three samples, with the basal plane as 4°, 41°, and 82° to the length. Measurements were made on each of these specimens at 30° and 75° over a pressure range of 12000 kg. The potentiometer method was used as usual. Terminals were soldered to the specimens, so that the following results must be corrected by the change of dimensions to obtain the true relative specific resistances. The accuracy of the measurements was that usual in this work; the average departure of a single reading from a smooth curve for all six runs with three samples was 0.10% of the maximum pressure effect, the extreme variations in accuracy being from 0.02% to 0.15%. Within these limits of error the results can be reproduced by a two-power series in the pressure; this was not the case with the previous measurements. The results follow, pressure being in kg/cm²:

TABLE II.

Angle between Basal Plane and Length	$\frac{\Delta R}{R(0.30^\circ)}$	$\frac{\Delta R}{R(0\ 75^\circ)}$
4°	$+1.047\times10^{-5}p+2.96\times10^{-10}p^2$	$+1.055\times10^{-5}p+2.67\times10^{-10}p^2$
41°	$+1.564 \qquad +4.93$	$+1.513 \qquad +3.92$
82°	$+2.011 \qquad +8.10$	$+1.975 \qquad +6.14$

The average temperature coefficients between 0° and 100° of these three samples were: 0.00441, 0.00431, and 0.00437 respectively. These values are nearly as high as the best which I have ever obtained for the purest electrolytic bismuth (00441), and are evidence of the high purity of this material. Before the war, the best bismuth which I could get from Kahlbaum had a coefficient of only 0.00332; it is evident that the quality of Kahlbaum's bismuth has improved, or possibly the process of very slow solidification to produce the single crystal itself brings about further purification.

In general, the new results under pressure bear out the results found previously in a narrower range. The pressure coefficient nearly doubles as the cleavage plane changes from parallel to perpendicular to the direction of flow, and except for orientations very close to parallel, the pressure coefficient is less at the higher temperatures. The effect of pressure is therefore to accentuate the excess of resistance across the cleavage planes over that parallel to the planes; one would perhaps be inclined to expect the opposite behavior.

Discussion of Various Questions Connected with the Thermo-Electric Behavior of Crystals.

On the Connection between the Symmetry of the Peltier and Thomson Heats. The conclusion reached in my former paper was that the *total* thermal e.m.f. of a crystal couple need not satisfy Kelvin's law. The total e.m.f. may be analyzed into contributions made by the Peltier heat and the Thomson heat. The conclusion reached in that paper was that the Thomson heat does satisfy Kelvin's law, but that the Peltier heat does not. This conclusion is also substantiated by the new data of this paper, since in the two cases which did not satisfy Kelvin's law, bismuth and tin, the deviation from linearity, which determines the second temperature derivative and so the Thomson heat, was nevertheless linear against $\cos^2\theta$ (or against the specific resistance). We now have to inquire whether this difference between the symmetry of P and σ is a real difference, or whether it is merely apparent, due to the unavoidably greater error in the determination of σ.* There is, of course, a thermodynamic connection between P and σ, namely:

$$\frac{d}{d\tau}\left(\frac{P}{\tau}\right) = \frac{\sigma}{\tau}.$$

Let us now assume that σ satisfies Kelvin's relation, so that we may put $\sigma = f_1(\tau) + f_2(\tau)\cos^2\theta$, and ask what is thereby involved about the symmetry of P. The equation may be integrated at once, giving:

$$\frac{P}{\tau} = \int_0^\tau \frac{f_1(\tau)}{\tau}\,d\tau + \cos^2\theta \int_0^\tau \frac{f_2(\tau)}{\tau}\,d\tau + \psi(\theta).$$

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* I had gone through considerations of this sort a number of years ago, but had not published them. The importance of the matter was recalled to my attention by a conversation with Dr. Goens in 1926.

$\psi(\theta)$ is an arbitrary function of integration, so that as far as the differential relation between P and σ is concerned, there is no reason why P should be linear in $\cos^2\theta$ at constant temperature merely because σ is.*

But now the third law of thermodynamics suggests that

$$\lim_{\tau=0}\left(\frac{P}{\tau}\right) = 0.$$

Call

$$\lim_{\tau=0}\int_0^\tau \frac{f_1(\tau)}{\tau}\,d\tau = A_1, \quad \text{and} \quad \lim_{\tau=0}\int_0^\tau \frac{f_2(\tau)}{\tau}\,d\tau = A_2,$$

and we have

$$\psi(\theta) = -A_1 - A_2\cos^2\theta,$$

so that in general:

$$P = \tau\left\{\left(\int_0^\tau \frac{f_1(\tau)}{\tau}\,d\tau - A_1\right) + \cos^2\theta\left(\int_0^\tau \frac{f_2(\tau)}{\tau}\,d\tau - A_2\right)\right\},$$

and at constant temperature this is linear in $\cos^2\theta$. It seems, then, that the third law demands that the Peltier heat satisfy Kelvin's law if the Thomson heat does, and the explanation of the apparent difference in symmetry found experimentally is the greater error of the Thomson heat.

Although the conclusion reached by the argument in the last paragraph appears to me very probable, I do not believe that it can be regarded as entirely rigorous. The question comes in inferring from the third law that P/τ goes to 0 at 0° Abs. Strictly, I suppose that the third law means only that the total entropy change accompanying any finite change in a condensed system at 0° Abs. should vanish. Now consider a couple at 0° Abs. composed of two crystal rods of different orientations. A current may be made to flow in such a system, but the total heat absorbed by the current is necessarily zero because a Peltier heat at one junction is exactly neutralized by a Peltier cold at the other. If the system is so arranged that the Peltier heat arises from the transfer of static charge so that this compensation does not occur, then the system is left charged after the passage of current, and the surface heats of charging make contributions to the total heat absorbed, so that again the total heat absorbed would be zero.

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* The integration as given is not rigorously exact, because in a non-cubic crystal in which the thermal expansion is different along the different axes $\cos\theta$ is a function of temperature, but the effect is very small.

On the Transverse Thermal Effect, and on Kelvin's Axiom of the Superposition of Heating Effects. Kelvin based his proof of the symmetry of the Peltier heat when a current leaves a crystal surface, and also his proof of the existence and the symmetry of the transverse heating effect,[14] on his so-called "Principle of the Superposition of Thermo-Electric Action," which is: "Each of any number of co-existing systems of electric currents produces the same reversible thermal effect in any locality as if it existed alone." In the paper[15] in which I announced my experimental discovery of the transverse effect,* I stated that Kelvin's principle was internally inconsistent, and that therefore his prediction of the existence of the transverse effect could not be justified. I now find that this statement of mine was not correct, but that if due account is taken of the transverse effect itself, the principle may be maintained and the symmetry relations given by Kelvin are a consequence of the principle.

To straighten the matter out, it will be necessary to go back a considerable distance and correct another erroneous deduction which I made before I knew of the existence of the transverse effect, and which is essentially modified by it.[16] Consider the junction between two metals, one of them a single crystal and the other isotropic, shown in Figure 19. It is evident that thermodynamics requires that the total heat absorbed when a current flows from the single crystal, C, to the isotropic metal, A, in the region at constant temperature, t, be independent of the precise details of the transition from one metal to the other in the region, so that the total heat absorbed by the current is the same for the two arrangements of the figure. I drew from this the conclusion that the Peltier heat at the surface depends only on the direction of flow of the current within the body of the crystal with respect to the crystal axis, and is independent of the orientation of the surface by which it leaves the crystal. It is now obvious that this conclusion is erroneous, because it neglects the transverse effect at the free surfaces of the crystal. Heat is absorbed when the current passes, not only at the surface *bc* which separates the two metals, but also at the transverse surfaces *ab* and *dc*. In the upper arrangement of Figure 19, the surfaces *ab* and *dc* are equal in extent, and the heat absorbed at *ab* is exactly neutralized by the heat given out at *dc*. But in the lower arrangement, this neutralization does not take place. It is evident that the correct theorem is that the total heats absorbed at the surface AB and BC, Figure 20, is equal to that which would be absorbed if the crystal were cut on AC. The heat absorbed at

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* When I wrote that paper I supposed that I was the first to find the transverse effect experimentally. Dr. Borelius has, however, recently kindly called my attention to the fact that he and A. E. Lindh described the experimental investigation of this effect in 1917 (Ann. Phys. 53, 97, 1917), in a paper entitled "Resistance, Peltier Heat and Electrical After-Effects, outside and within a Magnetic Field, particularly in Crystalline Bismuth." They made quantitative measurements. I have compared their numerical results with those to be expected from my own measurements of difference of thermo-electric properties of bismuth in different directions, and find agreement within about 30%. The agreement is as good as could be expected in view of the unfavorable dimensions of their specimen, and there can be no doubt that they actually observed the effect. I am sorry that I did not know of their work when I wrote my paper. It is interesting that since I wrote my paper the experimental detection of the effect has also been announced by Terada and Tsutsiu in Japan.

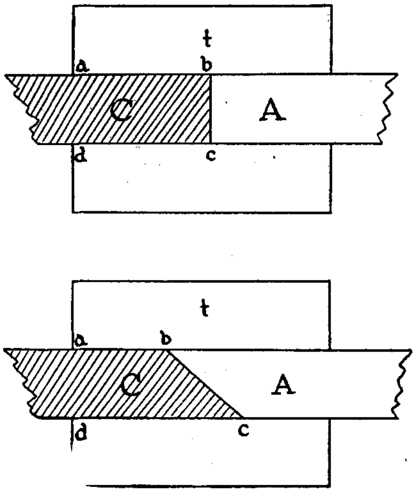

Figure 19. Two different methods of making the junction between an isotropic metal and a single crystal in a region at constant temperature. The total heat absorbed by unit quantity of electricity in passing from crystal to isotropic metal is the same in the two arrangements of the figure.

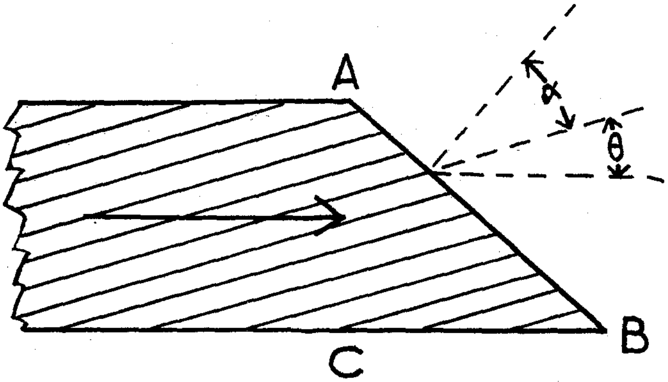

FIGURE 20. Diagram for the calculation of the relation between the reversible heats at various surfaces.

AB is proportional to the total quantity of electricity crossing the surface; hence if we take AC = 1, the cross section of the crystal equal to unity, and denote the current density by i, the heat will be merely proportional to i. This heat evidently involves two parameters: θ, the angle which the current within the crystal makes with the crystal axis, and α, the angle between the crystal axis and the normal to the surface. We may, therefore, put for the heat absorbed in unit time at AB, i P (θ, α). The transverse heat absorbed at BC by current parallel to the surface is evidently proportional to current density and the extent of the surface. Call the heat per unit current density per unit area T. T is evidently a function of the angle between the crystal axis and the surface. There is a sign convention that must be observed here. T reverses sign accordingly as the rotation through an acute angle which carries the current (or the surface CB) into the axis carries one inside or outside the crystal. There must also be a sign convention for α. The relation of heat equality is now evidently (since BC = tan $(\alpha + \theta)$):

$$P(\theta, \alpha) + T(\theta) \tan(\alpha + \theta) \text{ is independent of } \alpha.$$

The sum of these two expressions, or the value of P when $\alpha = -\theta$ $[= P(\theta, -\theta)]$, is what is given by the usual experiments with thermo-couples. In order to find *P* and *T* separately, it would be necessary to experiment with surfaces so large that the heat could be localized.

The conclusion of this analysis is, therefore, that the local heat absorbed when a current leaves a crystal and enters an isotropic conductor is a function both of the direction of flow with respect to the crystal axis in the crystal and the orientation of the surface. Now apply this to Kelvin's principle of the superposition of heating effects. My former proof that this cannot be correct rested on the assumption that the heat absorbed on crossing the surface AB by any of the components into which the current i was resolved was a function only of the direction of the component with respect to the crystal axis, and did not involve the orientation of AB with respect to the axis. But this assumption is erroneous. On using the correct and more general expression above, and also using Kelvin's explicit expressions for P and T

$$\left[\begin{array}{ll} P \equiv P(\theta, -\theta) & = P_{\perp} \cos^2\theta + P_{11} \sin^2\theta \\ T(\theta) & = (P_{\perp} - P_{11}) \cos\theta \sin\theta \end{array}\right],$$

everything is found to carry through consistently, the total heat is independent of the way in which the current is resolved into components, and the principle of superposition is logically possible.

The only question remaining is, therefore, whether as an experimental fact the symmetry relations of Kelvin are satisfied? We have seen that they are approximately satisfied, but in the case of bismuth and tin, the departures are apparently beyond experimental error. As far as I can see, there is no reason for thinking that the principle must necessarily be satisfied. It is without doubt true, however, that the principle holds to a first approximation in crystals hitherto examined, and in constructing a theory of these effects, it would doubtless be legitimate to begin with this assumption as a first approximation.

The fact that Kelvin's principle can be logically maintained involves a serious modification of a conclusion which I had drawn from my previous experimental results.[17] Granting my previous argument, neglecting the transverse effects, then it was easy to show that Kelvin's principle could be maintained only for one special method of resolving the current, namely parallel and perpendicular to the crystal axis. From the fact that experimentally Kelvin's symmetry relations were approximately satisfied I drew the conclusion that the electrons which constitute the current are actually compelled to travel either perpendicular or parallel to the crystal axis, by some mechanism more or less of the nature of channels. This conclusion must now be abandoned; from the approximate validity of Kelvin's relation no conclusion whatever can be drawn as to the paths of the electrons.

SUMMARY.

An improved method of making single metal crystals has been developed by which it is possible to cast from the same melt a number of single crystal rods of a wide range of orientation. An apparatus has been developed by which the thermal e.m.f. of 16 rods may be measured simultaneously, so that the only variable factor in the results is the crystal orientation. The thermal e.m.f. between 20° and 88° C of single crystal Zn, Cd, Sb, Sn, and Bi against Cu has been measured, and from the results the thermal e.m.f., Peltier heat, and difference of Thomson heats between rods of the same metal of different orientations is calculated. It is emphasized that in the case of Sb and Bi the heat absorbed by an electron when its direction of motion changes from perpendicular to the axis to parallel to the axis is very large, being respectively 0.2 and 0.4 of the energy of a gas molecule at the same temperature. This is a difficult point for any theory like the recent one of Sommerfeld, in which the part of the energy of an electron which varies with the temperature is supposed small compared with the classical amount. The Kelvin-Voigt law that thermal e.m.f. is a linear function of $\cos^2\theta$, θ being the angle between crystal axis and length of the rod, is verified for Zn, Cd, and Sb, but there are deviations for Sn and Bi which seem distinctly greater than possible experimental error, so that it would appear that in general the Kelvin-Voigt law is only an approximation. Within experimental error the Thomson heat does satisfy the law in all cases; it is only the contribution to total e.m.f. made by the Peltier heat that fails to satisfy the law within experimental error. In the case of Bi a special examination was made of the Kelvin-Voigt assumption that thermal e.m.f. has rotational symmetry about the crystal axis, and no appreciable deviation was found.

The specific resistance of these metals has also been studied as a function of orientation. The possible error from distortion is so great in the case of Cd that a special method of measurement had to be used. The Kelvin-Voigt symmetry relation for resistance is

satisfied within experimental error. Especial attention was given to the resistance of Bi, and the previous low value perpendicular to the cleavage plane verified. The pressure coefficient of resistance of Sb and Bi was redetermined, so that now this quantity is known over the entire range of orientation.

In the theoretical discussion it is shown that the third law of thermodynamics gives considerable plausibility, although not complete certainty, to the thesis that the symmetry of the Peltier and the Thomson heat must be the same; this means that the apparent experimental difference in the symmetry of the two heats is merely an effect of the much greater error in the Thomson heat. It is shown that, contrary to the statements previously made by me, Kelvin's axiom of the superposition of thermal effects of currents is not internally inconsistent, so that his proofs of the symmetry relations and of the existence of a transverse temperature effect are logically possible. Experiment, however, seems opposed to the truth of the axiom. The revised discussion further shows that the surface heat where a current leaves a metal crystal is a function both of the direction of flow with respect to the crystal axis within the crystal and of the orientation of the surface, contrary to my former statement. It is also shown that these crystal phenomena no longer offer a basis for a proof that the electrons must move in the crystal along something analogous to fixed channels.

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THE EFFECT OF PRESSURE ON THE RIGIDITY OF STEEL AND SEVERAL VARIETIES OF GLASS.

By P. W. Bridgman.

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INTRODUCTION.

THE change of rigidity or shearing modulus under pressure seems to have never been determined. The measurement of this effect is not easy, and for a number of years I have been searching for a suitable method. With the development of the sliding contact potentiometer method of measuring small displacements,[1] which I have already applied to the measurement of various small motions under pressure, it was evident that a means was at hand, and in this paper the application of such a method is described. When I first approached the problem, I felt that a method which would give merely the order of magnitude of the effect would be worth while, and my attack was conducted in this spirit. It seemed probable that the effect of pressure on the shearing modulus of a very compressible substance such as glass would be much higher than on such an incompressible substance as steel, and that to a first approximation the effect on steel could be neglected in comparison with that on glass. The first method adopted was a differential method, therefore, which gave the difference between the effects on steel and glass; by setting the effect on steel equal to zero, I hoped to get a first approximation to the effect on glass. The numerical magnitudes obtained were not, however, in the expected range, so that it appeared very questionable whether the assumption of a vanishingly small effect on steel was justifiable. It became necessary, therefore, to devise a second method allowing an absolute determination on the steel which had been used as the standard of comparison. It turned out that the magnitude of the effect on glass does not differ by a very large amount from that on steel, so that there was no real necessity in having made the first measurements on a substance of so comparatively little physical interest as glass. I plan now to extend the measurements to other metals besides steel.

EXPERIMENTAL METHODS.

In view of the smallness of the effect, it was necessary that the displacement, the change of which under pressure was to be measured, should be as large as possible. The helical spring satisfies this requirement, and also the necessary limitations imposed by the small size of the pressure apparatus, better than any other arrangement, such, for example, as the twist in a long slender rod. Furthermore, the extension of a helical spring of ordinary dimensions under load involves to a sufficient approximation only the geometrical dimensions and the shearing modulus, so that the required information could be calculated simply from the change of extension of the spring. The apparatus adopted for measuring the differential effect is shown in Figure 1. The glass spring G and the steel spring A are coupled together by a wire of manganin w. The springs are stretched to perhaps twice their normal length and attached at either end to a steel frame. The manganin wire slides over a contact C fixed to the steel frame, and there is a potentiometer terminal t attached to the wire. The entire assembly is exposed to hydrostatic pressure; if there is a change under pressure of the relative stiffness

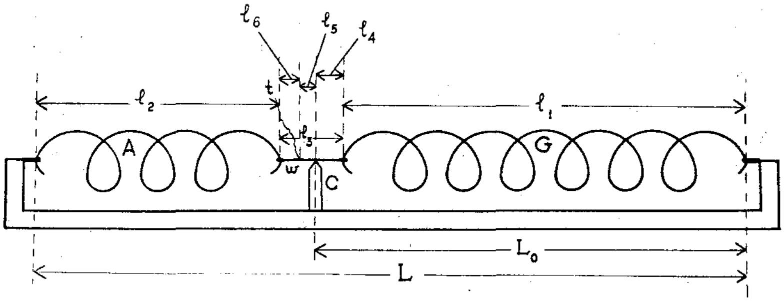

FIGURE 1. Scheme of the apparatus for determining the difference of the effect of pressure on the stiffness of two springs. If there is a change of relative stiffness under pressure, there is a shift of the point of coupling.

of the springs, there will be a motion of the point of coupling. If A becomes relatively stiffer than G, A shortens while G is lengthened, and conversely. The amount of this motion is measured in the conventional way on a potentiometer, current passing lengthwise in w, and the difference of potential between C and t being determined.

The method for measuring the absolute effect, shown in Figure 2, is even simpler. The spring of steel is suspended vertically, carrying at the bottom end a weight P, separated from the spring by a length of manganin wire w, which carries the lead wire t and, as in the first method, slides on the contact C rigidly connected to the upper support of the spring. The whole arrangement is immersed in the liquid by which hydrostatic pressure is transmitted. If the shearing modulus increases or decreases under pressure the spring shortens or lengthens accordingly, and the amount of motion is obtained electrically from the difference of potential between C and t.

It is obvious that a number of experimental precautions must be observed to make such apparatus function properly, and there are a number of corrections to be applied in making the calculations. Perhaps the most obvious and important difficulty is friction at the sliding contact C. The changes are so slight, the whole motion under 12000 kg. being of the order of 0.5 mm., that very slight friction here would entirely mask the effect. To avoid error from friction, the differential apparatus was mounted horizontally in a pressure cylinder which could be rotated through 180° about the horizontal axis of the springs. The rotatable pressure apparatus which this demands had been already constructed, and was the same as that which had been used in measuring the viscosity of liquids; it could be used again for this purpose with only minor alterations. While the pressure was being changed, the cylinder was rotated through 180°, so that the contact C, Figure 1, was over the wire. In this position the weight of the springs and the wire produce a slight amount of sag, so that the contact was broken, and there is therefore no frictional resistance to the springs taking the exact position of equilibrium. To make the reading, the cylinder was rotated back through 180°, the wire w dropping back into contact with C, without encountering any friction. The contact so made was often so light that there were various coherer effects from outside disturbances, which made readings difficult. The resistance of the contact was decreased by passing through it current from a small bell-ringing magneto, in the same way that I have previously done in making resistance measurements.

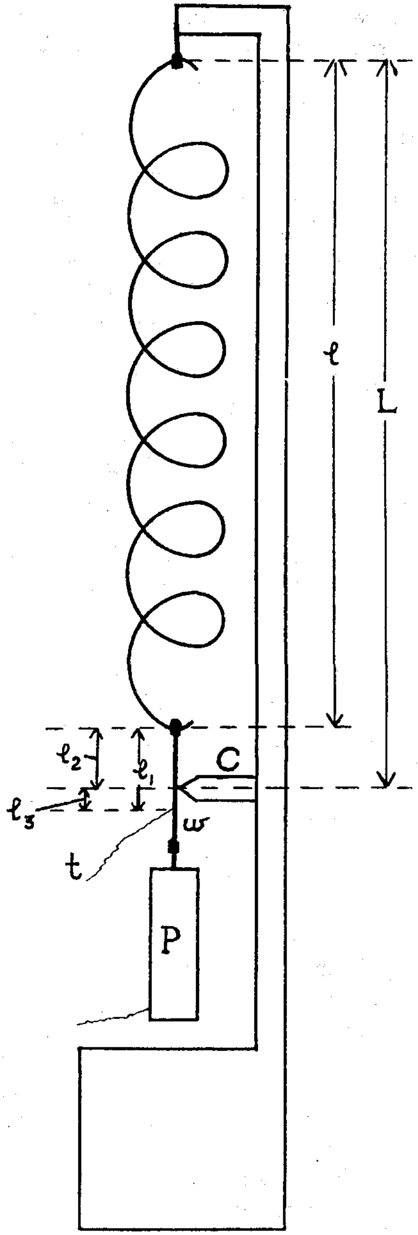
ℓ
L
ℓ_2
ℓ_1
C
ℓ_3
w
t
P

Figure 2. Scheme of the apparatus for determining the effect of pressure on the stiffness of a spring. The spring changes length when its stiffness changes.

In the case of the absolute apparatus (Figure 2), friction at the contact C during change of pressure was avoided by so mounting the apparatus that it could be very slightly tilted from the vertical, thus breaking contact.

It is evident that the various leads must be so flexible as to have negligible stiffness compared with the spring. The leads, t, were made of copper wire .004 cm. in diameter, wound into a long helix. The current connection at the glass spring end of the manganin wire in Figure 1 was a helix of this same wire, lying inside and concentric with the glass spring. The other current connection was made through the steel spring, which was grounded to the apparatus. In the absolute apparatus (Figure 2), the steel spring, grounded to the apparatus, was used for one current lead, and the other was a helix of the same fine wire grounded to the weight.

The measurements were made in a straight forward manner, and do not require detailed description. The pressure range was 12000 kg./cm.^{2} The temperature of most of the measurements was 30°, which was maintained with a thermostat. Rough measurements were made on two of the specimens to find whether there was any large temperature effect. Measurements were usually made at 2000 kg. intervals in the order: 0, 4000, 8000, 12000, 10000, 6000, 2000, and 0. The accuracy was not sufficient to justify an attempt to determine departures of the effect from linearity.

Corrections and Method of Calculation.

The corrections play an important part, so that it will be necessary to describe them in considerable detail. Consider first the differential method.

The force exerted by each spring is proportional to its extension from its unstressed length, and the condition of equilibrium when the two springs are coupled together is that the force exerted by the two springs should be equal. These conditions give

$$F_1 = k_1(l_1 - l_1'), \quad F_2 = k_2(l_2 - l_2'),$$

with the equilibrium condition

$$k_1(l_1 - l_1') = k_2(l_2 - l_2').$$

Besides the quantities which are sufficiently explained in Figure 1, l_1' and l_2' are the lengths of the two springs under no extensive force. The condition of equilibrium holds both before and after the application of pressure, so that we also have

$$(l_1 - l_1')dk_1 + k_1(dl_1 - dl_1') = (l_2 - l_2')dk_2 + k_2(dl_2 - dl_2'),$$

where the differentials are the increments produced by a given increment of pressure. Call now the linear compressibility of the two springs χ_1 and χ_2, that of the manganin χ_3, and that of the steel frame χ_4 (we choose the subscript 2 to denote the steel spring, so that for this particular apparatus χ_2 and χ_4 are to a sufficient approximation equal to each other). Using the condition that $l_1 + l_2 + l_3 = L$, and also that $dl_1' = - l_1'\chi_1 dp_1$, etc., a first equation is obtained:

$$(l_1 - l_1')dk_1 + k_1(dl_1 + l_1'\chi_1 dp)$$
$$= (l_2 - l_2')dk_2 + k_2\{(l_2'\chi_2 + l_3\chi_3 - L\chi_4)dp - dl_1\}. \quad (1)$$

In this equation, l_1, l_1', l_2, and l_2' are known from geometrical measurements of the apparatus; χ_2, χ_3, and χ_4 are known by independent experiment; k_1 and k_2 must be found by measurements of the extension of the spring under known weights; dk_2 is supposed known from the auxiliary experiments on the absolute pressure effect by the second method, and dp is the arbitrary increment of pressure, so that the only unknowns are dl_1 and dk_1. It is our next task to calculate dl_1 in terms of the measured change of resistance between the points C and t, thus permitting a determination of dk_1, and thus eventually of $d\mu_1$, the change of shearing modulus.

To connect dl_1 with the measured change of resistance, we have the relations $l_1 + l_4 = L_0$, and the similar equation obtained by differentiating this with respect to pressure; $l_4 + l_5 + l_6 = l_3$, and the corresponding differentiated equation; and $l_5 = R_0/\rho_0$ and the corresponding equation at pressure dp, which is

$$l_5 + dl_5 = \frac{R_0 + \Delta R_0}{\rho_0(1 + \alpha dp)}.$$

Combining these equations gives finally:

$$dl_1 = \frac{\Delta R_0}{\rho_0} + dp\left\{\chi_3(l_3 - l_6) - \chi_4 L_0 - \alpha\frac{R_0 + \Delta R_0}{\rho_0}\right\}. \quad (2)$$

In this equation, α is the pressure coefficient of the resistance of the wire w per unit length. It is the pressure coefficient of resistance measured in the usual way with terminals rigidly attached to the wire, corrected by the linear compressibility.

Let us next examine how to calculate dk_2 from the measurements with the absolute apparatus. By writing the equations $l + l_2 = L$, $l_2 = l_1 - l_3$, $l_3 = R_0 / \rho_0$, and the corresponding equations after pressure has been applied (see Figure 2 for notation), we find for the change of length of the spring in terms of the change of resistance, etc.:

$$dl = -\chi pL + \chi_m l_1 dp + \Delta R_0/\rho_0 - \{(R_0 + \Delta R_0)\alpha dp\}/\rho_0.$$

χ is the linear compressibility of the frame, and χ_m of the manganin wire.

To connect with the change in shearing modulus, we have the formula for a helical spring given by Miller:[2]

$$\mu = \frac{2Ps^3 \cos^2\alpha}{\pi a^4 \phi_0^2 (l - l_1)}.$$

Here

μ = shearing modulus of spring.
a = radius of the wire of the spring.
P = total longitudinal pull.
l' = initial length along axis of helix.
l = length under load along axis of helix.
α = angle between spires of spring and horizontal.
s = total length of spring wire ($l = s \sin \alpha$).
ϕ_0 = total angular twist of unstretched spring, in radians.

Differentiate this equation logarithmically, obtaining:

$$\frac{d\mu}{\mu} = \frac{dP}{P} + 3\frac{ds}{s} - 4\frac{da}{a} + 2\frac{d(\cos\alpha)}{\cos\alpha} - 2\frac{d\phi_0}{\phi_0} - \frac{d(l - l')}{l - l'}.$$

We have the relation

$$\frac{ds}{s} = \frac{da}{a} = -\chi dp.$$

If the material of which the helix is composed is in a state of ease, as we assume is approximately the case, then $d\phi_0 = 0$, because the effect of a change of pressure is merely a change of linear dimensions without a change of angle.

Using the connection between l and s, gives

$$\frac{dl}{l} = \frac{ds}{s} + \frac{\cos\alpha}{\sin\alpha} d\alpha.$$

This enables us finally to write:

$$\frac{d\mu}{\mu} = \frac{dP}{P} + \chi dp - 2\tan^2\alpha\left(\frac{dl}{l} + \chi dp\right) - \frac{dl}{l - l'} - \frac{l'\chi dp}{l - l'}.$$

On the right hand side everything may be found. dP is the change in the stretching force on the spring exerted by the weight. To calculate it, the density of the weight must be known and its compressibility, the density of the transmitting liquid, and its change under pressure.

Next to connect the change of shearing modulus, $d\mu$, with the change of stiffness of the spring, we find from the formula for μ that

$$k = \frac{\pi}{2} \frac{a^4 \mu \phi_0{}^2}{s^3 \cos^2 \alpha}.$$

Differentiate this logarithmically, using the various relations already employed, and we get

$$\frac{dk}{k} = -\chi dp(1 - 2\tan^2\alpha) + 2\tan^2\alpha \frac{dl}{l'} + \frac{d\mu}{\mu}. \qquad (3)$$

There are two relations of this kind, with the appropriate subscripts, one for the glass spring and one for the steel spring. For a given increment of pressure, $d\mu/\mu$ is, of course, determined entirely by the material, and does not depend on the particular geometry of the individual. For the steel spring, $d\mu/\mu$ is supposed known from the measurements with the absolute apparatus, so that for any particular experiment on the differential effect between glass and steel, dk/k for the steel spring (that is, dk_2/k_2) may be found. We are now in a position to return to equation 1, in which everything is now known except dk_1 for the glass spring. Solving this equation for dk_1, we now go back to equation (3) and solve for $d\mu/\mu$ for the glass, the quantity finally desired.

The magnitude of some of the correction terms is as great as that of the uncorrected effect. Thus in formula (2) for dl_1, if we call the uncorrected dl, $\Delta R_0/\rho_0$, the corrected dl will in some cases, where the pressure effect is comparatively small, be found to be of the opposite sign and greater numerically than the uncorrected dl. The largest part of the correction in this case arises from the term $\chi_4 L_0 dp$, that is, the term for the change of dimensions under pressure of the frame which holds the springs. More usually, however, the corrected dl differed from the uncorrected dl by something of the order of 20%. In finding $d\mu/\mu$ for steel by the direct method, we may call the term $dl/(l - l')$ the uncorrected effect. The actual effect was a little more than one half the uncorrected effect. By far the largest part of the correction here arises from the change with pressure of the buoyancy of the weight. The difference between $d\mu/\mu$ and

dk/k for the steel spring was of the order of 10%. In most cases $d\mu/\mu$ for the glass spring differed from its dk/k by something of the order of 20%.

Detailed Description of Experiments.

The Steel. Different steel springs were made for the absolute measurements and for each of the differential measurements. Greatest sensitiveness demands that the steel spring have such a stiffness that its total extension when coupled against the glass is the same as that of the glass, and since the stiffness of the glass springs varies greatly, it was necessary to vary the steel springs also. The general order of magnitude of the dimensions of the steel springs was: outside diameter 0.75 cm., length 1.5 cm., with 30 turns. The springs were wound in a lathe over mandrels of varying diameters; they were all made from the same coil of hard drawn piano wire, 0.025 cm. in diameter. Presumably the steel had a carbon content of about 1.25%.

The Glass. I am much indebted to Dr. Littleton, of the Research Laboratory of the Corning Glass Works, for providing six different varieties of glass. The compositions, supplied to me by W. C. Taylor, the chief chemist of the Corning Glass Works, were approximately as follows:

A is a potash lead silicate of very high lead content.
B is the same as Pyrex. A typical composition for this is:
SiO_2 81.4; B_2O_2 11.5; Na_2O 4.0; Al_2O_3 2.1; CaO 0.2; MgO 0.3.
C is a soda potash lime silicate.
D is a soda zinc borosilicate.
E is a soda lead borosilicate, opacified with calcium and aluminum fluorides.
F is a soda lime silicate containing a small percentage of boric oxide.

The glass was furnished in the form of solid rods of circular section, from 6 to 8 mm. in diameter. For converting these rods into helical springs I am very much indeed indebted to Professor Harold Pender, of the University of Pennsylvania, who developed the apparatus by which this was done, and to Dr. Charles Weyl, also of the University of Pennsylvania, who kindly supervised the actual work during the absence of Professor Pender in Europe. The glass rods were first drawn down to a diameter of the general order of 0.025 cm. The machine by which this is done consists essentially of a device by which the rod is fed through a brass casting maintained at the softening temperature of the glass, and is pulled out on the further side and wound up on a wheel (the rod is so flexible in small diameter that it can be wound on a wheel of large diameter without breaking), rotating at a definite speed with respect to the feeding speed. In this way a slender rod is produced whose section is a controllable fraction of the section of the original rod. The slender rod is now wound into a helix in another specially constructed machine, which consists of a core of carbon on which the rod is wound, the core being rotated inside a brass casting maintained at the proper temperature, and fed transversely as it rotates in order to give the helix the proper pitch. As furnished me, the helixes were open wound, with about 15 turns per cm. For my purpose it was better to convert these into closely wound helixes, which I did by slipping inside the helix a closely fitting core of aluminum, compressing the helix with a weight sliding on the core, and then slowly warming in an electric furnace to the softening temperature. By watching the heating, it is easy to stop the operation at such a point that the turns are closely in contact, but without sticking to each other. Finally, the ends of the helix were bent so as to give the proper means of attachment at the ends by very circumspect manipulation with a microscopic gas flame, issuing from a steel capillary such as is used in hypodermic needles.

Compressibility of the Glass. One of the corrections involves the linear compressibility of the glass. Since this correction may be important, and since the compressibility of glass varies greatly with the composition, it was necessary to make a direct determination of the compressibility of each variety of glass. This was done with the apparatus which I have described as "the lever apparatus for short specimens," and which has been used in measuring many other compressibilities.[3] The glass was cut from the original rod to a length of 2.7 cm., and the ends ground flat. In most cases the rod was of such a diameter that it could be used without further modification, but in one or two cases where the original diameter was too high, the diameter of these rods was reduced by grinding by the proper amount; I did this in order not to introduce the internal strains which might have been the result of drawing down to the proper size by heating. Although not immediately needed, the compressibility was determined at two temperatures, 30° and 75°; these measurements of the compressibility of glass have a certain interest for their own sake, and supplement determinations which I have already published for glass of other compositions.[4] The results are shown in Table I.

TABLE I.

COMPRESSIBILITY OF GLASS.

Designation of Glass	$\Delta V/V_0 = -(ap + bp^2)$, pressure in kg/cm²					
	30°			75°		
	a	*b*	Deviation*	*a*	*b*	Deviation*
A	30.54×10^{-7}	-24.3×10^{-12}	.09%	30.60×10^{-7}	-22.6×10^{-12}	.07%
B	30.12	+ 6.1	.10	29.72	+ 6.7	.11
C	24.69	−22.0	.44	25.44	−21.2	.42
D	25.97	+ 4.0	.10	26.30	+ .2	.08
E	27.78	+ 2.5	.10	27.71	+ 3.8	.10
F	23.29	− 6.1	.07	23.92	−10.1	.18

* This column shows the average deviations of a single reading from the smooth curve in terms of percentage of the maximum effect (at 12000 kg), and is an indication of accuracy.

Direct Measurement of the Shearing Modulus of Steel under Pressure. The steel spring for this determination had 48 turns, an outside diameter of 7.5 mm., and a length, when extended by the weight in a position to begin measurements, of 6.35 cm. The constant k was determined by hanging varying weights on the spring and observing the extension with a cathetometer; its value was 2.3, force being measured in gms., and extension in cm.

The liquid by which pressure is transmitted must be a perfect insulator, and should be relatively incompressible; furthermore its compressibility must be already known, or it must be specially determined. Kerosene was chosen as most nearly satisfying the several conditions. There is, however, a disadvantage in the use of kerosene, in that it becomes so viscous at high pressures that readings could not be made at pressures higher than 6000 kg., the contact refusing to open and shut when the apparatus was tilted at higher pressures. This defect could have been avoided by the use of petroleum ether, but the compressibility of this is much higher, and therefore the correction for the changing buoyancy of the weight is larger. Furthermore, the compressibility was not known with sufficient accuracy, so that, all things considered, the advantage seemed to lie with kerosene. The compressibility of kerosene was taken from a previous determination.[5]

The weight was composed of several metals, but was mostly of gold (7.073 gm. of gold, with 1.617 gm. of brass in the form of a holder for the gold, and 0.440 gm. of steel, the half weight of the spring). Gold was used in order to reduce the correction for the change of buoyancy with pressure to a low value. The correction was about 1% at the maximum pressure of 6000 kg. It is evident that with so small a correction the demands on the accuracy of the compressibility of the kerosene are not high. In addition to the correction arising from the compressibility of the kerosene, there is a correction arising from the compressibility of the metals of the weight; this was so small as to be just on the verge of the perceptible.

In Figure 3 all the observations are shown, the readings of the potentiometer slide wire from which the extension of the spring was obtained being plotted against the setting of the slider on the bridge by which pressure was measured in terms of the change of resistance of the manganin gauge. It will be seen that the relation between pressure and change of deformation of the spring is linear within a small margin of error. The change of length of the spring calculated from these readings was a shortening of 0.77 mm. at the

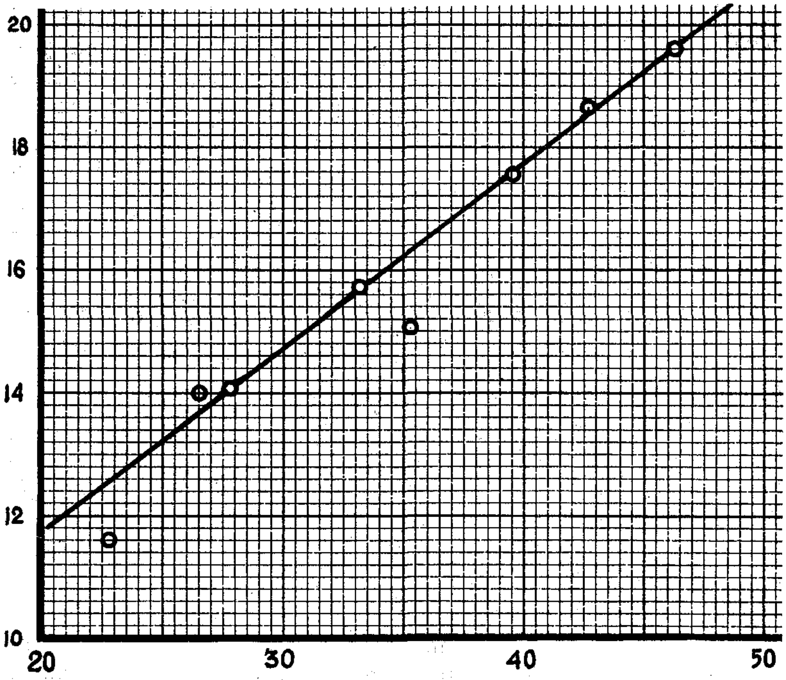

FIGURE 3. Reproduction of readings for the change of stiffness of a steel spring obtained with the apparatus of Figure 2. The ordinates are the settings in cm. on the potentiometer, and the abscissas the pressure in arbitrary units, the range of pressure between the extreme points being about 6000 kg. The extreme effect corresponds to an increase of shearing modulus of steel of about 1 per cent.

maximum pressure of 6000. From this, by means of the formulas already given, the change of shearing modulus is found to have the value:

$$\frac{1}{\mu}\left(\frac{\partial\mu}{\partial p}\right)_\tau = +\,2.16 \times 10^{-6},$$

pressure being expressed in kg./cm.^{2} This means an increase of a little over 2% under a pressure of 10000 kg./cm.^{2}

An attempt was made to obtain readings at 75° as well as at 30°. It was possible to run to higher pressures at 75°, because the viscosity of the kerosene is so much lower that there was no trouble from sticking contacts. Readings were made at 12000, 10000, and 8000, but at lower pressures nothing could be obtained because of electrical disturbances due to the chattering contacts of the thermal regulator. At higher pressures the viscosity of the kerosene prevented this difficulty. The difficulty was not serious, but to have remedied it, troublesome changes would have been necessary, and it did not seem worth while, especially since the subject is to be taken up again, and the value of the temperature coefficient, if it had been obtained, could not have been used in connection with the measurements on glass. The three points obtained at 75° lay on a straight line of 40% smaller slope than the points at 30°, so that it is probable that at 75° the increase of shearing modulus of steel under pressure is materially less than at 30°, but one cannot be certain of this until all the corrections at the higher temperature have been more carefully determined.

Differential Measurements on Glass. The differential measurements were made, as has been already explained, with the axis of the springs in a horizontal position, so that the correction for the changing buoyancy of the transmitting liquid could be neglected. This made it possible to transmit pressure with petroleum ether, which offers the advantage that the increase of viscosity under pressure is so small that the measurements could be pushed to 12000. Readings were usually made only at 30°, but for two of the varieties of glass, readings were also made at 75°. The results for the various kinds of glass differed greatly in regularity; this is partly to be explained by differences in the absolute magnitude of the effect, and partly by the dimensions of the springs, the springs which were wound out of smaller diameter rods being obviously more sensitive to disturbing effects. A set of observations is reproduced in Figure 4; this is one of the better ones, although not the best.

In order to get as much idea as possible of the elastic properties of the various kinds of glass, the shearing modulus was calculated from the constants of the springs. The accuracy of this determination is low, and the results must be used only for orienting purposes. The chief source of error was in the diameter of the glass. This enters as the fourth power into the formula for the modulus, and could be determined only with low accuracy. The diameter of the actual glass of the spring could not be conveniently measured; instead I determined the mean diameter of several of the straight lengths left over from the winding operation, and these might vary by as much as 10% in one or two cases. As an additional check, the absolute shearing modulus of the steel was determined from the constants of the steel springs. There were six of these springs,

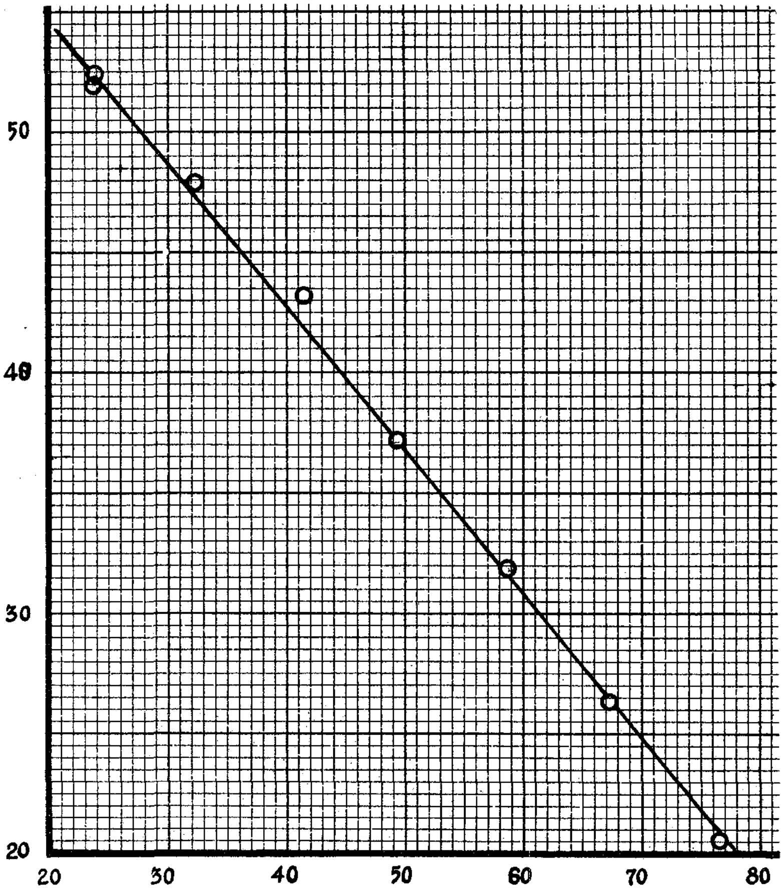

FIGURE 4. Reproduction of readings for the change of relative stiffness of steel and glass E, obtained with the apparatus of Figure 1. The ordinates are potentiometer settings in cm,, and the abscissas are pressures in arbitrary units, the range of pressure between the extreme points being about 12000 kg. The extreme effect corresponds to a decrease of rigidity of the glass with respect to the steel of about 10 per cent.

all of different dimensions, which were used to give the constant. The diameter of the steel wire may be safely assumed uniform since the wire was all from the same spool, but the chief source of error with the steel was in the outside diameter of the helix, which enters as the third power into the modulus. The outside diameter was measured with a micrometer, but since the spring is very flexible, it was difficult to be sure that it was correctly obtained. The following values were found for the shearing modulus of the steel springs used with glass springs A, B, C, D, E, and F respectively: 6.77×10^{11}, 6.94, 7.99, 8.02, 7.30, and 7.45, average 7.4×10^{11}. Kaye and Laby's Tables gives for the shearing modulus of steel of 1% carbon content 8.12×10^{11}.

The final results obtained with the different varieties of glass are shown in Table II. The negative sign of the effect was a surprise to me; by very crude analogy with the action of pressure in enormously increasing the viscosity of liquids, I had expected a rather large increase of shearing modulus. On reflection, however, the negative sign does not seem so strange in view of the fact that the compressibility of a number of different kinds of glass has been shown to increase with increasing pressure.[4] In fact, turning to the table of compressibilities, it will be seen that just those glasses, B, D, and E, which have the abnormal increase of compressibility with pressure also have the largest decrease of shearing modulus, and the two glasses, A and C, which are most normal in their decrease of compressibility, also have the smallest numerical change of shearing modulus under pressure.

The temperature effect was measured on samples B and E. The only conclusion that can be drawn is that the effect is not large. B at 75° showed a displacement of the contact point 8% less than at 30°, and the displacement of the contact of E at 75° was 6% greater than at 30°. The effect of temperature on the various corrections was not determined, so that the statement above, that the temperature coefficient of the pressure coefficient is small, seems to be all that is justified.

EFFECT OF PRESSURE ON OTHER ELASTIC CONSTANTS.

Since an isotropic substance has only two independent elastic constants, we are now in a position to find the effect of pressure on Young's modulus, E, and Poisson's ratio, σ. We have the relations:

$$\sigma = \frac{3 - 2\mu c}{6 + 2\mu c},$$

$$E = \frac{9\mu}{3 + \mu c}.$$

TABLE II.

Designation of Glass	Shearing Modulus of Glass, in Abs. C. G. S. units	Pressure Coefficient of Shearing Modulus in kg. units, $\frac{1}{\mu}\left(\frac{\partial\mu}{\partial p}\right)_\tau$.	Probable Accuracy as Shown by Deviations of a Single Reading from a Smooth Curve	
			(a) in terms of percentage of maximum measured effect	(b) in terms of cms. displacement of contact point
A	2.60×10^{11}	-0.62×10^{-6}	15.2%	.0012 cm.
B	2.31	-8.45	.8	.0009
C	2.33	-2.15	4.8	.0017
D	3.14	-8.02	5.4	.0032
E	2.54	-8.80	1.4	.0010
F	(6.64)	-3.86	16.	.0021

Here μ is the shearing modulus, as before, and c is the volume compressibility, defined by the relation

$$c = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_\tau .$$

Differentiation of these equations gives:

$$\frac{1}{\sigma}\frac{d\sigma}{dp} = \frac{-9\mu c}{(3-2\mu c)(3+\mu c)}\left\{\frac{1}{c}\frac{dc}{dp}+\frac{1}{\mu}\frac{d\mu}{dp}\right\},$$

$$\frac{1}{E}\frac{dE}{dp} = \frac{1}{3+\mu c}\left\{3\frac{1}{\mu}\frac{d\mu}{dp} - \mu c\frac{1}{c}\frac{dc}{dp}\right\}.$$

$1/c\ dc/dp$ may be found from the formulas for compressibility, as in Table I, to have the value

$$a + \frac{2b}{a} \cdot$$

$\frac{1}{\mu}\frac{d\mu}{dp}$ is known, so that all the quantities are known which are required for the calculation of the pressure derivatives.

The velocity of a wave of shear is also of interest. This is given by

$$w = \sqrt{\frac{\mu}{\rho}},$$

whence:

$$\frac{1}{w}\frac{dw}{dp} = \frac{1}{2}\left[\frac{1}{\mu}\frac{d\mu}{dp} - c\right].$$

The results are contained in Table III. The calculation was not carried through for glass F because of the great uncertainty in the absolute value of its shearing modulus.

In general it appears that the changes in the various elastic constants are of the same order of magnitude as the change of compressibility already found. There is no general rule about the sign. The fact that the velocity of a wave of shear in glass decreases with increasing pressure may be of some geological interest.

I am indebted to my assistant, Mr. Stephen Stark, for most of the readings of this paper.

The Jefferson Physical Laboratory,
Harvard University, Cambridge, Mass.

TABLE III.

Effect of Pressure on Various Elastic Constants.

Substance	E Young's Modulus Abs. C. G. S.	σ Poisson's Ratio	$\frac{1}{E}\frac{dE}{dp}$ (p is in kg. /cm.^{2})	$\frac{1}{\sigma}\frac{d\sigma}{dp}$	$\frac{1}{w}\frac{dw}{dp}$ w is velocity of a wave of shear
Steel	20.8×10^{11}	.30	$+2.8 \times 10^{-6}$	$+2.6 \times 10^{-6}$	$+.78 \times 10^{-6}$
Glass A	6.2	.19	$+2.2$	$+18.1$	-1.8
" B	5.6	.22	-8.2	$+1.5$	-5.7
" C	5.9	.26	$+0.7$	$+13.7$	-2.3
" D	7.4	.18	-7.5	$+3.3$	-5.3
" E	6.2	.21	-8.0	$+4.6$	-5.8

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By Edward V. Huntington.

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TABLES OF LAGRANGEAN COEFFICIENTS FOR INTERPOLATING WITHOUT DIFFERENCES.

BY EDWARD V. HUNTINGTON.

TABLES OF LAGRANGEAN COEFFICIENTS FOR INTERPOLATING WITHOUT DIFFERENCES.*

By Edward V. Huntington.

Introduction.

The tables presented in this paper are intended to facilitate the process of high-order interpolation in any given table in which the argument advances by equal intervals, provided a computing machine is available.

The tables in Part I are for use in interpolating for single or isolated values. Those in Part II are for use in continuous or systematic interpolation. In either case the result is obtained directly from the given tabulated values, without the necessity of forming columns of differences. Unless the columns of differences are desired for other purposes, the saving of time thus effected will be found to be substantial.

The tables are based on the well-known formula of Lagrange (see Appendix below) and are believed to be the first of their kind which are sufficiently extensive for practical use.** The figures given are exact in all cases, except in Tables 5, 5a, and 5b; in these tables, the entries in every tenth line are exact, the other entries being correct to eight places of decimals. For most work, a smaller number of decimals will of course be sufficient.

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* Presented to the American Academy of Arts and Sciences, Boston, November 14, 1928. (An abstract of this paper was presented to the American Mathematical Society, April 28, 1923.)

** A very brief table of this sort is given by J. Burn and E. H. Brown in *Elements of Finite Differences*, 1915, page 31. Excellent tables for use with the formulas of Newton, Stirling, Bessel, Everett, etc., are available in many places (see, for example, J. W. Glover's *Tables of Applied Mathematics*, 1923); but all these formulas require the preliminary computation of the columns of differences. An extended study of the whole theory will be found in Karl Pearson's Tracts for Computers, No. II, On the Construction of Tables and on Interpolation, Part I, Uni-variate Tables (Cambridge University Press, 1920). The coefficients in Pearson's formulas v to xii ("Four-point Mid-panel Lagrangians"), on page 31, are the same as the entries opposite $n = 0.6$, 0.8, 0.4, 0.2 in the present Tables 3 and 3a; and the entries in Pearson's Table A ("Six-ordinate Mid-panel Lagrangians"), facing page 45, are the same as the entries opposite $n = 0.2$, 0.4, 0.6, 0.8 in the present Tables 5, 5a, and 5b. Pearson also gives, at similar intervals, a Table B for "Eight-Entry Lagrangians" and a Table C for "Ten Entry Lagrangians," facing page 46.

PART I. SINGLE INTERPOLATIONS.

Let . . . Y_{-1}, Y_0, Y_1, Y_2, . . . be a given set of tabulated values, corresponding to the given arguments . . . X_{-1}, X_0, X_1, X_2, . . . (these latter values being supposed to advance by equal intervals); and let it be required to find by interpolation the value Y_n corresponding to any given value X_n between X_0 and X_1. (Here n is any positive fraction between 0 and 1.) If 3rd degree interpolation is sufficiently accurate, use table 3 or 3a; if 5th degree interpolation is necessary, use table 5 or 5a or 5b; the process being a very simple one, as follows:

*Table 3. Third degree interpolation in main part of table.**

If the interval between X_0 and X_1 lies in the main part of the given table (so that values on both sides of Y_0 and Y_1 are known), find from table 3 the factors A_{-1}, A_0, A_1, A_2 corresponding to the given value of n. Multiply these factors by the given values Y_{-1}, Y_0, Y_1, Y_2 respectively, and add the products. The result will be Y_n, according to the formula

$$Y_n = A_{-1}Y_{-1} + A_0Y_0 + A_1Y_1 + A_2Y_2$$

(where the A's are functions of n). The computing machine will accumulate the products as formed—the positive ones by addition, the negative ones by subtraction—so that nothing need be written down except the final result, Y_n.

To test the precision of the result of this 3rd degree interpolation, first find the value of $D_4 = Y_{-1} - 4Y_0 + 6Y_1 - 4Y_2 + Y_3$. Then the correction E_3 which would have to be added to the result of the 3rd degree interpolation to make it equal the result of a 4th degree interpolation will certainly lie between 0 and 0.024 D_4.

[More precisely, $E_3 = (n + 1)_4 D_4$, where

$$(n + 1)_4 = (n + 1)\ (n)\ (n - 1)\ (n - 2)/(4!).$$

From a table of binomial coefficients it is seen that the values of $(n + 1)_4$ for values of n between 0 and 1 will be positive and less than 0.024.]

---

* Here by "third degree interpolation" is meant interpolation by means of a polynomial of the third degree, the coefficients being so chosen that the curve representing the polynomial passes exactly through the four points nearest the interval in question; in this case through the four points (X_{-1}, Y_{-1}), (X_0, Y_0), (X_1, Y_1), (X_2, Y_2). Similarly, a "fifth degree interpolation" means interpolation by a polynomial of the fifth degree, passing exactly through six points.

Table 3a. Third degree interpolation at beginning of table.

If the interval from X_0 to X_1 is the first interval in the given table, the value Y_{-1} is not known, and Table 3 cannot be used. In this case, find the factors a_0, a_1, a_2, a_3, from Table 3a, and compute the required value Y_n from the formula

$$Y_n = a_0 Y_0 + a_1 Y_1 + a_2 Y_2 + a_3 Y_3.$$

To test the precision in this case, first find $d_4 = Y_0 - 4Y_1 + 6Y_2 - 4Y_3 + Y_4$. Then the correction e_3 which would have to be added to the result of the 3rd degree interpolation to make it equal to the result of a 4th degree interpolation will lie between 0 and $-$ 0.042 d_4.

[More precisely, $e_3 = (n)_4 d_4$ where

$$(n)_4 = (n)\ (n-1)\ (n-2)\ (n-3)/4!]$$

Note.—If the interval between X_0 to X_1 is the *last* interval in the given table, we have only to imagine the table written in the reverse order (so that X_1 becomes X_0, X_0 becomes X_1, etc.) and to use $1 - n$ in place of n.

Table 5. Fifth degree interpolation in main part of table.

If fifth degree interpolation is required, and the interval between X_0 and X_1 lies in the main part of the given table, we have

$$Y_n = B_{-2} Y_{-2} + B_{-1} Y_{-1} + B_0 Y_0 + B_1 Y_1 + B_2 Y_2 + B_3 Y_3,$$

where the factors B_{-2}, B_{-1}, B_0, B_1, B_2, B_3, are taken from Table 5.

The correction E_5 which would have to be added to the result of this fifth degree interpolation to make it equal to the result of a sixth degree interpolation lies between 0 and $-$ 0.005 D_6, where $D_6 = Y_{-2} - 6Y_{-1} + 15Y_0 - 20Y_1 + 15Y_2 - 6Y_3 + Y_4$.

[More precisely, $E_5 = (n+2)_6 D_6$, where

$$(n+2)_6 = (n+2)\ (n+1)\ (n)\ (n-1)\ (n-2)\ (n-3)\ /\ 6!]$$

Table 5a. Fifth degree interpolation at beginning of table.

If the interval between X_0 and X_1 is the first interval in the table, we have

$$Y_n = b_0 Y_0 + b_1 Y_1 + b_2 Y_2 + b_3 Y_3 + b_4 Y_4 + b_5 Y_5,$$

where the factors b_0, b_1, b_2, b_3, b_4, b_5, are taken from Table 5a.

The "correction" e_5 now lies between 0 and $-$ 0.024 d_6 where

$$d_6 = Y_0 - 6Y_1 + 15Y_2 - 20Y_3 + 15Y_4 - 6Y_5 + Y_6$$

[More precisely, $e_5 = (n)_6 d_6$, where

$$(n)_6 = (n)(n-1)(n-2)(n-3)(n-4)(n-5)/6!]$$

Table 5b. Fifth degree interpolation in second interval of table.

If the interval between X_0 and X_1 is the second interval in the table, we have

$$Y_n = c_1 Y_{-1} + c_0 Y_0 + c_1 Y_1 + c_2 Y_2 + c_3 Y_3 + c_4 Y_4$$

where the factors c_{-1}, c_0, c_1, c_2, c_3, c_4 are taken from Table 5b.

The "correction" e_5' now lies between 0 and 0.007 d_6', where

$$d_6' = Y_{-1} - 6Y_0 + 15Y_1 - 20Y_2 + 15Y_3 - 6Y_4 + Y_5$$

[More precisely, $e_5' = (n+1)_6 d_6'$, where

$$(n+1)_6 = (n+1)(n)(n-1)(n-2)(n-3)(n-4)/6!]$$

Note. If the interval in question is the last, or next to the last, interval in the table, we have only to imagine the table written in the reverse order, and to use $1 - n$ in place of n.

PART II. CONTINUOUS INTERPOLATION.

Let it be required to expand a given table, by reducing the tabular interval to 1/10 of its size.

It will be found desirable to begin by interpolating a mid-value in each interval, using Table 3 or 3a (for third degree interpolation) or Table 5 or 5a or 5b (for fifth degree interpolation), with $n = 0.50$, and then to subdivide the resulting intervals into fifths.*

In subdividing into fifths, it is well to use a cumulative process (Table 30 or 30a, or Table 50 or 50a or 50b), in which each result is left in the machine to serve as the starting point of the next computation. By this method, a valuable check is secured at the end of each block of five.

* Since on a modern machine division is as easy as multiplication, it may often be convenient to replace the entries opposite $n = 0.5$ in Tables 3 and 3a by the equivalent common fractions, as follows:

(Table 3) $Y_{0.5} = (-Y_{-1} + 9Y_0 + 9Y_1 - Y_2)/16$

(Table 3a) $Y_{0.5} = (5Y_0 + 15Y_1 - 5Y_2 + Y_3)/16$

Many formulas of this type are given in Pearson's Tract.

APPENDIX.

The theory of the Lagrangean interpolation formula is extremely simple. For example, for third degree interpolation in the main part of the table, we form the following function of n:

$$\begin{aligned} F(n) = {} & [-(n)\ (n-1)\ (n-2)\ /\ (3!)]\ Y_{-1} \\ & + [(n+1)\ (n-1)(n-2)\ /\ (1!\,2!)]\ Y_0 \\ & + [-(n+1)\ (n)\ (n-2)\ /\ (2!\ 1!)]\ Y_1 \\ & + [(n+1)\ (n)\ (n-1)\ /\ (3!)]\ Y_2 \end{aligned}$$

where Y_{-1}, Y_0, Y_1, Y_2 are given tabular values.

By inspection, we see that this expression is a polynomial of the third degree in n; also, that when $n = -1$, $F(n) = Y_{-1}$; when $n = 0$, $F(n) = Y_0$; when $n = 1$, $F(n) = Y_1$; and when $n = 2$, $F(n) = Y_2$. Hence, when n is a fraction between 0 and 1, $F(n)$ will be a suitable value to take as the interpolated value Y_n. Now the quantities in brackets, [ ], depend only on n, and can be tabulated once for all; these are precisely the quantities given as A_{-1}, A_0, A_1, A_2 in Table 3.

Similarly, for third degree interpolation at the beginning of the table, we form the following function of n:

$$\begin{aligned} F(n) = {} & [-(n-1)\ (n-2)\ (n-3)\ /\ (3!)]\ Y_0 \\ & + [(n)\ (n-2)\ (n-3)\ /\ (1!\,2!)]\ Y_1 \\ & + [-(n)\ (n-1)\ (n-3)\ /\ (2!\,1!)]\ Y_2 \\ & + [(n)\ (n-1)\ (n-2)\ /\ (3)]\ Y_3 \end{aligned}$$

and tabulate the coefficients in brackets as a_0, a_1, a_2, a_3 in Table 3a.

If we build analogous expressions for the fourth degree, and subtract from them the expressions just written, we obtain the "corrections" given above.

The formulas underlying Tables 5, 5a, and 5b are obtained in a similar way.

TABLE 3.

For 3rd Degree Interpolation, in Main Part of Table.

n	A_{-1}	A_0	A_1	A_2
0.00	−0.0000	1.0000	0.0000	−0.0000
.01	−.0032835	0.9945005	.0100495	−.0016665
.02	−.0064680	.9896040	.0201960	−.0033320
.03	−.0095545	.9841135	.0304365	−.0049955
.04	−.0125440	.9784320	.0407680	−.0066560
.05	−.0154375	.9725625	.0511875	−.0083125
.06	−.0182360	.9665080	.0616920	−.0099640
.07	−.0209405	.9602715	.0722785	−.0116095
.08	−.0235520	.9538560	.0829440	−.0132480
.09	−.0260715	.9472645	.0936855	−.0148785
0.10	−0.0285	0.9405	0.1045	−0.0165
.11	−.0308385	.9335655	.1153845	−.0181115
.12	−.0330880	.9264640	.1263360	−.0197120
.13	−.0352495	.9191985	.1373515	−.0213005
.14	−.0373240	.9117720	.1484280	−.0228760
.15	−.0393125	.9041875	.1595625	−.0244375
.16	−.0412160	.8964480	.1707520	−.0259840
.17	−.0430355	.8885565	.1819935	−.0275145
.18	−.0447720	.8805160	.1932840	−.0290280
.19	−.0464265	.8723295	.2046205	−.0305235
0.20	−0.0480	0.8640	0.2160	−0.0320
.21	−.0494935	.8555305	.2274195	−.0334565
.22	−.0509080	.8464240	.2388760	−.0348920
.23	−.0522445	.8381835	.2503665	−.0363055
.24	−.0535040	.8293120	.2618880	−.0376960
.25	−.0546875	.8203125	.2734375	−.0390625
.26	−.0557960	.8111880	.2850120	−.0404040
.27	−.0568305	.8019415	.2966085	−.0417195
.28	−.0577920	.7925760	.3082240	−.0430080
.29	−.0586815	.7830945	.3189555	−.0442685
0.30	−0.0595	0.7735	0.3315	−0.0455
.31	−.0602485	.7637955	.3431545	−.0467015
.32	−.0609280	.7539840	.3548160	−.0478720
.33	−.0615395	.7440685	.3664815	−.0490105
.34	−.0620840	.7340520	.3781480	−.0501160
.35	−.0625625	.7239375	.3898125	−.0511875
.36	−.0629760	.7137280	.4014720	−.0522240
.37	−.0633255	.7034265	.4131235	−.0532245
.38	−.0636120	.6930360	.4247640	−.0541880
.39	−.0638365	.6825595	.4363905	−.0551135
0.40	−0.0640	0.6720	0.4480	−0.0560
.41	−.0641035	.6613605	.4595895	−.0568465
.42	−.0641480	.6506440	.4711560	−.0576520
.43	−.0641345	.6398535	.4826965	−.0584155
.44	−.0640640	.6289920	.4942080	−.0591360
.45	−.0639375	.6180625	.5056875	−.0598125
.46	−.0637560	.6070680	.5171320	−.0604440
.47	−.0635205	.5960115	.5285385	−.0610295
.48	−.0632320	.5848960	.5399040	−.0615680
.49	−.0628915	.5737245	.5512255	−.0620585
0.50	−0.0625	0.5625	0.5625	−0.0625

$$Y_n = A_{-1}Y_{-1} + A_0Y_0 + A_1Y_1 + A_2Y_2$$

TABLE 3 (*continued*).

n	A_{-1}	A_0	A_1	A_2
0.50	−0.0625	0.5625	0.5625	−0.0625
.51	−.0620585	.5512255	.5737245	−.0628915
.52	−.0615680	.5399040	.5848960	−.0632320
.53	−.0610295	.5285385	.5960115	−.0635205
.54	−.0604440	.5171320	.6070680	−.0637560
.55	−.0598125	.5056875	.6180625	−.0639375
.56	−.0591360	.4942080	.6289920	−.0640640
.57	−.0584155	.4826965	.6398535	−.0641345
.58	−.0576520	.4711560	.6506440	−.0641480
.59	−.0568465	.4595895	.6613605	−.0641035
0.60	−0.0560	0.4480	0.6720	−0.0640
.61	−.0551135	.4363905	.6825595	−.0638365
.62	−.0541880	.4247640	.6930360	−.0636120
.63	−.0532245	.4131235	.7034265	−.0633255
.64	−.0522240	.4014720	.7137280	−.0629760
.65	−.0511875	.3898125	.7239375	−.0625625
.66	−.0501160	.3781480	.7340520	−.0620840
.67	−.0490105	.3664815	.7440685	−.0615395
.68	−.0478720	.3548160	.7539840	−.0609280
.69	−.0467015	.3431545	.7637955	−.0602485
0.70	−0.0455	0.3315	0.7735	−0.0595
.71	−.0442685	.3189555	.7830945	−.0586815
.72	−.0430080	.3082240	.7925760	−.0577920
.73	−.0417195	.2966085	.8019415	−.0568305
.74	−.0404040	.2850120	.8111880	−.0557960
.75	−.0390625	.2734375	.8203125	−.0546875
.76	−.0376960	.2618880	.8293120	−.0535040
.77	−.0363055	.2503665	.8381835	−.0522445
.78	−.0348920	.2388760	.8469240	−.0509080
.79	−.0334565	.2274195	.8555305	−.0494935
0.80	−0.0320	0.2160	0.8640	−0.0480
.81	−.0305235	.2046205	.8723295	−.0464265
.82	−.0290280	.1932840	.8805160	−.0447720
.83	−.0275145	.1819935	.8885565	−.0430355
.84	−.0259840	.1707520	.8964480	−.0412160
.85	−.0244375	.1595625	.9041875	−.0393125
.86	−.0228760	.1484280	.9117720	−.0373240
.87	−.0213005	.1373515	.9191985	−.0352495
.88	−.0197120	.1263360	.9264640	−.0330880
.89	−.0181115	.1153845	.9335655	−.0308385
0.90	−0.0165	0.1045	0.9405	−0.0285
.91	−.0148785	.0936855	.9472645	−.0260715
.92	−.0132480	.0829440	.9538560	−.0235520
.93	−.0116095	.0722785	.9602715	−.0209405
.94	−.0099640	.0616920	.9665080	−.0182365
.95	−.0083125	.0511875	.9725625	−.0154375
.96	−.0066560	.0407680	.9784320	−.0125440
.97	−.0049955	.0304365	.9841135	−.0095545
.98	−.0033320	.0201960	.9896040	−.0064680
.99	−.0016665	.0100495	0.9945005	−.0032835
1.00	−0.0000	0.0000	1.0000	−0.0000

Correction required to take into account 4th differences lies between 0 and $+.024\,[Y_{-1} - 4Y_0 + 6Y_1 - 4Y_2 + Y_3.]$

TABLE 3a.

For 3rd Degree Interpolation, at Beginning of Table.

n	a_0	a_1	a_2	a_3
0.00	1.0000	0.0000	−0.0000	0.0000
.01	.9817665	.0297505	−.0148005	.0032835
.02	.9637320	.0590040	−.0292040	.0064680
.03	.9458955	.0877635	−.0432135	.0095545
.04	.9282560	.1160320	−.0568320	.0125440
.05	.9108125	.1438125	−.0700625	.0154375
.06	.8935640	.1711080	−.0829080	.0182360
.07	.8765095	.1979215	−.0953715	.0209405
.08	.8596480	.2242560	−.1074560	.0235520
.09	.8429785	.2501145	−.1191645	.0260715
0.10	0.8265	0.2755	−0.1305	0.0285
.11	.8102115	.3004155	−.1414655	.0308385
.12	.7941120	.3248640	−.1520640	.0330880
.13	.7782005	.3488485	−.1622985	.0352495
.14	.7624760	.3723720	−.1721720	.0373240
.15	.7469375	.3954375	−.1816875	.0393125
.16	.7315840	.4180480	−.1908480	.0412160
.17	.7164145	.4402065	−.1996565	.0430355
.18	.7014280	.4619160	−.2081160	.0447720
.19	.6866235	.4831795	−.2162295	.0464265
0.20	0.6720	0.5040	−0.2240	0.0480
.21	.6575565	.5243805	−.2314305	.0494935
.22	.6432920	.5443240	−.2385240	.0509080
.23	.6292055	.5638335	−.2452835	.0522445
.24	.6152960	.5829120	−.2517120	.0535040
.25	.6015625	.6015625	−.2578125	.0546875
.26	.5880040	.6197880	−.2635880	.0557960
.27	.5746195	.6375915	−.2690415	.0568305
.28	.5614080	.6549760	−.2741760	.0577920
.29	.5483685	.6719445	−.2789945	.0586815
0.30	0.5355	0.6885	−0.2835	0.0595
.31	.5228015	.7046455	−.2876955	.0602485
.32	.5102720	.7203840	−.2915840	.0609280
.33	.4979105	.7357185	−.2951685	.0615395
.34	.4857160	.7506520	−.2984520	.0620840
.35	.4736875	.7651875	−.3014375	.0625625
.36	.4618240	.7793280	−.3041280	.0629760
.37	.4501245	.7930765	−.3065265	.0633255
.38	.4385880	.8064360	−.3086360	.0636120
.39	.4272135	.8194095	−.3104595	.0638365
0.40	0.4160	0.8320	−0.3120	0.0640
.41	.4049465	.8442105	−.3132605	.0641035
.42	.3940520	.8560440	−.3142440	.0641480
.43	.3833155	.8675035	−.3149535	.0641345
.44	.3727360	.8785920	−.3153920	.0640640
.45	.3623125	.8893125	−.3155625	.0639375
.46	.3520440	.8996680	−.3154680	.0637560
.47	.3419295	.9096615	−.3151115	.0635205
.48	.3319680	.9192960	−.3144960	.0632320
.49	.3221585	.9285745	−.3136245	.0628915
0.50	0.3125	0.9375	−0.3125	0.0625

$$Y_n = a_0 Y_0 + a_1 Y_1 + a_2 Y_2 + a_3 Y_3$$

TABLE 3a (*continued*).

n	a_0	a_1	a_2	a_3
0.50	0.3125	0.9375	−0.3125	0.0625
.51	.3029915	.9460755	−.3111255	.0620585
.52	.2936320	.9543040	−.3095040	.0615680
.53	.2844205	.9621885	−.3076385	.0610295
.54	.2753560	.9697320	−.3055320	.0604440
.55	.2664375	.9769375	−.3031875	.0598125
.56	.2576640	.9838080	−.3006080	.0591360
.57	.2490345	.9903465	−.2977965	.0584155
.58	.2405480	0.9965560	−.2947560	.0576520
.59	.2322035	1.0024395	−.2914895	.0568465
0.60	0.2240	1.0080	−0.2880	0.0560
.61	.2159365	1.0132405	−.2842905	.0551135
.62	.2080120	1.0181640	−.2803640	.0541880
.63	.2002255	1.0227735	−.2762235	.0532245
.64	.1925760	1.0270720	−.2718720	.0522240
.65	.1850625	1.0310625	−.2673125	.0511875
.66	.1776840	1.0347480	−.2625480	.0501160
.67	.1704395	1.0381315	−.2575815	.0490105
.68	.1633280	1.0412160	−.2524160	.0478720
.69	.1563485	1.0440045	−.2470545	.0467015
0.70	0.1495	1.0465	−0.2415	0.0455
.71	.1427815	1.0487055	−.2357555	.0442685
.72	.1361920	1.0506240	−.2298240	.0430080
.73	.1297305	1.0522585	−.2237085	.0417195
.74	.1233960	1.0536120	−.2174120	.0404040
.75	.1171875	1.0546875	−.2109375	.0390625
.76	.1111040	1.0554880	−.2042880	.0376960
.77	.1051445	1.0560165	−.1974665	.0363055
.78	.0993080	1.0562760	−.1904760	.0348920
.79	.0935935	1.0562695	−.1833195	.0334565
0.80	0.0880	1.0560	−0.1760	0.0320
.81	.0825265	1.0554705	−.1685205	.0305235
.82	.0771720	1.0546840	−.1608840	.0290280
.83	.0719355	1.0536435	−.1530935	.0275145
.84	.0668160	1.0523520	−.1451520	.0259840
.85	.0618125	1.0508125	−.1370625	.0244375
.86	.0569240	1.0490280	−.1288280	.0228760
.87	.0521495	1.0470015	−.1204515	.0213005
.88	.0474880	1.0447360	−.1119360	.0197120
.89	.0429385	1.0422345	−.1032845	.0181115
0.90	0.0385	1.0395	−0.0945	0.0165
.91	.0341715	1.0365355	−.0855855	.0148785
.92	.0299520	1.0333440	−.0765440	.0132480
.93	.0258405	1.0299285	−.0673785	.0116095
.94	.0218360	1.0262920	−.0580920	.0099640
.95	.0179375	1.0224375	−.0486875	.0083125
.96	.0141440	1.0183680	−.0391680	.0066560
.97	.0104545	1.0140865	−.0295365	.0049955
.98	.0068680	1.0095960	−.0197960	.0033320
.99	.0033835	1.0048995	−.0099495	.0016665
1.00	0.0000	1.0000	−0.0000	0.0000

Correction required to take into account 4th differences lies between 0 and $-.042\,[Y_0 - 4Y_1 + 6Y_2 - 4Y_3 + Y_4.]$

TABLE 5.

For 5th Degree Interpolation, in Main Part of Table

n	B_{-2}	B_{-1}	B_0	B_1	B_2	B_3
0.00	0.000000	−0.000000	1.000000	0.000000	−0.000000	0.000000
.01	.00049579	−.00493338	0.99654209	.01006608	−.00250387	.00033329
.02	.00098301	−.00973369	.99283671	.02026197	−.00501433	.00066633
.03	.00146141	−.01440126	.98888645+	.03058412	−.00752959	.00099888
.04	.00193077	−.01893642	.98469396	.04102892	−.01004790	.00133067
.05	.00239088	−.02333957	.98026195+	.05159273	−.01256746	.00166146
.06	.00284153	−.02761113	.97559318	.06227190	−.01508649	.00199101
.07	.00328153	−.03175156	.97069045−	.07306272	−.01760319	.00231906
.08	.00371368	−.03576136	.96555663	.08396145−	−.02011576	.00264536
.09	.00413481	−.03964106	.96019466	.09496431	−.02262239	.00296967
0.10	0.00454575	−0.04339125	0.95460750	0.10606750	−0.02512125	0.00329175
.11	.00494634	−.04701252	.94879818	.11726719	−.02761053	.00361134
.12	.00533643	−.05050552	.94276977	.12855951	−.03008840	.00392821
.13	.00571588	−.05387093	.93652539	.13994058	−.03255302	.00424210
.14	.00608456	−.05710945+	.93006822	.15140646	−.03500257	.00455278
.15	.00644234	−.06022184	.92340148	.16295320	−.03743520	.00486001
.16	.00678910	−.06320886	.91652844	.17457684	−.03984906	.00516354
.17	.00712474	−.06607133	.90945239	.18627338	−.04224232	.00546314
.18	.00744917	−.06881009	.90217669	.19803879	−.04461313	.00575857
.19	.00776228	−.07142601	.89470475+	.20986902	−.04695964	.00604961
0.20	0.008064	−0.073920	0.887040	0.221760	−0.049280	0.006336
.21	.00835426	−.07629299	.87918592	.23370765−	−.05157296	.00661753
.22	.00863298	−.07854595+	.87114603	.24570785+	−.05383487	.00689396
.23	.00890011	−.08067988	.86292388	.25775648	−.05606568	.00716507
.24	.00915560	−.08269578	.85452308	.26984940	−.05826294	.00743064
.25	.00939941	−.08459473	.84594727	.28198242	−.06042480	.00769043
.26	.00963151	−.08637779	.83720010	.29415138	−.06254943	.00794424
.27	.00985185+	−.08804607	.82828528	.30635209	−.06463498	.00819183
.28	.01006143	−.08960072	.81920655+	.31858033	−.06667960	.00843301
.29	.01025723	−.09104288	.80996769	.33083187	−.06868147	.00866755+
0.30	0.01044225	−0.09237375	0.80057250	0.34310250	−0.07063875	0.00889525
.31	.01061548	−.09359454	.79102481	.35538796	−.07254961	.00911590
.32	.01077694	−.09470648	.78132849	.36768399	−.07441224	.00932930
.33	.01092665−	−.09571085−	.77148742	.37998634	−.07622481	.00953524
.34	.01106461	−.09660891	.76150554	.39229074	−.07798551	.00973353
.35	.01119087	−.09740199	.75138680	.40459289	−.07969254	.00992398
.36	.01130545+	−.09809142	.74113516	.41688852	−.08134410	.01010639
.37	.01140841	−.09867854	.73075462	.42917335−	−.08293841	.01028058
.38	.01149978	−.09916475−	.72024921	.44144307	−.08447367	.01044636
.39	.01157962	−.09955143	.70962298	.45369338	−.08594813	.01060356
0.40	0.011648	−0.099840	0.698880	0.465920	−0.087360	0.010752
.41	.01170498	−.10003191	.68802435+	.47811862	−.08870754	.01089151
.42	.01175063	−.10012861	.67706015−	.49028493	−.08998901	.01102191
.43	.01178504	−.10013159	.66599152	.50241465+	−.09120266	.01114305−
.44	.01180828	−.10004234	.65482260	.51450348	−.09234678	.01125476
.45	.01182045−	−.09986238	.64355758	.52654711	−.09341965−	.01135690
.46	.01182164	−.09959325−	.63220062	.53854126	−.09441957	.01144930
.47	.01181195+	−.09923649	.62075591	.55048166	−.09534486	.01153183
.48	.01179150+	−.09879368	.60922767	.56236401	−.09619384	.01160434
.49	.01176040	−.09826640	.59762013	.57418404	−.09696485+	.01166669
0.50	0.01171875	−0.09765625	0.58593750	0.58593750	−0.09765625	0.01171875

$$Y_n = B_{-2}Y_{-2} + B_{-1}Y_{-1} + B_0Y_0 + B_1Y_1 + B_2Y_2 + B_3Y_3$$

TABLE 5 (*continued*).

n	B_{-2}	B_{-1}	B_0	B_1	B_2	B_3
.50	0.01171875	−0.09765625	0.58593750	0.58593750	−0.09765625	0.01171875
.51	.01166669	−.09696485+	.57418404	.59762013	−.09826640	.01176040
.52	.01160434	−.09619384	.56236401	.60922767	−.09879368	.01179150+
.53	.01153183	−.09534486	.55048166	.62075591	−.09923649	.01181195+
.54	.01144930	−.09441957	.53854126	.63220062	−.09959325−	.01182164
.55	.01135690	−.09341965−	.52654711	.64355758	−.09986238	.01182045−
.56	.01125476	−.09234678	.51450348	.65482260	−.10004234	.01180828
.57	.01114305−	−.09120266	.50241465+	.66599152	−.10013159	.01178504
.58	.01102191	−.08998901	.49028493	.67706015−	−.10012861	.01175063
.59	.01089151	−.08870754	.47811862	.68802435+	−.10003191	.01170498
.60	0.010752	−0.087360	0.465920	0.698880	−0.099840	0.011648
.61	.01060356	−.08594813	.45369338	.70962298	−.09955143	.01157962
.62	.01044636	−.08447367	.44144307	.72024921	−.09916475−	.01149978
.63	.01028058	−.08293841	.42917335−	.73075462	−.09867854	.01140841
.64	.01010639	−.08134410	.41688852	.74113516	−.09809142	.01130545+
.65	.00992398	−.07969254	.40459289	.75138680	−.09740199	.01119087
.66	.00973353	−.07798551	.39229074	.76150554	−.09660891	.01106461
.67	.00953524	−.07622481	.37998634	.77148742	−.09571085−	.01092665−
.68	.00932930	−.07441224	.36768399	.78132849	−.09470648	.01077694
.69	.00911590	−.07254961	.35538796	.79102481	−.09359454	.01061548
0.70	0.00889525	−0.07063875	0.34310250	0.80057250	−0.09237375	0.01044225
.71	.00866755+	−.06868147	.33083187	.80996769	−.09104288	.01025723
.72	.00843301	−.06667960	.31858033	.81920655+	−.08960072	.01006143
.73	.00819183	−.06463498	.30635209	.82828528	−.08804607	.00985185+
.74	.00794424	−.06254943	.29415138	.83720010	−.08637779	.00963151
.75	.00769043	−.06042480	.28198242	.84594727	−.08459473	.00939941
.76	.00743064	−.05826294	.26984940	.85452308	−.08269578	.00915560
.77	.00716507	−.05606568	.25775648	.86292388	−.08067988	.00890011
.78	.00689396	−.05383487	.24570785+	.87114603	−.07854595+	.00863298
.79	.00661753	−.05157296	.23370765−	.87918592	−.07629299	.00835426
0.80	0.006336	−0.049280	0.221760	0.887040	−0.073920	0.008064
.81	.00604961	−.04695964	.20986902	.89470475+	−.07142601	.00776228
.82	.00575857	−.04461313	.19803879	.90217669	−.06881009	.00744917
.83	.00546314	−.04224232	.18627338	.90945239	−.06607133	.00712474
.84	.00516354	−.03984906	.17457684	.91652844	−.06320886	.00678910
.85	.00486001	−.03743520	.16295320	.92340148	−.06022184	.00644234
.86	.00455278	−.03500257	.15140646	.93006822	−.05710945+	.00608456
.87	.00424210	−.03255302	.13994058	.93652539	−.05387093	.00571588
.88	.00392821	−.03008840	.12855951	.94276977	−.05050552	.00533643
.89	.00361134	−.02761053	.11726719	.94879818	−.04701252	.00494634
0.90	0.00329175	−0.02512125	0.10606750	0.95460750	−0.04339125	0.00454575
.91	.00296967	−.02262239	.09496431	.96019466	−.03964106	.00413481
.92	.00264536	−.02011576	.08396145−	.96555663	−.03576136	.00371368
.93	.00231906	−.01760319	.07306272	.97069045−	−.03175156	.00328153
.94	.00199101	−.01508649	.06227190	.97559318	−.02761113	.00284153
.95	.00166146	−.01256746	.05159273	.98026195+	−.02333957	.00239088
.96	.00133067	−.01004790	.04102892	.98469396	−.01893642	.00193077
.97	.00099888	−.00752959	.03058412	.98888645+	−.01440126	.00146141
.98	.00066633	−.00501433	.02026197	.99283671	−.00973369	.00098301
.99	.00033329	−.00250387	.01006608	0.99654209	−.00493338	.00049579
1.00	0.000000	−0.000000	0.000000	1.000000	−0.000000	0.000000

Correction required to take into account 6th differences lies between zero and

$$-0.005\ [Y_{-2} - 6Y_{-1} + 15Y_0 - 20Y_1 + 15Y_2 - 6Y_3 + Y_4]$$

TABLE 5a.

FOR 5TH DEGREE INTERPOLATION, AT BEGINNING OF TABLE.

n	b_0	b_1	b_2	b_3	b_4	b_5
0.00	1.000000	0.000000	−0.000000	0.000000	−0.000000	0.000000
.01	0.97735398	.04936129	−.04911324	.03268741	−.01224754	.00195862
.02	.95507820	.09745691	−.09647249	.06409917	−.02399693	.00383565+
.03	.93316907	.14430440	−.14210687	.09425944	−.03525825+	.00563281
.04	.91162225+	.18992118	−.18604524	.12319212	−.04604150−	.00735179
.05	.89043862	.23432449	−.22831617	.15092086	−.05635652	.00899427
.06	.86959907	.27753147	−.26894802	.17746910	−.06621309	.01056193
.07	.84911454	.31955911	−.30796888	.20286005−	−.07562086	.01205639
.08	.82897598	.36042424	−.34540657	.22711665−	−.08458936	.01347928
.09	.80917938	.40014360	−.38128866	.25026163	−.09312805−	.01483221
0.10	0.78972075	0.43873375	−0.41564250	0.27231750	−0.10124625	0.01611675
.11	.77059653	.47621114	−.44849515−	.29330652	−.10895319	.01733448
.12	.75180222	.51259208	−.47987343	.31325071	−.11625800	.01848693
.13	.73333394	.54789273	−.50980393	.33217190	−.12316968	.01957563
.14	.71518784	.58212915−	−.53831298	.35009166	−.12969717	.02060210
.15	.69736007	.61531723	−.56542664	.36703133	−.13584926	.02156782
.16	.67984684	.64747274	−.59117076	.38301204	−.14163466	.02247426
.17	.66264436	.67861134	−.61557094	.39805470	−.14706199	.02332287
.18	.64594887	.70874851	−.63865251	.41217998	−.15213973	.02411509
.19	.62915665+	.73789965+	−.66044057	.42540834	−.15687630	.02485324
0.20	0.612864	0.766080	−0.680960	0.437760	−0.161280	0.025536
.21	.59686765−	.79330467	−.70023541	.44925497	−.16535902	.02616746
.22	.58116337	.81958865−	−.71829117	.45991305+	−.16912147	.02674808
.23	.56574757	.84494679	−.73515144	.46975381	−.17257534	.02727920
.24	.55061668	.86939382	−.75084012	.47879660	−.17572854	.02776216
.25	.53576715−	.89294434	−.76538086	.48706055−	−.17858887	.02819824
.26	.52119544	.91561281	−.77879710	.49456458	−.18116403	.02858875+
.27	.50689806	.93741359	−.79111205−	.50132741	−.18346164	.02893496
.28	.49287154	.95836088	−.80234865−	.50736753	−.18544920	.02923813
.29	.47911241	.97846878	−.81252963	.51270320	−.18725413	.02949948
0.30	0.46561725	0.99775125	−0.82167750	0.51735250	−0.18876375	0.02972025
.31	.45238307	1.01622212	−.82981452	.52133328	−.19002528	.02990163
.32	.43940594	1.03389512	−.83696271	.52466319	−.19104584	.03004482
.33	.42668251	1.05078382	−.84314390	.52735967	−.19183247	.03015097
.34	.41420949	1.06690169	−.84837966	.52943994	−.19239211	.03022125−
.35	.40198360	1.08226207	−.85269133	.53092102	−.19273160	.03025679
.36	.38000159	1.09687818	−.85610004	.53181972	−.19285770	.03025871
.37	.36826022	1.11076312	−.85862671	.53215267	−.19277707	.03022811
.38	.35675628	1.12392985+	−.86029199	.53193627	−.19249627	.03016608
.39	.34544866	1.13639124	−.86111634	.53118671	−.19202179	.03007369
0.40	0.344448	1.148160	−0.861120	0.529920	−0.191360	0.029952
.41	.33363772	1.15924875+	−.86032297	.52815194	−.19051720	.02980204
.42	.32305222	1.16966999	−.85874505+	.52589813	−.18949961	.02962483
.43	.31268830	1.17943607	−.85640581	.52317398	−.18831332	.02942138
.44	.30254296	1.18855926	−.85332460	.51999468	−.18696438	.02919268
.45	.29261318	1.19705168	−.84952055−	.51637523	−.18545871	.02893971
.46	.28289597	1.20492535+	−.84501258	.51233046	−.18380217	.02866342
.47	.27338837	1.21219217	−.83981941	.50787498	−.18200052	.02836476
.48	.26408746	1.21886392	−.83395953	.50302321	−.18005944	.02804466
.49	.25499017	1.22495227	−.82745120	.49778937	−.17798452	.02770402
0.50	0.24609375	1.23046875	−0.82031250	0.49218750	−0.17578125	0.02734375

$$Y_n = b_0Y_0 + b_1Y_1 + b_2Y_2 + b_3Y_3 + b_4Y_4 + b_5Y_5$$

TABLE 5a (*Continued*).

n	b_0	b_1	b_2	b_3	b_4	b_5
0.50	0.24609375	1.23046875	−0.82031250	0.49218750	−0.17578125	0.02734375
.51	.23739568	1.23542481	−.81256128	.48623145+	−.17345506	.02696523
.52	.22889253	1.23983176	−.80421519	.47993487	−.17101128	.02656782
.53	.22058149	1.24370080	−.79529167	.47331124	−.16845515	.02615389
.54	.21245978	1.24704303	−.78580794	.46637382	−.16579185−	.02572376
.55	.20452463	1.24986941	−.77578102	.45913570	−.16302645−	.02527826
.56	.19677330	1.25219082	−.76522772	.45160980	−.16016394	.02481820
.57	.18920306	1.25401800	−.75416467	.44380884	−.15720925	.02434437
.58	.18181121	1.25536159	−.74260827	.43574535−	−.15416721	.02885755+
.59	.17459508	1.25623212	−.73057471	.42743168	−.15104257	.02385851
0.60	0 167552	1.256640	−0.718080	0.418880	−0.147840	0.022848
.61	.16067975+	1.25659564	−.70513994	.41010231	−.14456409	.02232675+
.62	.15397515+	1.25610893	−.69177013	.40111041	−.14121935−	.02179550−
.63	.14743564	1.25519025+	−.67798598	.39191594	−.13781021	.02125494
.64	.14105867	1.25384950−	−.66380268	.38253036	−.13434102	.02070577
.65	.13484171	1.25209652	−.64923523	.37296492	−.13081605+	.02014868
.66	.12878227	1.24994109	−.63429846	.36323074	−.12723951	.01958433
.67	.12287785+	1.24739286	−.61900698	.35333875−	−.12361551	.01901338
.68	.11712600	1.24446136	−.60337521	.34329969	−.11994808	.01843646
69	.11152427	1.24115605−	−.58741737	.33312413	−.11624120	.01785422
0.70	0.10607025	1.23748625	−0.57114750	0.32282250	−0.11249875	0.01726725
.71	.10076194	1.23346119	−.55457945+	.31240502	−.10872454	.01667617
.72	.09559640	1.22909000	−.53772687	.30188175+	−.10492232	.01608155+
.73	.09057129	1.22438168	−.52060324	.29126260	−.10109574	.01548398
.74	.08568431	1.21934517	−.50322182	.28055730	−.09724839	.01488403
.75	.08093316	1.21398926	−.48559570	.26077539	−.09338399	.01428223
.76	.07631557	1.20832266	−.46773780	.25892628	−.08950538	.01367912
.77	.07182928	1.20235399	−.44966084	.24801921	−.08561654	.01307524
.78	.06747207	1.19609173	−.43137735−	.23706323	−.08172055+	.01247110
.79	.06324170	1.18954430	−.41289968	.22606724	−.07782066	.01186719
0.80	0.059136	1.182720	−0.394240	0.215040	−0.073920	0.011264
.81	.05515319	1.17562702	−.37541031	.20399008	−.07002167	.01066201
.82	.05129057	1.16827347	−.35642241	.19292589	−.06612869	.01006169
.83	.04754600	1.16066734	−.33728794	.18185571	−.06224399	.00946347
.84	.04391742	1.15281654	−.31801836	.17078764	−.05837046	.00886782
.85	.04040274	1.14472887	−.29862492	.15972961	−.05451090	.00827515−
.86	.03699992	1.13641203	−.27911874	.14868942	−.05066805+	.00768588
.87	.03370691	1.12787364	−.25951075−	.13767472	−.04684459	.00710042
.88	.03052171	1.11912120	−.23981169	.12669297	−.04304312	.00651915+
.89	.02744232	1.11016213	− 22003213	.11575150+	−.03926618	.00594247
0.90	0.02446675	1.10100375	−0.20018250	0.10485750	−0.03551625	0.00537075
.91	.02159346	1.09165328	−.18027302	.09402005+	−.03179573	.00480434
.92	.01881995+	1.08211784	−.16031375+	.08324319	−.02810696	.00424360
.93	.01614432	1.07240447	−.14031460	.07253379	−.02445222	.00368886
.94	.01356469	1.06252011	−.12028530	.06189354	−.02083373	.00314045+
.95	.01107920	1.05247160	−.10023539	.05134398	−.01725363	.00259870
.96	.00868600	1.04226570	−.08017428	.04087652	−.01371402	.00206389
.97	.00638329	1.03190907	−.06011121	.03050239	−.01021692	.00153634
.98	.00416924	1.02140827	−.04005523	.00022767	−.00676429	.00101633
.99	.00204207	1.01076979	−.02001524	.01005828	−.00335804	.00050412
1.00	0.000000	1.000000	−0.000000	0.000000	−0.000000	0.000000

Correction required to take into account 6th differences lies between zero and

$$-0.024\,[Y_0 - 6Y_1 + 15Y_2 - 20Y_3 + 15Y_4 - 6Y_5 + Y_6]$$

TABLE 5b.

FOR 5TH DEGREE INTERPOLATION, IN SECOND INTERVAL OF TABLE.

n	c_{-1}	c_0	c_1	c_2	c_3	c_4
0.00	−0.000000	1.000000	0.000000	−0.000000	0.000000	−0.000000
.01	−.00195862	0.98910520	.01998192	−.00994076	.00330966	−.00049579
.02	−.00383565+	.97809161	.03992211	−.01975943	.00656693	−.00098301
.03	−.00563281	.96696532	.05981229	−.03945052	.00977030	−.00146141
.04	−.00735179	.95573228	.07964436	−.03900948	.01291830	−.00193077
.05	−.00899427	.94439871	.09941039	−.04843070	.01600949	−.00239088
.06	−.01056193	.93297017	.11910258	−.05770950−	.01904249	−.00284153
.07	−.01205639	.92145252	.13871328	−.06684112	.02201595+	−.00328253
.08	−.01347928	.90985144	.15823503	−.07582095+	.02492856	−.00371368
.09	−.01483221	.89817252	.17766050−	−.08464453	.02777907	−.00463481
0.10	−0.01611675	0.88642125	0.19698250	−0.09330750	0.03056625	−0.00454575
.11	−.01733448	.87460306	.21619401	−.10180565−	.03329100	−.00494634
.12	−.01848693	.86272328	.23528817	−.11013489	.03594936	−.00533643
.13	−.01957563	.85078715+	.25425823	−.11829126	.03854037	−.00571588
.14	−.02060210	.83879985−	.27300762	−.12629094	.04106313	−.00 08456
.15	−.02156782	.82676645−	.29179992	−.13407023	.04351676	−.00647234
.16	−.02247426	.81469194	.31035884	−.14168556	.04590042	−.00678910
.17	−.02332287	.80258125+	.32876822	−.14911346	.04821332	−.00712474
.18	−.02411509	.79043921	.34702209	−.15635061	.05045469	−.00744917
.19	−.02485324	.77827057	.36511459	−.16339382	.05262381	−.00776228
0.20	−0.025536	0.766080	0.383040	−0.170240	0.054720	−0.008064
.21	−.02616746	.75387209	.40079276	−.17688619	.05674467	−.00835426
.22	−.02674808	.74165135−	.41836743	−.18332955−	.05869439	−.00863298
.23	−.02727920	.72942221	.43575872	−.18956735+	.06056871	−.00890011
.24	−.02776216	.71718902	.45296148	−.19559700	.06236726	−.00915560
.25	−.02819824	.70495605+	.46997070	−.20141602	.06408965−	−.00939941
.26	−.02858875+	.69272751	.48678150−	−.20702202	.06573555+	−.00963151
.27	−.02893496	.68050751	.50338912	−.21241275−	.06730467	−.00985185+
.28	−.02923813	.66830008	.51978895+	−.21758607	.06879672	−.01006143
.29	−.02949948	.65610921	.53597653	−.22253996	.07021148	−.01025723
0.30	−0.02972025	0.64393875	0.55194750	−0.22727250	0.07154875	−0.01044225
.31	−.02990163	.63179254	.56769765	−.23178188	.07281042	−.01061548
.32	−.03004482	.61967432	.58322289	−.23606641	.07399352	−.01077694
.33	−.03015097	.60758774	.59851926	−.24012449	.07509809	−.01092665−
.34	−.03022125−	.59553639	.61358294	−.24395466	.07612419	−.01106461
.35	−.03025679	.58352379	.62841023	−.24755555−	.07707191	−.01119087
.36	−.03025871	.57155338	.64299756	−.25092588	.07794138	−.01130545+
.37	−.03022811	.55962854	.65734146	−.25406449	.07873274	−.01140841
.38	−.03016608	.54775255+	.67143861	−.25697033	.07944615+	−.01149978
.39	−.03007369	.53592866	.68528582	−.25964245+	.08008183	−.01157962
0.40	−0.029952	0.524160	0.698880	−0.262080	0.080640	−0.011648
.41	−.02980204	.51244967	.71221819	−.26428222	.08112299	−.01170498
.42	−.02962483	.50080069	.72529755−	−.26624847	.08152825−	−.01175063
.43	−.02942138	.48921599	.73811535+	−.26797819	.08185623	−.01178504
.44	−.02919268	.47769846	.75066900	−.26947092	.08210742	−.01180828
.45	−.02893971	.46625090	.76295602	−.27072633	.08228230	−.01182045−
.46	−.02866342	.45487605+	.77497402	−.29174414	.08238141	−.01182164
.47	−.02836476	.44357659	.78672075−	−.27252418	.08240528	−.01181195+
.48	−.02804466	.43235512	.79819407	−.27306639	.08235448	−.01179150+
.49	−.02770402	.42121418	.80939196	−.27337080	.08222960	−.01176040
0.50	−0.02734375	0.41015625	0.82031250	−0.27343750	0.08203125	−0.01171875

$$Y_n = c_{-1}Y_{-1} + c_0Y_0 + c_1Y_1 + c_2Y_2 + c_3Y_3 + c_4Y_4$$

Table 5b (*continued*).

n	c_{-1}	c_0	c_1	c_2	c_3	c_4
0.50	−0.02734375	0.41015625	0.82031250	−0.27343750	0.08203125	−0.01171875
.51	−.02696523	.39918373	.83095388	−.27326671	.08176213	−.01166669
.52	−.02656982	.38829896	.84131441	−.27285873	.08142008	−.01160434
.53	−.02615389	.37750422	.85139249	−.27221393	.08100590	−.01153183
.54	−.02572376	.36680173	.86118666	−.27133278	.08052045−	−.01144930
.55	−.02527826	.35619363	.87069555−	−.27021586	.07996457	−.01135690
.56	−.02481820	.34568202	.87991788	−.26886380	.07933914	−.01125476
.57	−.02434437	.33526892	.88885249	−.26727732	.07864505+	−.01114305−
.58	−.02385755+	.32495629	.89749833	−.26545725+	.07788321	−.01102191
.59	−.02335851	.31474604	.90585445+	−.26340449	.07705455−	−.01089151
0.60	−0.022848	0.304640	0.913920	−0.261120	0.076160	−0.010752
.61	−.02232675+	.29463996	.92169422	−.25860485+	.07520260	−.01060356
.62	−.02179550−	.28474763	.92917647	−.25586019	.07418051	−.01044636
.63	−.02125494	.27496467	.93636619	−.25288722	.07309485	−.01028059
.64	−.02070577	.26529270	.94326292	−.24968724	.07194678	−.01010639
.65	−.02014868	.25593324	.94986633	−.24626164	.07073746	−.00992398
.66	−.01958433	.24628779	.95617614	−.24261186	.06946807	−.00973353
.67	−.01901338	.23695970	.96219218	−.23873941	.06813980	−.00953524
.68	−.01843646	.22974456	.96991439	−.23464591	.06675384	−.00932930
.69	−.01785422	.21864947	.97334280	−.23033303	.06531142	−.00911590
0.70	−0.01726725	0.20967375	0.97847750	−0.22580250	0.06381375	−0.00889525
.71	−.01667617	.20081861	.98331871	−.22105614	.06226415+	−.00866755+
.72	−.01608155+	.19208520	.98786673	−.21609585−	.06066104	−.00843301
.73	−.01548398	.18347460	.99212193	−.21092356	.05900582	−.00819183
.74	−.01488403	.17498787	.99608478	−.20554130	.05729991	−.00794424
.75	−.01428223	.16662598	0.99975586	−.19995117	.05554473	−.00769043
.76	−.01367912	.15838986	1.00313580	−.19415532	.05374170	−.00743064
.77	−.01307524	.15028041	1.00622532	−.18815595+	.05189227	−.00716507
.78	−.01247110	.14229843	1.00902525+	−.18195537	.04999783	−.00689396
.79	−.01186719	.13444472	1.01153649	−.17555592	.04805996	−.00661753
0.80	−0.011264	0.126720	1.013760	−0.168960	0.046080	−0.006336
.81	−.01066201	.11912494	1.01569685+	−.16217009	.04406152	−.00604961
.82	−.01006169	.11166019	1.01734819	−.15518871	.04200317	−.00575857
.83	−.00946347	.10432626	1.01871522	−.14801845+	.03990656	−.00546314
.84	−.00886782	.09712374	1.01979924	−.14066196	.03777334	−.00516354
.85	−.00827515−	.09005809	1.02060164	−.13312195+	.03560512	−.00486001
.86	−.00768588	.08311473	1.02112386	−.12540118	.03340353	−.00455278
.87	−.00710042	.07630906	1.02136741	−.11750245−	.03117022	−.00424210
.88	−.00651915+	.06963640	1.02133391	−.10942863	.02890680	−.00392821
.89	−.00594247	.06309705+	1.02102503	−.10118266	.02661493	−.00361134
0.90	−0.00537075	0.05669125	1.02044250	−0.09276750	0.02429625	−0.00329175
.91	−.00480434	.05041919	1.01958814	−.08418618	.02195447	−.00296967
.92	−.00424360	.04428104	1.01846385−	−.07544177	.01958840	−.00264536
.93	−.00368886	.03829689	1.01707156	−.06653739	.01719983	−.00231906
.94	−.00314045+	.03240681	1.01541330	−.05747622	.01479057	−.00199101
.95	−.00259870	.02667082	1.01349117	−.04826148	.01236238	−.00166146
.96	−.00206389	.02106890	1.01130732	−.03889644	.00991706	−.00133067
.97	−.00153634	.01560099	1.00886395+	−.02938439	.00745639	−.00099888
.98	−.00101633	.01026697	1.00616337	−.01972869	.00498213	−.00066633
.99	−.00050412	.00506671	1.00320792	−.00993275+	.00249608	−.00033329
1.00	−0.000000	0.000000	1.000000	−0.000000	0.000000	−0.000000

Correction required to take into account 6th differences lies between zero and $+0.007\,[Y_{-1} - 6Y_0 + 15Y_1 - 20Y_2 + 15Y_3 - 6Y_4 + Y_5]$

TABLE 30a.

(CUMULATIVE) 3RD DEGREE. FOR USE AT BEGINNING OF TABLE.

m	d_0	d_1	d_2	d_3
0	−0.000	+0.000	−0.000	+0.000
⅕	−0.328	+0.504	−0.224	+0.048
⅖	−0.256	+0.328	−0.088	+0.016
⅗	−0.192	+0.176	+0.024	−0.008
⅘	−0.136	+0.048	+0.112	−0.024
1	−0.088	−0.056	+0.176	−0.032

$$Y_m = Y_{m-1/5}+d_0Y_0+d_1Y_1+d_2Y_2+d_3Y_3$$
$$(m = \tfrac{1}{5}, \tfrac{2}{5}, \tfrac{3}{5}, \tfrac{4}{5}, 1)$$

TABLE 30.

(CUMULATIVE) 3RD DEGREE. FOR USE IN MAIN PART OF TABLE.

m	D_{-1}	D_0	D_1	D_2
0	−0.000	−0.000	+0.000	−0.000
⅕	−0.048	−0.136	+0.216	−0.032
⅖	−0.016	−0.192	+0.232	−0.024
⅗	+0.008	−0.224	+0.224	−0.008
⅘	+0.024	−0.232	+0.192	+0.016
1	+0.032	−0.216	+0.136	+0.048

$$Y_m \quad Y_{m-1/5}+D_{-1}Y_{-1}+D_0Y_0+D_1Y_1+D_2Y_2$$
$$(m = \tfrac{1}{5}, \tfrac{2}{5}, \tfrac{3}{5}, \tfrac{4}{5}, 1)$$

TABLE 50a.

(CUMULATIVE) 5TH DEGREE. FOR USE AT BEGINNING OF TABLE.

m	g_0	g_1	g_2	g_3	g_4	g_5
0	−0.000000	+0.000000	−0.000000	+0.000000	−0.000000	+0.000000
⅕	−0.387136	+0.766080	−0.680960	+0.437760	−0.161280	+0.025536
⅖	−0.268416	+0.382080	−0.180160	+0.092160	−0.030080	+0.004416
⅗	−0.176896	+0.108480	+0.143040	−0.111040	+0.043520	−0.007104
⅘	−0.108416	−0.073920	+0.323840	−0.203840	+0.073920	−0.011584
1	−0.059136	−0.182720	+0.394240	−0.215040	+0.073920	−0.011264

$$Y_m = Y_{m-1/5} + g_0Y_0 + g_1Y_1 + g_2Y_2 + g_3Y_3 + g_4Y_4 + g_5Y_5 \qquad (m = \tfrac{1}{5}, \tfrac{2}{5}, \tfrac{3}{5}, \tfrac{4}{5}, 1)$$

TABLE 50b.

(Cumulative) 5th Degree. For Use in Second Interval of Table.

m	h_{-1}	h_0	h_1	h_2	h_3	h_4
0	−0.000000	−0.000000	+0.000000	−0.000000	+0.000000	−0.000000
$1/5$	−0.025536	−0.233920	+0.383040	−0.170240	+0.054720	−0.008064
$2/5$	−0.004416	−0.241920	+0.315840	−0.091840	+0.025920	−0.003584
$3/5$	+0.007104	−0.219520	+0.215040	+0.000960	−0.004480	+0.000896
$4/5$	+0.011584	−0.177920	+0.099840	+0.092160	−0.030080	+0.004416
1	+0.011264	−0.126720	−0.013760	+0.168960	−0.046080	+0.006336

$Y_m = Y_{m-1/5} + h_{-1}Y_{-1} + h_0 Y_0 + h_1 Y_1 + h_2 Y_2 + h_3 Y_3 + h_4 Y_4$ $(m = 1/5, 2/5, 3/5, 4/5, 1)$

TABLE 50.

(Cumulative) 5th Degree. For Use in Main Part of Table.

m	G_{-2}	G_{-1}	G_0	G_1	G_2	G_3
0	+0.000000	−0.000000	−0.000000	+0.000000	−0.000000	+0.000000
$1/5$	+0.008064	−0.073920	−0.112960	+0.221760	−0.049280	+0.006336
$2/5$	+0.003584	−0.025920	−0.188160	+0.244160	−0.038080	+0.004416
$3/5$	−0.000896	+0.012480	−0.232960	+0.232960	−0.012480	+0.000896
$4/5$	−0.004416	+0.038080	−0.244160	+0.188160	+0.025920	−0.003584
1	−0.006336	+0.049280	−0.221760	+0.112960	+0.073920	−0.008064

$Y_m = Y_{m-1/5} + G_{-2}Y_{-2} + G_{-1}Y_{-1} + G_0 Y_0 + G_1 Y_1 + G_2 Y_2 + G_3 Y_3$ $(m = 1/5, 2/5, 3/5, 4/5, 1)$

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RECORDS OF MEETINGS

One thousand one hundred and fifty-ninth meeting.

OCTOBER 19, 1927.—STATED MEETING.

The Academy met at its House at 8.15 P. M.

The PRESIDENT in the Chair.

There were present seventy-five Fellows and Associates, two Foreign Honorary Members, and nine guests.

The Records of the Annual Meeting, of May 11, were read and approved.

The following letters were presented by the Corresponding Secretary: from K. D. Blackfan, R. P. Blake, W. J. Crozier, Philip Drinker, G. M. Fair, W. C. Forbes, J. L. Gamble, Wallace Goodrich, H. M. Hall, A. N. Holcombe, E. A. Hooton, Charles Hopkinson, P. R. Howe, I. M. Johnston, A. V. Kidder, D. H. McLaughlin, J. H. Means, A. L. Melander, G. J. Peirce, F. H. Pratt, J. H. Pratt, E. A. Robinson, A. W. Rowe, George Sarton, A. W. Sellards, J. C. Slater, W. L. Sperry, H. T. Stetson, E. C. Tarbell, Langdon Warner, J. T. Wearn, R. S. Weston, W. W. Willoughby, and Quincy Wright, accepting Fellowship; from F. C. Burkitt, H. H. Dale, Peter Debye, Léon Duguit, Willem Einthoven, Ejnar Hertzsprung, Victor Laloux, Friedrich von Müller, Gilbert Murray, Henri Rabaud, Joseph Redlich, and Richard Wettstein, accepting Foreign Honorary Membership; from N. T. Kidder, accepting Associate Membership; and from W. H. P. Faunce, resigning Fellowship.

The Corresponding Secretary reported the receipt from Mr. William C. Lane of a notice issued for a meeting of the Council of the Academy on October 17, 1784.

He also announced that the following biographical notices had been received: of William Healey Dall, by G. H. Parker; James Furman Kemp, by B. K. Emerson; and Charles Doolittle Walcott, by Charles Schuchert.

The President announced the death of nine Fellows: Henry Larcom Abbot (Class I, Section 4), Thomas Willing Balch (Class III, Section 1), Bertram Borden Boltwood (Class I, Section 3), Francis

Tiffany Bowles (Class I, Section 4), James Kendall Hosmer (Class III, Section 4), Alexander Crombie Humphreys (Class I, Section 4), Charles Frederic Mabery (Class I, Section 3), Francis Weld Peabody (Class II, Section 4), Henry Paul Talbot (Class I, Section 3); of two Foreign Honorary Members: Svante August Arrhenius (Class I, Section 2), Willem Einthoven (Class II, Section 3); and of one Associate, Edwin Hale Abbot.

The newly elected Fellows and Associates were then presented.

The following communications were presented:

Mr. Edward M. East: "The World Population Congress at Geneva and the Fifth International Congress on Genetics at Berlin."

Mr. Willem J. Luyten: "Forty Seconds at Jokkmokk."

Sir Charles S. Sherrington: Some Remarks.

Two papers were presented by title:

"The Compressibility and Pressure Coefficient of Resistance of Eleven Elements," by P. W. Bridgman.

"The Viscosity of Mercury under Pressure," by P. W. Bridgman.

The meeting was dissolved at 9.55 P. M.

One thousand one hundred and sixtieth meeting.

NOVEMBER 9, 1927.—STATED MEETING.

The Academy met at its House at 8.15 P. M.

The PRESIDENT in the Chair.

There were present fifty-two Fellows and Associates and eight guests.

The Records of the meeting of October 19 were read and approved.

The President reported the death of two Fellows: Alfred Hemenway (Class III, Section 1) and John Collins Warren (Class II, Section 4).

The President announced that the next meeting on December 14 will be preceded by a subscription dinner.

The Rumford Medals were presented to Professor Arthur Holly Compton, of the University of Chicago, for his researches on Röntgen rays.

The following communication was presented:

Mr. Arthur Holly Compton: "X-Rays as a Branch of Optics."

The meeting was dissolved at 9.45 P. M.

One thousand one hundred and sixty-first meeting.

DECEMBER 14, 1927.—STATED MEETING.

The Academy met at its House at 8.15 P. M.

The PRESIDENT in the Chair.

There were present thirty-three Fellows and Associates and thirty guests, including ladies.

The meeting was preceded by a subscription dinner held in the Reading Room of the Academy. This was attended by thirty-two Fellows and Associates and thirty guests, including ladies.

The reading of the Records of the meeting of November 9 was suspended.

The President reported the death of two Fellows: Samuel McChord Crothers (Class III, Section 4) and Nathan Matthews (Class III, Section 1).

The following communications were presented:

Mr. Charles Burton Gulick: "A Cookery Book of the Second Century."

Mr. Willem J. Luyten: "Some Remarks on the Skjellerup Comet."

The meeting was dissolved at 9.30 P. M.

One thousand one hundred and sixty-second meeting.

JANUARY 11, 1928.—STATED MEETING.

The Academy met at its House at 8.15 P. M.

The PRESIDENT in the Chair.

In the absence of the Recording Secretary Vice-President Kennelly was requested to assume his duties.

There were present fifty-three Fellows and Associates and thirteen guests.

The Records of the meeting of December 14 were read and approved.

The Corresponding Secretary reported the receipt of a biographical notice of Francis Weld Peabody, by M. J. Rosenau.

The following communications were presented:

Dr. George R. Minot and (by invitation) Dr. William P. Murphy: "The Treatment of Pernicious Anemia."

Dr. William B. Castle: "The Effect of Liver Feeding on the Blood-forming Organs in Pernicious Anemia; Studies Commenced by Dr. Francis W. Peabody."

Dr. Edwin J. Cohn: "The Present Status of the Chemical Problems Raised by the Study of Pernicious Anemia."

The following papers were presented by title:

"Magnetostriction Oscillators Applied to the Production, Control, and Measurement of Audio and Radio Frequencies," by George W. Pierce.

"Magnetostriction Oscillators Applied to the Measurement of the Electric Constants of Metals and Alloys," by George W. Pierce.

"A Dynamic Study of Magnetostriction," by K. C. Black. Presented by George W. Pierce.

The meeting was dissolved at 9.45 P. M.

One thousand one hundred and sixty-third meeting.

FEBRUARY 8, 1928.—STATED MEETING.

The Academy met at its House at 8.15 P. M.

The PRESIDENT in the Chair.

There were present thirty-three Fellows and five guests.

The Records of the meeting of January 11 were read and approved.

The Corresponding Secretary reported the receipt of a biographical notice of Willem Einthoven, by W. B. Cannon.

He also announced that James Haughton Woods had been appointed delegate to represent the Academy at the International Congress of Orientalists at Oxford in August, 1928, and William Morton Wheeler its representative at the Fourth International Congress of Entomologists at Ithaca, August 12–18, 1928.

The President announced the death of three Fellows: John Spencer Bassett (Class III, Section 3), Archibald Cary Coolidge (Class III, Section 3), George Washington Goethals (Class I, Section 4); and of one Foreign Honorary Member: Thomas Hardy (Class III, Section 4).

The following communications were presented:

Mr. Arthur E. Kennelly, Chairman Rumford Committee: "Recommendation for an Award of the Rumford Medal," with an Experimental Demonstration by Mr. William S. Franklin.

Mr. William Yandell Elliott: "The Principles of Political Science."

The meeting was dissolved at 9.55 P. M.

One thousand one hundred and sixty-fourth meeting.

MARCH 14, 1928.—STATED MEETING.

The Academy met at its House at 8.30 P. M.

The PRESIDENT in the Chair.

There were present thirty-six Fellows and Associates and three guests.

The Records of the meetings of February 8 and November 9, 1927, were read and approved.

On recommendation of the Council the following appropriations were made for the ensuing year:

From the income of the General Fund, $7,700, to be used as follows:

for General and Meeting expenses	$ 800
for Library expenses	2,000
for Books, Periodicals, and Binding	1,200
for House expenses	2,850
for Treasurer's expenses	850
	$7,700

From the income of the Publication Fund, $5,091.93, to be used for publication.

From the income of the Rumford Fund, $3,887.87, to be used as follows:

for Research	$1,000.00
for Books, Periodicals, and Binding	200.00
for Publication	600.00
for use at the discretion of the Committee	2,087.87
	$3,887.87

From the income of the C. M. Warren Fund, $1,173.07, to be used at the discretion of the Committee.

The President reported the death of Hendrik Antoon Lorentz, Foreign Honorary Member in Class I, Section 2, and of Harris Hawthorne Wilder, Fellow in Class II, Section 3.

The President appointed the Nominating Committee as follows:

Arthur B. Lamb, of Class I
James L. Gamble, of Class II
John O. Sumner, of Class III

The following communication was presented:

Mr. L. Denis Peterkin: "The New Novel."

The meeting was dissolved at ten o'clock.

One thousand one hundred and sixty-fifth meeting.

APRIL 11, 1928.—STATED MEETING.

The Academy met at its House at 8.15 P. M.

The PRESIDENT in the Chair.

There were present twenty-nine Fellows and five guests.

In the absence of the Recording Secretary, the Corresponding Secretary was requested to assume his duties.

The Records of the meeting of March 14 were read and approved.

The President announced the death of two Fellows: Gaetano Lanza (Class I, Section 4) and Theodore William Richards (Class I, Section 3).

On motion of Mr. Kennelly, it was

Voted, That steps be taken to determine whether an increase in the statutory limit of the number of Foreign Honorary Members is desirable, and if so, how great the increase should be, and that a committee should be appointed to consider this question, and if an increase be found desirable, to frame an amendment to the Statutes for consideration at the next stated meeting of the Academy.

The President appointed on this committee Vice-Presidents Kennelly, Parker, and Kittredge.

The following communication was presented:

Mr. Warren M. Persons: "The Statistical Basis for Forecasting Business Conditions."

The following paper was presented by title: "The Petrology of a Portion of the White Mountains, N. H.," by Marland P. Billings, presented by E. S. Larsen, Jr.

The meeting was dissolved at 10.25 P. M.

One thousand one hundred and sixty-sixth meeting.

MAY 9, 1928.—ANNUAL MEETING.

The Academy met at its House at 8.30 P. M.

The PRESIDENT in the Chair.

There were present forty-two Fellows and four guests.

The Records of the meeting of April 11 were read and approved.

The following report of the Council was presented:

REPORT OF THE COUNCIL.

Since the last report of the Council there have been reported the deaths of nineteen Fellows: Henry Larcom Abbot, Thomas Willing Balch, John Spencer Bassett, Bertram Borden Boltwood, Francis Tiffany Bowles, Archibald Cary Coolidge, Samuel McChord Crothers, George Washington Goethals, Alfred Hemenway, James Kendall Hosmer, Alexander Crombie Humphreys, Gaetano Lanza, Charles Frederic Mabery, Nathan Matthews, Francis Weld Peabody, Theodore William Richards, Henry Paul Talbot, John Collins Warren, Harris Hawthorne Wilder; of four Foreign Honorary Members: Svante August Arrhenius, Willem Einthoven, Thomas Hardy, Hendrik Antoon Lorentz; and of one Associate, Edwin Hale Abbot.

Thirty-five Fellows, thirteen Foreign Honorary Members, and one

Associate were elected by the Council and announced to the Academy in May, 1927.

The roll now includes 569 Fellows, 67 Foreign Honorary Members, and 16 Associates (not including those elected in May, 1928).

The annual report of the Treasurer, Ingersoll Bowditch, was read, of which the following is an abstract.

General Fund.

Receipts.

From Investments	$2,876.50	
From Assessments	3,380.00	
From Admissions	270.00	
From Interest on Deposits	151.35	
From Fund for Current Expenses	1,926.13	
		$8,603.98

Expenditures.

Expenses of Library	$1,857.16		
Treasurer's Expenses	899.45		
Books and Binding	1,241.48		
General Expenses	894.19		
House Expenses	3,026.13		
President's Expenses	45.00		
		$7,963.41	
Interest on Bonds bought	112.50		
Transferred to Publication Funds	250.00		
Income transferred to Principal	278.07		
		640.57	
			$8,603.98

Rumford Fund.

Receipts.

Income on hand April 1, 1927	$13,918.46	
From Investments	4,229.82	
		$18,148.28

Expenditures.

Purchase and Binding of Books	$ 233.94		
Publications	71.73		
Research	2,743.94		
Medals	301.94		
Tables of Constants, two years	400.00		
		$3,751.55	
Income transferred to Principal		204.63	
			$3,956.18

Warren Fund.

Receipts.

Income on hand April 1, 1927		$ 460.32	
From Investments		1,173.50	
			$1,633.82

Expenditures.

Research	$ 625.00		
Vault Rent, part	3.00		
		$ 628.00	
Interest on Notes bought	8.13		
Charged to cancel premiums on Notes bought	5.00		
Income transferred to Principal	58.18		
		71.31	
			$ 699.31

Publication Fund.

Receipts.

Income on hand April 1, 1927		$2,555.60	
From Income Appleton Fund	$ 698.75		
From Income Centennial Fund	2,542.77		
From Authors' Reprints	177.04		
From Sale of Publications	556.95		
From General Fund Income	250.00		
		4,225.51	
			$6,781.11

Expenditures.

Publications	$1,768.25		
Vault Rent, part	10.00		
		$1,778.25	
Interest on Bonds bought	9.13		
Charged to cancel premium on Notes	2.50		
Income transferred to Principal	152.41		
		164.04	
			$1,942.29

FRANCIS AMORY FUND.

Receipts.

From Investments	$2,055.50

Expenditures.

Publishing Statement	$ 47.55		
Interest on Bonds bought	24.38		
Charged to cancel premium on Bonds bought	10.00		
		$ 81.93	
Income transferred to Principal		1,973.57	
			$2,055.50

The following reports were also presented:

REPORT OF THE LIBRARY COMMITTEE.

During the year 127 volumes and 17 unbound numbers of periodicals have been borrowed by 22 Fellows and 12 libraries, and many more have been consulted at the Academy. All books taken out have been returned or satisfactorily accounted for.

The number of books on the shelves at the time of the last report was 40,884. During the year 355 volumes were added, making the number now 41,239. This includes 55 purchased from the General Fund, seven from the Rumford Fund, and 293 received by gift or exchange. The number of pamphlets added was 143.

The committee has made a general revision of the periodical subscription list and has decided to discontinue subscriptions to the following journals, beginning with January, 1928:

Annales des Mines
Central-Zeitung für Optik und Mechanik
Chemical News
Engineering
Revue Générale des Sciences
Scientia

while "Biological Abstracts" was added to the list.

With a balance of $438.85 left from last year, an appropriation of $3,000, and $10.36 received from the sale of duplicates, the Library Committee has had at its disposal the sum of $3,449.21, apart from the sums derived from the Rumford Fund. The total expenditure from this has been $3,109, leaving an unexpended balance on April 1, 1928, of $340.21. In addition there was available from the Rumford Fund for the purchase and binding of books on light and heat the sum of $347.88, of which $233.94 has been expended, leaving a balance of $113.94.

The expenses charged to the Library during the financial year ending April 1, 1928, are:

Salaries	$1,800.00
Binding:	
General Fund	699.95
Rumford Fund	68.50
Purchase of Books and Periodicals:	
General Fund	551.89
Rumford Fund	165.44
Miscellaneous	57.16
Total	$3,342.94

H. M. Goodwin, *Librarian.*

May 9, 1928.

Report of the Rumford Committee.

The Committee held six meetings during the year (Oct. 19, Nov. 9, Dec. 7, 1927, and Jan. 11, Apr. 11, May 9, 1928).

The Committee made the following grants during the academic year 1927–28:

	1927		
No. 262.	Oct. 9	Prof. Sebastian Albrecht, Dudley Observatory, Albany, N. Y., for the purchase of a spectrophotometer in the measurement of stellar wavelengths	$500
No. 263.	Oct. 19	Prof. Paul Kirkpatrick, University of Hawaii, Honolulu, for apparatus in researches on X-Ray Reflection	250
No. 264.	Oct. 19	Prof. Harlow Shapley, Harvard College Observatory, towards the construction of a stellar comparator for use in researches on the proper motion and variability of stars	500
No. 265.	Nov. 9	Prof. Norton A. Kent, Boston University, for apparatus to be used in researches on spectrum analysis	150
No. 266.	Dec. 7	Prof. John R. Roebuck, University of Wisconsin, Madison, Wis., a further grant for apparatus to be used in the measurement of the Mechanical Equivalent of Heat	500
No. 267.	Dec. 7	Annual contribution to the Preparation of the International Annual Tables of Constants	200
No. 268.	Dec. 7	Prof. Raymond T. Birge, University of California, Berkeley, Cal., for aid in researches on spectrum analysis	200
No. 269.	Dec. 7	Prof. G. W. Pierce, Rumford Professor of Physics, Harvard University, for apparatus to be used in measurements on the velocity of light	300
			$2,600

The Committee voted unanimously, first on December 7, 1927, and again on January 11, 1928, to recommend the award of the Rumford premium to Edward Leamington Nichols, for his researches in spectrophotometry.

Progress reports concerning researches aided by Rumford grants have been received from Messrs. S. Albrecht, P. W. Bridgman, G. S. Forbes, G. H. Harrison, J. C. Hubbard, N. A. Kent, G. W. Pierce, J. R. Roebuck, F. A. Saunders.

The following recent publications relate to researches that have been aided by Rumford grants:

"Doublet Separation and Fine Structure of the Balmer Lines of Hydrogen," by Kent, Taylor and Pearson, Sept. 1927.

"Compressibility and Pressure Coefficient of Resistance of Ten Elements," by P. W. Bridgman. Proc. Am. Ac. Arts & Sc.

"Spectral Energy Characteristics of the Constricted Mercury Vapor Lamp—An Extremely Concentrated Source of Ultraviolet Illumination," by George Shannon Forbes and George R. Harrison. Jour. Optical Soc. of America, Vol. 11, No. 2, Aug. 1925.

"Relations Involving Internal Pressure, Intensity, Mercury Transfer, Cross Section and Electrical Conditions in Mercury-Vapor Lamps," by George Shannon Forbes and Albert Leighton. Jour. Optical Soc. of America, Vol. 12, No. 1, Jan. 1926.

Respectfully submitted,
A. E. Kennelly, *Chairman.*

May 9, 1928.

Report of the C. M. Warren Committee.

The Committee had at its disposal at the beginning of the fiscal year 1927–1928, $1,511.24, of which $405.92 was a balance from the previous year. Six hundred and twenty-five dollars ($625) has been appropriated during the year, leaving a balance of $886.24. The estimated available income for the year 1928–1929 is $1,173.07.

Since the last annual report grants have been made as follows:

December 22, 1927: to Professor Ben Corson, Middlebury College, $200, a renewal of a previous grant to continue his work on the condensation reactions of malonitrile.

December 22, 1927: to Dr. Louis Fieser, Bryn Mawr College, $150, to be used in a research on the synthesis of polycyclic anthracene and anthraquinone derivatives.

Applications for grants amounting to $1,200 are in the hands of the Committee and will be considered at its next meeting in May.

Reports of progress have been received from Professor C. James, Professor Ralph Bullard, Dr. Ben Corson and Dr. Charles F. H. Allen.

The papers listed below, recently published, describe the results of investigations aided by the Warren Fund. In each case there is an acknowledgment by the author of the assistance received.

Charles F. H. Allen and Harry B. Rosener: "The Synthesis of Alpha, Beta-Diacylstyrenes and Their Reaction with Hydrogen Bromide." (J. Amer. Chem. Soc., *49*, 2110 (1927).)

B. B. Corson, R. K. Hazen, and J. S. Thomas: "The Mechanism Underlying the Reaction between Aldehydes or Ketones and Tautometric Substances of the Keto-Enol Type. II. The Condensation of Oxomalonic Esters with Cyano-acetic and Malonic Esters." (J. Amer. Chem. Soc., *50*, 913 (1928).)

B. B. Corson, Ruth A. Dodge, S. A. Harris, and R. K. Hazen: "Ethyl Benzoylformate." (Organic Syntheses, Vol. VII.)

James F. Norris, *Chairman.*

May 9, 1928.

Report of the Committee of Publication.

Since May 1927 five numbers aggregating 105 pages have been printed for Volume 62 of the Proceedings, and Volume 62 will soon be completed with the printing of the records number.

Nos. 1 and 2 of Volume 63 have just come off the press and No. 3 of Volume 63 is now in press.

Nos. 1 and 2 of Volume 63, aggregating 67 pages, cost $516, which is at the rate of $7.70 per page.

Many requests for publication of papers by non-members have come to the Committee, but in every case to date these requests have been declined because of lack of funds except in the case of No. 3 of Volume 63, which represents work done by a non-member which is published without cost to the Academy at the request of Professor Reginald A. Daly.

A request has just come in from Professor Franz Boas asking if the Academy can publish the material collected under the auspices of a committee of which he is chairman. The committee has been arranging texts, grammars, and dictionaries of Indian languages, and has had the support of the Carnegie Corporation. If we are to undertake the publication of this material we will need additional funds. The funds available for next year will no more than cover the usual papers which will come in from our members.

WM. S. FRANKLIN, *Chairman.*

May 9, 1928.

REPORT OF THE HOUSE COMMITTEE.

The House Committee has had funds at its disposal amounting to $3,633.74, made up as follows:

Balance from last year	$ 103.74
Appropriation for 1927–28	3,235.00
Received for use of rooms	295.00
	$3,633.74

Of this amount, the sum of $2,648.24 has been spent for the usual routine expenses, janitor, light, power, heat, telephone, etc., and $672.89 has been spent for upkeep, making a total of $3,321.13, and leaving an unexpended balance of $312.61.

Meetings have been held as follows:

The Academy	
Stated meetings	8
Open meetings	1
American Antiquarian Society	1
Archaeological Institute	2
Boston Biological Society	4
Boston Geological Society	1
Chamber Music Club	8
English-Speaking Union	1
Harvard-Technology Chemical Club	6
Mediaeval Academy of America	1
National Women's Farm and Garden Association	1
Needlework Guild of America	1
New England Botanical Club	9
New England Farm and Garden Association	3
Zeta Phi Fraternity	4
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The Council Chamber has been used for Academy Council and Committee meetings, and also by the Mediaeval Academy, the Trustees of the Children's Museum, the Thursday Evening Club, etc.

A detailed list of expenditures follows:

Janitor			$ 958.00
Electricity	Light	.183.26	
	Power	66.64	
			249.90
Coal			1,084.00
Care of elevator			88.55
Gas			65.16
Water			35.42
Telephone			110.81
Ash tickets			40.20
Upkeep			672.89
Janitor's supplies and sundries			16.20
Total expenditure			$3,321.13

William H. Lawrence, *Chairman.*

May 9, 1928.

On the recommendation of the Treasurer, it was

Voted, That the Annual Assessment be $10.00.

It was

Voted, To consider the proposed amendment to the Statutes to increase the number of Foreign Honorary Members from seventy-five to one hundred, with not more than thirty-five in any one class.

It was

Voted, there being no objection to this procedure, That the Corresponding Secretary be instructed to cast one ballot for the amendment.

On the recommendation of the Rumford Committee, it was

Voted, To award the Rumford Premium to Edward Leamington Nichols, for his researches in spectrophotometry.

The medals were then presented to Mr. Nichols by the Presiden t

The annual election resulted in the choice of the following officers and committees:

EDWIN B. WILSON, *President*
ARTHUR E. KENNELLY, *Vice-President for Class I*
GEORGE H. PARKER, *Vice-President for Class II*
GEORGE L. KITTREDGE, *Vice-President for Class III*
ROBERT P. BIGELOW, *Corresponding Secretary*
CHARLES B. GULICK, *Recording Secretary*
INGERSOLL BOWDITCH, *Treasurer*
HARRY M. GOODWIN, *Librarian*
WILLIAM S. FRANKLIN, *Editor*

Councillors for Four Years.

CHARLES A. KRAUS, *of Class I*
GEORGE R. MINOT, *of Class II*
W. CAMERON FORBES, *of Class III*

Finance Committee.

THOMAS BARBOUR
PAUL J. SACHS
FREDERICK P. FISH

Rumford Committee.

ARTHUR E. KENNELLY
ELIHU THOMSON
PERCY W. BRIDGMAN
HARRY M. GOODWIN
CHARLES L. NORTON
HARLOW SHAPLEY
FREDERICK A. SAUNDERS

C. M. Warren Committee.

JAMES F. NORRIS
GREGORY P. BAXTER
WALTER L. JENNINGS
ARTHUR D. LITTLE
FREDERICK G. KEYES
REID HUNT
CHARLES A. KRAUS

Publication Committee.

(Editor) WILLIAM S. FRANKLIN, *ex officio Chairman*
EDWIN C. KEMBLE, *of Class I*
HERBERT V. NEAL, *of Class II*
GEORGE F. MOORE, *of Class III*

Library Committee.

(Librarian) HARRY M. GOODWIN, *ex officio Chairman*
RAYMOND C. ARCHIBALD, *of Class I* THOMAS BARBOUR, *of Class II*
WILLIAM C. LANE, *of Class III*

House Committee.

S. BURT WOLBACH, *Chairman*
ROBERT P. BIGELOW WILLIAM H. LAWRENCE

Committee on Meetings.

THE PRESIDENT
THE RECORDING SECRETARY
GEORGE H. PARKER GREGORY P. BAXTER WILLIAM C. GREENE

Auditing Committee.

GEORGE R. AGASSIZ JOHN E. THAYER

The Council reported that the following gentlemen were elected members of the Academy:

FELLOWS.

CLASS I.

Section 1.

Jacob David Tamarkin, Providence, R. I.

Section 2.

Adelbert Ames, Jr., Hanover, N. H.
Arthur Holly Compton, Chicago, Ill.
Manuel Sandoval Vallarta, Brookline.

Section 3.

Louis John Gillespie, Cambridge.
George Scatchard, Cambridge.

Section 4.

Johnson O'Connor, Boston.
Landgon Pearse, Chicago, Ill.
John F. Stevens, New York, N. Y.
Charles Terzaghi, Cambridge.

CLASS II.

Section 2.

Richard Thornton Fisher, Petersham.
Willis Linn Jepson, Berkeley, Calif.

Section 3.

John Wymond Miller Bunker, Belmont.
Roy Graham Hoskins, Brookline.
Thomas Hunt Morgan, New York, N. Y.

Section 4.

Warfield Theobald Longcope, Baltimore, Md.
William Lorenzo Moss, Boston.
John Howard Mueller, Boston.

CLASS III.

Section 1.

Edgar Sheffield Brightman, Newton Centre.
Walter Fenno Dearborn, Cambridge.
John Dewey, New York, N. Y.
Albert Cornelius Knudson, Cambridge.
Ralph Barton Perry, Cambridge.
Henry Bradford Washburn, Cambridge.

Section 2.

William Albert Nitze, Chicago, Ill.
Joshua Whatmough, Cambridge.

Section 3.

James Phinney Baxter, 3d, Cambridge.
Arthur Harrison Cole, Cambridge.
William Yandell Elliott, Cambridge.
Clarence Henry Haring, Cambridge.
Warren Milton Persons, Cambridge.
Arthur Meier Schlesinger, Cambridge.

Section 4.

Curtis Hidden Page, Hanover, N. H.

Foreign Honorary Members.

CLASS I.

Section 2.

Friedrich Paschen, Berlin.

Section 3.

Sören Peter Lauritz Sörensen, Copenhagen.

Section 4.

Guglielmo Marconi, Bologna.

CLASS II.

Section 2.

Carl Hansen Ostenfelt, Copenhagen.
Alfred Barton Rendle, London.

Section 3.

Francis Arthur Bather, London.
Louis Dollo, Brussels.
Wolfgang Köhler, Berlin.
Charles Tate Regan, London.
D'Arcy Wentworth Thompson, St. Andrews.

Section 4.

Mikinosuke Miyajima, Tokyo.

CLASS III.

Section 1.

Henri Louis Bergson, Paris.
Benedetto Croce, Naples.
Richard Burdon Haldane, Viscount Haldane of Cloan, London.
Edmund Husserl, Freiburg i.B.

Section 3.

Karl Pearson, London.
Hanns Oertel, Munich.
Arthur Cecil Pigou, Cambridge, England.

ASSOCIATES.

Franklin Greene Balch, Jamaica Plain.
Charles Francis Dorr Belden, Jamaica Plain.
Charles Knowles Bolton, Boston.
Roland William Boyden, Beverly.
Frederick Pickering Cabot, Boston.
George Alexander Philips Haldane-Duncan, Earl of Camperdown, Boston.
John Candler Cobb, Milton.
Howard Coonley, Milton.
William Lusk Webster Field, Milton.
Thomas Goddard Frothingham, Boston.
Edward Jackson Holmes, Boston.
George Snell Mandell, Boston.
Robert Lincoln O'Brien, Dedham.
Thomas Nelson Perkins, Westwood.
Arthur Stanwood Pier, Milton.
Alfred Lawrence Ripley, Andover.
Odin Roberts, Boston.
Jeremiah Smith, Jr., Cambridge.
Payson Smith, Brookline.
Eliot Wadsworth, Boston.
Edmund Allen Whitman, Cambridge.

The following communication was presented:

Mr. Edward L. Nichols: "Reminiscences of Fifty Years' Work in Spectro-Photometry."

The following paper was presented by title:

"Studies on the Fauna of Hot Springs in the Western United States and the Biology of Thermophilous Animals," by Charles T. Brues.

The meeting was dissolved at 9.55 P. M.

BIOGRAPHICAL NOTICES.

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JAMES FURMAN KEMP (1859–1926).

Fellow in Class II, Section 1, 1921.

James Furman Kemp (born in New York on August 14, 1859, died November 17, 1926) received, as will appear from "Who's Who," vol. 14, all the honors an American geologist could expect, and was for thirty-five years head of the Department of Geology at Columbia University, New York City. The complete bibliographic list is long and will be found appended to the memoir in vol. 39 of the Bulletin of the Geological Society of America.

Longer personal tributes will be found in "Science," Dec. 11, 1926, vol. LXIV, no. 1670, pp. 639-642, and the "Columbia Alumni News," vol. XVIII, no. 10, Dec. 3, 1926, and the Proceedings of the American and Canadian Mining Institutes; and many other journals testify to the affection for him and the regret for his loss. To these accounts of his relations with colleagues and pupils a personal word may be added:

"Kemp was with me two years as a pupil, and for a time after graduation, and we drove and tramped together over the hills. He was a cheerful and devoted worker, and the best of companions. A few years ago I was his near neighbor for a long time, and learned to know him in his maturer years, and came to know his pleasant family. He was as always genial and faithful to his friends and his work, and I was delighted that he had not only become a widely known scholar, but a devoted and much loved teacher. It was a beautiful life."

B. K. Emerson.

FRANCIS WELD PEABODY (1881–1927).

Fellow in Class II, Section 4, 1921.

Francis Weld Peabody was successful as teacher, director of a laboratory, administrator of a hospital service, and practitioner of medicine. He made useful contributions to the science of medicine and was beloved for his humanity to his patients and his friends. He was endowed with an inspiring personality that radiated loving kindness. He faced Fate with a calm courage that will remain an example.

Detailed notices of his activities will be found in a minute of the records of the Faculty of Medicine of Harvard University; in the Boston Medical and Surgical Journal of October 20th and December 15th, 1927; the Journal of the American Medical Association, October 22, 1927, page 1443; the Harvard Graduates Magazine, January 1928.

M. J. Rosenau.

The following is reprinted from the Journal of Clinical Investigation, vol. 5, no. 1, Dec. 1927:

Francis Weld Peabody was born at Cambridge, Massachusetts, November 24, 1881, and died at the same place, October 13, 1927. His ancestors were New England people of high character and distinguished ability. His father was formerly a professor at Harvard University and later Dean of the Harvard Divinity School. He received through inheritance a sound body and a good brain, and from his early environment, which was the best that this country offers, he undoubtedly acquired the "gentleness, and unshaken adherence to judgments deliberately formed; indifference to outward show and compliment, industry and assiduity," that were to distinguish him throughout his life.

He graduated from Harvard College in 1903, and the same year he entered the Harvard Medical School where he showed conspicuous ability. While still an undergraduate, he undertook with his teacher, Dr. J. H. Pratt, the solution of a bacteriological problem relating to typhoid fever, and this was published the month that he graduated, in the Journal of the American Medical Association. Thus he began the practice of investigation and publication, which he continued to the very end of his life. He received the degree of Doctor of Medicine in 1907, and the following year he served as interne in the Massachusetts General Hospital.

Thus far he had followed the beaten path. But now he departed from the course which at that time was almost universal in America. Instead of entering at once on the practice of his profession, he undertook his "Wander Jahre," a custom which, earlier at least, was frowned upon in America, but which in Europe has for centuries been held to offer the greatest rewards to the student in search of an education.

He spent two years in the Johns Hopkins Medical School at Baltimore, first as Assistant Resident Physician under Dr. Thayer, and later as Fellow in Pathology under Dr. Welch. In April of 1910, he went to Berlin and worked in the laboratory of Emil Fischer on organic chemistry. In the autumn, he returned to this country and became Assistant Resident Physician in the Hospital of the Rockefeller Institute, then newly opened. He remained there for almost two years, leaving in the spring for another trip to Europe, where he visited a number of clinics in Germany, made a short stay in Russia, and worked for six weeks on a physiological problem in the laboratory of Professor Krogh in Copenhagen.

He now returned to Boston after an absence of over four years. He did not return, however, with the idea that his education had been completed, that his student days were over. The Peter Bent Brigham Hospital had just been erected on ground adjacent to the Harvard Medical School, and it was intended that this hospital should be organized as a medical clinic of the school. Dr. Peabody was offered the position of Resident Physician and accepted this proposal for the reason, as he said, that this new institution provided facilities and equipment and teachers that would permit him to continue his studies and investigations under favorable conditions, and also because it gave him an opportunity to aid in the development of a modern medical clinic in which teaching and research should be important functions. At that time new ideas concerning medical education were in the air, and he was one of the small group of young men in this country who recognized the need of developing a scientific atmosphere in the medical clinic if the medical schools were to perform their proper function, and if medicine in this country was to keep abreast of that existing in other parts of the world.

He continued in residence at the Peter Bent Brigham Hospital for three years, working in the clinic, developing his ability as a teacher; busy in the wards, widening his experience with the maladies that afflict man, and increasing his sympathetic understanding of those who suffer; active in the laboratory, constantly carrying on investigations with methods of precision, and thus extending his knowledge of the nature of disease and increasing his power as a man of science. During all these years which had passed since graduation, he showed a remarkable spirit of restraint and patience by persisting in the course of training on which he had, years before, voluntarily and independently resolved.

But these years were not merely years of self development and selfish acquisition of knowledge. He had been of great benefit to all the patients who came under his care, he had been helpful to the students with whom he had been in contact, he had been useful to his university to which he was devoted, and he had been of service to humanity through the contributions he had already made to medical knowledge. But in still other ways he was of service, for in 1914 he was given leave of absence to join a commission of the Rockefeller Foundation which went to China to give advice regarding the new school of medicine which it was proposed to create in Peking. As the medical member of this commission, he undoubtedly had much influence in shaping the future policy of the Peking Union Medical School, and he was later made a member of the China Medical Board of the Rockefeller Foundation.

In 1915 he gave up his residence at The Peter Bent Brigham Hospital but continued as Physician to that institution, and was appointed Associate Professor of Medicine in Harvard University. He now continued for five years to teach, to practice, and to investigate. During these years, however, there were many other professional activities. In 1917 he served as a member of the American Red Cross Commission to Roumania, and later to Russia. On the entry of the United States into the war, he became a member of the Army Medical Corps, and served in hospitals in this country, later going to France as Medical Consultant.

In 1921 he was appointed Professor of Medicine in Harvard University and, at the same time, became Director of the Thorndyke Memorial Laboratory. In Dr. Peabody's words, this laboratory "is a reasearch department of the Boston City Hospital. Its establishment is due to the conviction of the Board of Trustees that the responsibility of the City Hospital is not limited to the treatment and care of individual patients, but includes also medical teaching, preventive medicine, and medical research." This attitude on the part of the authorities of this institution marked a decided departure from the traditional policy of American municipal hospitals, and was of much significance for future medical education in this country. It was very fitting that the Trustees should have turned to Dr. Peabody to undertake the direction of this new department. Brief mention can only be made here of the splendid manner in which he carried out the plans of its organization, arranging that its equipment should permit the prosecution of fundamental studies concerning disease, and gathering together for its staff a group of well trained young men, eager for research. Here Dr. Peabody continued his teaching and investigation, at the same time stimulating and aiding his co-workers. Within a short time important contributions from the staff began to appear.

Dr. Peabody began his own career as an investigator of disease by undertaking studies concerning typhoid fever, the first of which has already been mentioned. During his residence at the Johns Hopkins Hospital, he began his studies of heart disease. Later, at the Rockefeller Institute, he undertook the investigation of problems relating to poliomyelitis and acute lobar pneumonia. At the Peter Bent Brigham Hospital, he began a long series of investigations concerning metabolism and the function of respiration. While Director of the Thorndyke Laboratory, he undertook the study of pernicious anemia. To all these subjects he made definite contributions. In all of his studies he was obviously interested in the application of the results to practice. For instance, he made studies concerning the function of respiration which have much scientific interest and significance, and he then indicated the application of his results; he showed the importance of a decrease in the vital capacity of the lungs as a factor in the production of dyspnea in heart disease, and demonstrated how the determination of the vital capacity could be made without difficulty in patients. He thus introduced into clinical medicine a new method that has proved to be of distinct value.

It is impossible in this place to attempt to evaluate his contributions to the science of medicine. That will undoubtedly be done elsewhere. It is one of the hardships suffered by workers in medicine that the domain is so vast, and that, if one is to investigate disease as a whole, he must be acquainted with many kinds of technique, for in no disease is there a disturbance of only one function, the entire mechanism of the body is always involved. The science of medicine demands that its votaries shall have some knowledge of all the sciences. That

Dr. Peabody could apply himself in succession to various branches of internal medicine, employing different methods in each, and to each one of these subjects adding to knowledge, is the best evidence of his ability and mental capacity. During his years at work in the Thorndyke Laboratory, carrying on his duties as teacher, and, to a more limited extent, as practitioner, he found time to respond to many demands that were made upon him to give aid and council in projects of importance to medicine. When the Journal of Clinical Investigation was founded in 1924, he was made a member of the Editorial Board. His interest in, and sympathy with, the young men of the medical clinics who were making original contributions to the science of medicine, made him keenly desire that a special organ should be established in which their reports might be published. He was an active and useful member of the Board, and helped in shaping its policies. He served on the Council of the Association of American Physicians, and was also an officer in this organization. In 1926 he was made a member of the Board of Scientific Directors of the Rockefeller Institute for Medical Research.

His ability and training, his upright character, his academic interests, his success as a teacher and investigator, made him sought after by a number of universities to undertake the reorganization of their departments of medicine along modern lines. He refused, however, to leave his own university, believing that here he could be of the greatest service.

At the age of forty-five, he was at the height of his usefulness. With an ideal training, full of energy and resourcefulness, all those who were interested in medicine and its future looked upon him as one of its most able representatives, and as one upon whom would fall many of the responsibilities associated with shaping the development of medical education and practice in this country during the next decades. When, therefore, in the summer of 1926 his friends heard that, although apparently in the best of health, he was stricken with a serious disease, from which there was little chance of his recovering, they were shocked and dismayed; shocked that they were soon to lose a beloved associate, dismayed that the profession of medicine was to lose one of its most able defenders and promoters. But no consternation or apprehension was evident in Dr. Peabody himself. With at least an outward calmness and tranquility, he faced the inevitable. He decided not to let the knowledge of his fate hinder him from continuing his work or prevent him from leading his accustomed life. Consideration for his family and friends undoubtedly influenced him in coming to this decision. Not with a sad and sorrowful countenance, but with a bright and unflinching spirit he continued his daily tasks. For over a year he continued his work, a comfort to his patients, an inspiration to his students and associates, a champion of the science of medicine, and an instrument in promoting its progress.

He was not only interested in the development of the science of medicine, however; he understood that "the application of the principles of science to the diagnosis and treatment of disease is only one limited aspect of medical practice." Shortly before his death he wrote an essay on "The Care of the Patient," which not only is a beautiful example of simple, straightforward writing but is an exposition of his own attitude toward his fellow men. The essay closes with the effective lines, "One of the essential qualities of the physician is interest in humanity, for the secret of the care of the patient is in caring for the patient." And this is also one of the secrets of his own engaging personality. This Abou Ben Adhem was devoted to his friends, fond of his associates. He liked people. He had delightful social qualities, and concealed a seriousness of mind and a great earnestness of purpose under a delightful and winning humor. He was always—

"A square-set man and honest; and his eyes,
An out-door sign of all the warmth within,"

and even during his last days—

"Smiled with his lips—a smile beneath a cloud."

American Academy of Arts and Sciences.

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Class I.—*Mathematical and Physical Sciences.*—197.

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Austin Flint Rogers	Palo Alto, Cal.

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William Chase Greene	Cambridge
Charles Burton Gulick	Cambridge
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William Arthur Heidel	Middletown, Conn.
George Lincoln Hendrickson	New Haven, Conn.
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Eugene Xavier Louis Henry Hyvernat	Washington, D. C.
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James Richard Jewett	Cambridge
Alfred Vincent Kidder	Andover
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Chandler Rathfon Post	Cambridge
Edward Kennard Rand	Cambridge
George Andrew Reisner	Cambridge
Edward Robinson	New York, N. Y.

Fred Norris Robinson	Cambridge
Rudolph Schevill	Berkeley, Cal.
Herbert Weir Smyth	Cambridge
Herbert Joseph Spinden	Cambridge
Franklin Bache Stephenson	Washington, D. C.
Charles Cutler Torrey	New Haven, Conn.
Alfred Marston Tozzer	Cambridge
Langdon Warner	Cambridge
Joshua Whatmough	Cambridge
Clark Wissler	New York, N. Y.
James Haughton Woods	Cambridge

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Charles McLean Andrews	New Haven, Conn.
James Phinney Baxter, 3d	Cambridge
Carl Lotus Becker	Ithaca, N. Y.
Robert Pierpont Blake	Cambridge
Charles Jesse Bullock	Cambridge
Thomas Nixon Carver	Cambridge
Edward Channing	Cambridge
John Bates Clark	New York, N. Y.
Arthur Harrison Cole	Cambridge
Richard Henry Dana	Cambridge
Clive Day	New Haven, Conn.
Davis Rich Dewey	Cambridge
William Yandell Elliott	Cambridge
Ephraim Emerton	Cambridge
Henry Walcott Farnam	New Haven, Conn.
Max Farrand	San Gabriel, Cal.
William Scott Ferguson	Cambridge
Irving Fisher	New Haven, Conn.
William Cameron Forbes	Milton
Worthington Chauncey Ford	Cambridge
Edwin Francis Gay	Cambridge
Frank Johnson Goodnow	Baltimore, Md.
Evarts Boutell Greene	New York, N. Y.

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Westel Woodbury Willoughby	Baltimore, Md.
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George Lyman Kittredge	Cambridge
William Coolidge Lane	Cambridge
John Ellerton Lodge	Boston
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John Livingston Lowes	Cambridge
Charles Donagh Maginnis	Brookline
Albert Matthews	Boston
Harold Murdock	Brookline
William Allan Neilson	Northampton
Curtis Hidden Page	Hanover, N. H.
William Lyon Phelps	New Haven, Conn.
Arthur Kingsley Porter	Cambridge
Herbert Putnam	Washington, D. C.
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Denman Waldo Ross	Cambridge
Paul Joseph Sachs	Cambridge
Ellery Sedgwick	Boston
Henry Dwight Sedgwick	Boston
Richard Clipston Sturgis	Boston
Edmund C. Tarbell	Boston
Charles Howard Walker	Boston
Owen Wister	Philadelphia, Pa.
George Edward Woodberry	Beverly
Charles Henry Conrad Wright	Cambridge

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Jacques Salomon Hadamard	Paris
Godfrey Harold Hardy	Oxford
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Tullio Levi-Civita	Rome
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Friedrich Paschen	Berlin
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Sir Ernest Rutherford	Cambridge
Sir Joseph John Thomson	Cambridge

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Maurice d'Ocagne	Paris
Vsevolod Evgenievich Timonoff	Leningrad
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Viktor Goldschmidt	Heidelberg
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Raymond Poincaré	Paris
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Hanns Oertel	Munich
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Jean Adrien Antoine Jules Jusserand	Paris
Rudyard Kipling	Burwash
Victor Laloux	Paris
Gilbert Murray	Oxford
Henri Rabaud	Paris

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La Piana, G. F, III: 3
Larmor, Sir J. FHM, I: 2
Larsen, E. S. F, II: 1
Lawrence, W. F, III: 1
Lawrence, W. H. F, I: 4
Laws, F. A. F, I: 2
Lawson, A. C. F, II: 1
Lawton, F. F, III: 1
Le Chatelier, H. L. FHM, I: 3
Lee, R. I. F, II: 4
Lefavour, H. F, I: 2
Leith, C. K. F, II: 1
Levi-Civita, T. FHM, I: 1
Lewis, F. T. F, II: 3
Lewis, G. N. F, I: 3
Lewis, W. K. F, I: 3
Lillie, R. S. F, II: 3
Lindgren, W. F, II: 1
Lipman, J. G. F, II: 2
Little, A. D. F, I: 3
Livingston, B. E. F, II: 2
Lodge, J. E. F, III: 4
Loeffler, C. M. T. F, III: 4
Longcope, W. T. F, II: 4
Loomis, F. B. F, II: 1
Loring, W. C. F, III: 1
Lowell, A. L. F, III: 3
Lowes, J. L. F, III: 4
Lull, R. S. F, II: 3
Luyten, W. J. F, I: 1
Lyman, T. F, I: 2
Lyon, D. G. F, III: 2
McAdie, A. G. F, II: 1
MacDonald, W. F, III: 3
Macdonell, A. A. FHM, III: 2
McDougall, W. F, III: 1
McIlwain, C. H. F, III: 3
MacInnes, D. A. F, I: 3
McLaughlin, D. H. F, II: 1
Maginnis, C. D. F, III: 4
Main, C. T. F, I: 4
Mallory, F. B. F, II: 4
Margerie, E. de. FHM, II: 1
Mark, E. L. F, II: 3
Marks, L. S. F, I: 4
Martin, E. G. F, II: 3
Mather, K. F. F, II: 1
Mathews, E. B. F, II: 1
Matthews, A. F, III: 4
Maudslay, A. P. FHM, III: 2
Mayo, W. J. F, II: 4
Mead, A. D. F, II: 3
Means, J. H. F, II: 4
Melander, A. L. F, II: 3
Merriam, J. C. F, II: 1
Merrill, E. D. F, II: 2
Merriman, R. B. F, III: 3
Merritt, E. G. F, I: 2
Michelson, A. A. F, I: 2
Miller, D. C. F, I: 2
Miller, E. F. F, I: 4
Miller, G. A. F, I: 1
Miller, G. S. F, II: 3
Miller, J. A. F, I: 1
Miller, W. J. F, II: 1

Millikan, R. A. F, I: 2
Minot, G. R. F, II: 4
Mitchell, S. A. F, I: 1
Miyajima, M. FHM, II: 4
Molengraaff, G. A. F. FHM, II: 1
Moore, C. H. F, III: 2
Moore, C. L. E. F, I: 1
Moore, E. C. F, III: 1
Moore, E. H. F, I: 1
Moore, G. D. F, I: 3
Moore, G. F. F, III: 2
Moore, J. B. F, III: 1
Morgan, T. H. F, II: 3
Morison, S. E. F, III: 3
Morley, F. F, I: 1
Morse, H. W. F, I: 2
Morss, E. A
Moss, W. L. F, II: 4
Moulton, F. R. F, I: 1
Mueller, E. F, I: 3
Müller, F. von. FHM, II: 4
Mueller, J. H. F, II: 4
Mulliken, S. P. F, I: 3
Munro, W. B. F, III: 3
Munroe, C. E. F, I: 3
Murdock, H. F, III: 4
Murray, G. FHM, III: 4
Neal, H. V. F, II: 3
Neilson, W. A. F, III: 4
Nichols, C. L. F, III: 3
Nichols, E. L. F, I: 2
Nitze, W. A. F, III: 2
Norris, J. F. F, I: 3
Norton, C. L. F, I: 2
Noyes, A. A. F, I: 3
Noyes, W. A. F, I: 3
Nuttall, G. H. F. FHM, II: 3
O'Brien, R. L. A
d'Ocagne, M. FHM, I: 4
O'Connor, J. F, I: 4
Oertel, H. FHM, III: 2
Olmsted, F. L. F, I: 4
Osborn, H. F. F, II: 3
Osgood, R. B. F, II: 4
Ostenfelt, C. H. FHM, II: 2
Osterhout, W. J. V. F, II: 2
Ostwald, W. FHM, I: 3
Page, C. H. F, III: 4
Palache, C. F, II: 1
Palmer, G. H. F, III: 1
Park, C. E. F, III: 1
Park, C. F. F, I: 4
Parker, G. H. F, II: 3
Parks, L. F, III: 1
Parsons, W. B. F, I: 4
Paschen, F. FHM, I: 2
Patten, W. F, II: 3
Peabody, F. G. F, III: 1
Pearl, R. F, II: 3
Pearse, L. F, I: 4
Pearson, K. FHM, III: 3
Peirce, G. J. F, II: 2
Pender, H. F, I: 4
Pepper, G. W. F, III: 1
Perkin, W. H. FHM, I: 3
Perkins, T. N. A
Perry, R. B. F, III: 1
Persons, W. M. F, III: 3
Peters, A. J. A
Phelps, W. L. F, III: 4
Phillips, H. B. F, I: 1
Phillips, J. C. F, II: 3
Philpott, A. J. A
Picard, C. E. FHM, I: 1
Pickering, W. H. F, I: 1
Pidal, R. M. FHM, III: 3
Pier, A. S. A
Pierce, G. W. F, I: 2
Pigou, A. C. FHM, III: 3
Pilsbry, H. A. F, II: 3
Pirenne, H. FHM, III: 3
Planck, M. FHM, I: 2
Poincaré, R. FHM, III: 1
Pollock, Sir F. FHM, III: 1
Poor, C. L. F, I: 1
Porter, A. K. F, III: 4
Post, C. R. F, III: 2
Pound, R. F, III: 1

Pratt, F. H. F, II: 3
Pratt, J. H. F, II: 4
Prescott, S. C. F, I: 3
Pupin, M. I. F, I: 2
Putnam, H. F, III: 4
Rabaud, H. FHM, III: 4
Rand, E. K. F, III: 2
Rand, H. W. F, II: 3
Raymond, P. E. F, II: 1
Redfield, A. C. F, II: 3
Redlich, J. FHM, III: 1
Regan, C. T. FHM, II: 3
Rehder, A. F, II: 2
Reisner, G. A. F, III: 2
Rendle, A. B. FHM, II: 2
Richards, R. H. F, I: 3
Richardson, R. G. D. F, I: 1
Richardson, W. L. F, II: 4
Ripley, A. L. A
Ritter, W. E. F, II: 3
Roberts, O. A
Robinson, B. L. F, II: 2
Robinson, E. F, III: 2
Robinson, E. A. F, III: 4
Robinson, F. N. F, III: 2
Rogers, A. F. F, II: 1
Root, E. F, III: 1
Ropes, J. H. F, III: 1
Rosanoff, M. A. F, I: 3
Rosenau, M. J. F, II: 4
Ross, D. W. F, III: 4
Rostovtzeff, M. I. F, III: 3
Rowe, A. W. F, I: 3
Rugg, A. P. F, III: 1
Russell, H. N. F, I: 1
Rutherford, Sir E. FHM, I: 2
Ruthven, A. G. F, II: 3
Sachs, P. J. F, III: 4
Sarton, G. F, III: 1
Saunders, F. A. F, I: 2
Sauveur, A. F, I: 4
Sayles, R. W. F, II: 1
Scatchard, G. F, I: 3
Schaller, W. T. F, II: 1
Schevill, R. F, III: 2
Schlesinger, A. M. F, III: 3
Schlesinger, F. F, I: 1
Schuchert, C. F, II: 1
Scott, A. W. F, III: 1
Scott, W. B. F, II: 1
Sedgwick, E. F, III: 4
Sedgwick, H. D. F, III: 4
Sellards, A. W. F, II: 4
Setchell, W. A. F, II: 2
Seymour, A. B. F, II: 2
Shapley, H. F, I: 1
Shaw, Sir W. N. FHM, II: 1
Shepherd, F. J. FHM, II: 4
Sherrill, M. S. F, I: 3
Sherrington, Sir C. S. FHM, II: 4
Shimer, H. W. F, II: 1
Slater, J. C. F, I: 2
Slipher, V. M. F, I: 1
Slocum, F. F, I: 1
Smith, H. M. F, I: 3
Smith, J., Jr. A
Smith, P. A
Smith, T. F, II: 4
Smyth, H. L. F, I: 4
Smyth, H. W. F, III: 2
Snyder, V. F, I: 1
Sörensen, S. P. L. FHM, I: 3
Sperry, W. L. F, III: 1
Spinden, H. J. F, III: 2
Spofford, C. M. F, I: 4
Stakman, E. C. F, II: 2
Stebbins, J. F, I: 1
Stephenson, F. B. F, III: 2
Stetson, H. T. F, I: 1
Stevens, J. F. F, I: 4
Stieglitz, J. O. F, I: 3
Stiles, C. W. F, II: 4
Stiles, P. G. F, II: 3
Stone, J. S. F, I: 2
Storey, M. F, III: 1
Story, W. E. F, I: 1
Stratton, S. W. F, I: 4
Strong, R. P. F, II: 4

Sturgis, R. C. F, III: 4
Sturgis, W. C. F, II: 2
Sumner, J. O. F, III: 3
Swain, G. F. F, I: 4
Taber, H. F, I: 1
Taft, W. H. F, III: 1
Tamarkin, J. D. F, I: 1
Tarbell, E. C. F, III: 4
Taussig, F. W. F, III: 3
Taylor, C. H. A
Taylor, E. W. F, II: 4
Taylor, F. B. F, II: 1
Terzaghi, C. F, I: 4
Thaxter, R. F, II: 2
Thayer, J. E. F, II: 3
Thayer, W. S. F, II: 4
Thompson, D'A. W. FHM, II: 3
Thompson, M. deK. F, I: 2
Thomson, E. F, I: 2
Thomson, Sir J. J. FHM, I: 2
Timonoff, V. E. FHM, I: 4
Tolman, R. C. F, I: 3
Torrey, C. C. F, III: 2
Tozzer, A. M. F, III: 2
Trelease, W. F, II: 2
Trevelyan, Sir G. O. FHM, III: 3
Turner, F. J. F, III: 3
Tyler, H. W. F, I: 1
Tyzzer, E. E. F, II: 4
Unwin, W. C. FHM, I: 4
Urban, I. FHM, II: 2
Vallarta, M. S. F, I: 2
Vallée Poussin, C. J. de la. FHM, I: 1
Van Tyne, C. H. F, III: 3
Vaughan, T. W. F, II: 1
Veblen, O. F, I: 1
Verhoeff, F. H. F, II: 4
Vogt, J. H. L. FHM, II: 1
Vries, H. de. FHM, II: 2
Wadsworth, E. A
Wait, W. C. F, III: 1
Walcott, H. P. F, II: 4
Walker, C. H. F, III: 4
Walker, W. H. F, I: 3
Wambaugh, E. F, III: 1
Ward, R. De C. F, II: 1
Warner, E. P. F, I: 4
Warner, L. F, III: 2
Warren, C. H. F, II: 1
Warren, W. F, III: 1
Washburn, H. B. F, III: 1
Watson, J. B. F, II: 3
Wearn, J. T. F, II: 4
Webster, D. L. F, I: 2
Webster, E. S. A
Welch, W. H. F, II: 4
Weston, R. S. F, I: 4
Weston, W. H. F, II: 2
Weysse, A. W. F, II: 3
Wettstein, R. FHM, II: 2
Whatmough, J. F, III: 2
Wheeler, W. M. F, II: 3
White, B. F, II: 4
White, D. F, II: 1
Whitehead, A. N. F, I: 1
Whitlock, H. P. F, II: 1
Whitman, E. A. A
Whitney, W. R. F, I: 3
Williams, F. H. F, II: 4
Williams, R. S. F, I: 3
Willis, B. F, II: 1
Williston, S. F, III: 1
Willoughby, W. W. F, III: 3
Wilson, E. B. F, II: 3
Wilson, E. B. F, I: 2
Wilson, G. G. F, III: 3
Winship, G. P. F, III: 3
Winslow, A. F, II: 1
Wissler, C. F, III: 2
Wister, O. F, III: 4
Wolbach, S. B. F, II: 4
Wolff, J. E. F, II: 1
Wood, H. C. F, II: 4
Wood, R. W. F, I: 2
Woodberry, G. E. F, III: 4
Woodman, A. G. F, I: 3
Woods, F. A. F, II: 3
Woods, F. S. F, I: 1

Woods, J. H. F, III: 2
Worcester, J. R. F, I: 4
Wright, C. H. C. F, III: 4
Wright, F. E. F, II: 1
Wright, Q. F, III: 1
Yerkes, R. M. F, II: 3
Young, A. A. F, III: 3
Young, B. L. A
Zeleny, J. F, I: 2
Zinsser, H. F, II: 4

STATUTES AND STANDING VOTES.

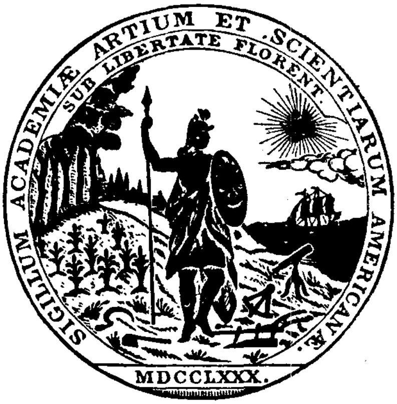
STATUTES.

Adopted November 8, 1911: *amended May* 8, 1912, *January* 8, *and May* 14, 1913, *April* 14, 1915, *April* 12, 1916, *April* 10, 1918, *May* 14, 1919, *February* 8, *April* 12, *and December* 13, 1922, *February* 14, *March* 14, *and October* 10, 1923, *and March* 10, 1926, *and May* 9, 1928.

CHAPTER I.

THE CORPORATE SEAL.

ARTICLE 1. The Corporate Seal of the Academy shall be as here depicted:


SIGILLUM ACADEMIÆ ARTIUM ET SCIENTIARUM AMERICANÆ
SUB LIBERTATE FLORENT
MDCCLXXX.

ARTICLE 2. The Recording Secretary shall have the custody of the Corporate Seal.

See Chap. v, art. 3; chap. vi, art. 2.

CHAPTER II.

Fellows and Foreign Honorary Members and Dues.

Article 1. The Academy consists of Fellows, who are either citizens or residents of the United States of America, and Foreign Honorary Members. They are arranged in three Classes, according to the Arts and Sciences in which they are severally proficient, and each Class is divided into four Sections, namely:

Class I. *The Mathematical and Physical Sciences*
- Section 1. Mathematics and Astronomy
- Section 2. Physics
- Section 3. Chemistry
- Section 4. Technology and Engineering

Class II. *The Natural and Physiological Sciences*
- Section 1. Geology, Mineralogy, and Physics of the Globe
- Section 2. Botany
- Section 3. Zoölogy and Physiology
- Section 4. Medicine and Surgery

Class III. *The Moral and Political Sciences*
- Section 1. Theology, Philosophy, and Jurisprudence
- Section 2. Philology and Archæology
- Section 3. Political Economy and History
- Section 4. Literature and the Fine Arts

Article 2. The number of Fellows shall not exceed Six hundred, of whom not more then Four hundred shall be residents of Massachusetts, nor shall there be more than Two hundred and ten in any one Class.

Article 3. The number of Foreign Honorary Members shall not exceed One hundred. They shall be chosen from among citizens of foreign countries most eminent for their discoveries and attainments in any of the Classes above enumerated. There shall not be more than Thirty-five in any one Class.

Article 4. If any person, after being notified of his election as Fellow, shall neglect for six months to accept in writing, or, if a Fellow resident within fifty miles of Boston shall neglect to pay his Admission Fee, his election shall be void; and if any Fellow resident within fifty miles of Boston shall neglect to pay his Annual Dues for six months after they are due, provided his attention shall have been called to this Article of the Statutes in the meantime, he shall cease to be a Fellow; but the Council may suspend the provisions of this Article for a reasonable time.

With the previous consent of the Council, the Treasurer may dispense (*sub silentio*) with the payment of the Admission Fee or of the Annual Dues or both whenever he shall deem it advisable. In the case of officers of the Army or Navy who are out of the Commonwealth on duty, payment of the Annual Dues may be waived during such absence if continued during the whole financial year and if notification of such expected absence be sent to the Treasurer. Upon similar notification to the Treasurer, similar exemption may be accorded to Fellows subject to Annual Dues, who may temporarily remove their residence for at least two years to a place more than fifty miles from Boston.

If any person elected a Foreign Honorary Member shall neglect for six months after being notified of his election to accept in writing, his election shall be void.

See Chap. vii, art. 2.

Article 5. Every Fellow resident within fifty miles of Boston hereafter elected shall pay an Admission Fee of Ten dollars.

Every Fellow resident within fifty miles of Boston shall, and others may, pay such Annual Dues, not exceeding Fifteen dollars, as shall be voted by the Academy at each Annual Meeting, when they shall become due; but any Fellow shall be exempt from the annual payment if, at any time after his admission, he shall pay into the treasury Two hundred dollars in addition to his previous payments. Any Fellow shall also be exempt from Annual Dues who has paid such dues for forty years, or, having attained the age of seventy-five has paid dues for twenty-five years.

All Commutations of the Annual Dues shall be and remain permanently funded, the interest only to be used for current expenses.

Any fellow not previously subject to Annual Dues who takes up his residence within fifty miles of Boston, shall pay to the Treasurer within three months thereafter Annual Dues for the current year, failing which his Fellowship shall cease; but the Council may suspend the provisions of this Article for a reasonable time.

Only Fellows who pay Annual Dues or have commuted them may hold office in the Academy or serve on the Standing Committees or vote at meetings.

Article 6. Fellows who pay or have commuted the Annual Dues and Foreign Honorary Members shall be entitled to receive gratis one copy of all Publications of the Academy issued after their election.

See Chap. xi, art. 2.

Article 7. Diplomas signed by the President and the Vice-President of the Class to which the member belongs, and countersigned by the Secretaries, shall be given to Foreign Honorary Members and to Fellows on request.

Article 8. If, in the opinion of a majority of the entire Council, any Fellow or Foreign Honorary Member shall have rendered himself unworthy of a place in the Academy, the Council shall recommend to the Academy the termination of his membership; and if three-fourths of the Fellows present, out of a total attendance of not less than fifty at a Stated Meeting, or at a Special Meeting called for the purpose, shall adopt this recommendation, his name shall be stricken from the Roll.

See Chap. iii; chap. vi, art. 1; chap. x, art. 1, 7; chap. xi, art. 2.

CHAPTER III.

Election of Fellows and Foreign Honorary Members

The procedure in the election of Fellows and Foreign Honorary Members shall be as follows:

Nominations to Fellowship or Foreign Honorary Membership in any Section must be signed by two Fellows of that Section, or by three Fellows of any Sections, and sent to the Corresponding Secretary accompanied by a statement of the qualifications of the nominee and brief biographical data.

Notice shall be sent to every Fellow not later than the fifteenth of January in each year, reminding him that all nominations must be in the hands of the Corresponding Secretary before the fifteenth of February following.

A list of the nominees, giving a brief account of each, with the names of the nominators, shall be sent to every Fellow with a request that he return the list with such confidential comments and indications of preference as he may choose to make.

All the nominations, with any comments thereon and with expressions of preference on the part of the Fellows, shall be referred to the appropriate Class Committees, which shall canvass them, and report their recommendations in writing to the Council before the Stated Meeting of the Academy in April.

Elections of Fellows and Foreign Honorary Members shall be made by the Council before the Annual Meeting in May, and announced at that meeting.

Persons nominated in any year, but not elected, may be carried over to the list of nominees for the next year at the discretion of the Council, but shall not be further continued unless renominated.

See Chap. ii; chap. vi, art. 1; chap. x, art. 1.

CHAPTER IV.

OFFICERS.

ARTICLE 1. The Officers of the Academy shall be a President (who shall be Chairman of the Council), three Vice-Presidents (one from each Class), a Corresponding Secretary (who shall be Secretary of the Council), a Recording Secretary, a Treasurer, a Librarian, and an Editor, all of whom shall be elected by ballot at the Annual Meeting, and shall hold their respective offices for one year, and until others are duly chosen and installed.

There shall be also twelve Councillors, one from each Section of each Class. At each Annual Meeting three Councillors, one from each Class, shall be elected by ballot to serve for the full term of four years and until others are duly chosen and installed. The same Fellow shall not be eligible for two successive terms.

The Councillors, with the other officers previously named, and the Chairman of the House Committee, *ex officio*, shall constitute the Council.

See Chap. xi, art. 1.

Article 2. If any officer be unable, through death, absence, or disability, to fulfil the duties of his office, or if he shall resign, his place may be filled by the Council in its discretion for any part or the whole of the unexpired term.

Article 3. At the Stated Meeting in March, the President shall appoint a Nominating Committee of three Fellows having the right to vote, one from each Class. This Committee shall prepare a list of nominees for the several offices to be filled, and for the Standing Committees, and file it with the Recording Secretary not later than four weeks before the Annual Meeting.

See Chap. vi, art. 2.

Article 4. Independent nominations for any office, if signed by at least twenty Fellows having the right to vote, and received by the Recording Secretary not less than ten days before the Annual Meeting, shall be inserted in the call therefor, and shall be mailed to all the Fellows having the right to vote.

See Chap. vi, art. 2.

Article 5. The Recording Secretary shall prepare for use in voting at the Annual Meeting a ballot containing the names of all persons duly nominated for office.

CHAPTER V.

The President.

Article 1. The President, or in his absence the senior Vice-President present (seniority to be determined by length of continuous fellowship in the Academy), shall preside at all meetings of the Academy. In the absence of all these officers, a Chairman of the meeting shall be chosen by ballot.

ARTICLE 2. Unless otherwise ordered, all Committees which are not elected by ballot shall be appointed by the presiding officer.

ARTICLE 3. Any deed or writing to which the Corporate Seal is to be affixed, except leases of real estate, shall be executed in the name of the Academy by the President or, in the event of his death, absence, or inability, by one of the Vice-Presidents, when thereto duly authorized.

See Chap. ii, art. 7; chap. iv, art. 1, 3; chap. vi, art. 2; chap. vii, art. 1; chap. x, art. 6; chap. xi, art. 1, 2; chap. xii, art. 1.

CHAPTER VI.

THE SECRETARIES.

ARTICLE 1. The Corresponding Secretary shall conduct the correspondence of the Academy and of the Council, recording or making an entry of all letters written in its name, and preserving for the files all official papers which may be received. At each meeting of the Council he shall present the communications addressed to the Academy which have been received since the previous meeting, and at the next meeting of the Academy he shall present such as the Council may determine.

He shall notify all persons who may be elected Fellows or Foreign Honorary Members, send to each a copy of the Statutes, and on their acceptance issue the proper Diploma. He shall also notify all meetings of the Council; and in case of the death, absence, or inability of the Recording Secretary he shall notify all meetings of the Academy.

Under the direction of the Council, he shall keep a List of the Fellows and Foreign Honorary Members, arranged in their several Classes and Sections. It shall be printed annually and issued as of the first day of July.

See Chap. ii, art. 7; chap. iii, art. 2, 3; chap. iv, art. 1; chap. x, art. 6; chap. xi, art. 1; chap. xii, art. 1.

ARTICLE 2. The Recording Secretary shall have the custody of the Charter, Corporate Seal, Archives, Statute-Book, Journals, and all literary papers belonging to the Academy.

Fellows borrowing such papers or documents shall receipt for them to their custodian.

The Recording Secretary shall attend the meetings of the Academy and keep a faithful record of the proceedings with the names of the Fellows present; and after each meeting is duly opened, he shall read the record of the preceding meeting.

He shall notify the meetings of the Academy to each Fellow and by mail at least seven days beforehand, and in his discretion may also cause the meetings to be advertised; he shall apprise Officers and Committees of their election or appointment, and inform the Treasurer of appropriations of money voted by the Academy.

After all elections, he shall insert in the Records the names of the Fellows by whom the successful nominees were proposed.

He shall send the Report of the Nominating Committee in print to every Fellow having the right to vote at least three weeks before the Annual Meeting.

See Chap. iv, art. 3.

In the absence of the President and of the Vice-Presidents he shall, if present, call the meeting to order, and preside until a Chairman is chosen.

See Chap. i; chap. ii, art. 7; chap. iv, art. 3, 4, 5; chap. x, art. 6; chap. xi, art. 1, 2; chap. xii, art. 1, 3.

Article 3. The Secretaries, with the Editor, shall have authority to publish such of the records of the meetings of the Academy as may seem to them likely to promote its interests.

CHAPTER VII.

The Treasurer and the Treasury.

Article 1. The Treasurer shall collect all money due or payable to the Academy, and all gifts and bequests made to it. He shall pay all bills due by the Academy, when approved by the proper officers, except those of the Treasurer's office, which may be paid without such approval; in the name of the Academy he shall sign all leases of real estate; and, with the written consent of a member of the Committee on Finance, he shall make all transfers of stocks, bonds, and other securities belonging to the Academy, all of which shall be in his official custody.

He shall keep a faithful account of all receipts and expenditures, submit his accounts annually to the Auditing Committee, and render them at the expiration of his term of office, or whenever required to do so by the Academy or the Council.

He shall keep separate accounts of the income of the Rumford Fund, and of all other special Funds, and of the appropriation thereof, and render them annually.

His accounts shall always be open to the inspection of the Council.

ARTICLE 2. He shall report annually to the Council at its March meeting on the expected income of the various Funds and from all other sources during the ensuing financial year. He shall also report the names of all Fellows who may be then delinquent in the payment of their Annual Dues.

ARTICLE 3. He shall give such security for the trust reposed in him as the Academy may require.

ARTICLE 4. With the approval of a majority of the Committee on Finance, he may appoint an Assistant Treasurer to perform his duties, for whose acts, as such assistant, he shall be responsible; or, with like approval and responsibility, he may employ any Trust Company doing business in Boston as his agent for the same purpose, the compensation of such Assistant Treasurer or agent to be fixed by the Committee on Finance and paid from the Funds of the Academy.

ARTICLE 5. At the Annual Meeting he shall report in print all his official doings for the preceding year, stating the amount and condition of all the property of the Academy entrusted to him, and the character of the investments.

ARTICLE 6. The Financial Year of the Academy shall begin with the first day of April.

ARTICLE 7. No person or committee shall incur any debt or liability in the name of the Academy, unless in accordance with a previous vote and appropriation therefor by the Academy or the Council, or sell or otherwise dispose of any property of the Academy, except cash or invested funds, without previous consent and approval of the Council.

See Chap. ii, art. 4, 5; chap. vi, art. 2; chap. x, art. 6; chap. xi, art. 1, 2, 3; chap. xii, art. 1.

CHAPTER VIII.

The Librarian and the Library.

Article 1. The Librarian shall have charge of the printed books, keep a correct catalogue thereof, and provide for their delivery from the Library.

At the Annual Meeting, as Chairman of the Committee on the Library, he shall make a Report on its condition.

Article 2. In conjunction with the Committee on the Library he shall have authority to expend such sums as may be appropriated by the Academy for the purchase of books, periodicals, etc., and for defraying other necessary expenses connected with the Library.

Article 3. All books procured from the income of the Rumford Fund or of other special Funds shall contain a book-plate expressing the fact.

Article 4. Books taken from the Library shall be receipted for to the Librarian or his assistant.

Article 5. Books shall be returned in good order, regard being had to necessary wear with good usage. If any book shall be lost or injured, the Fellow to whom it stands charged shall replace it by a new volume or by a new set, if it belongs to a set, or pay the current price thereof to the Librarian, whereupon the remainder of the set, if any, shall be delivered to the Fellow so paying, unless such remainder be valuable by reason of association.

Article 6. All books shall be returned to the Library for examination at least one week before the Annual Meeting.

ARTICLE 7. The Librarian shall have the custody of the Publications of the Academy. With the advice and consent of the President, he may effect exchanges with other associations.

See Chap. ii, art. 6; chap. xi, art. 1, 2.

CHAPTER IX.

THE EDITOR AND THE PUBLICATIONS.

ARTICLE 1. The Editor shall have charge of the conduct through the press of the Proceedings and the Memoirs, and all correspondence relative thereto, and shall have power to fix the price at which individual numbers of the Proceedings and Memoirs are sold.

ARTICLE 2. In conjunction with the Committee of Publication, he shall have authority to expend such sums as may be appropriated by the Academy for printing the publications and for defraying other expenses therwith connected.

ARTICLE 3. All publications which are financed in whole or in part from the income of the Rumford Fund or from the income of other special funds, and all publications of work done with the aid of the Rumford Fund or other special funds, shall contain a conspicuous statement of this fact.

ARTICLE 4. Two hundred extra copies of each paper printed in the Proceedings or Memoirs shall be placed at the disposal of the author without charge.

If, on account of the number of communications offered for publication, it shall be necessary to decline for publication communications otherwise acceptable, members of the Academy shall be given preference in each of the several Classes over non-members; but whenever it shall be necessary to exercise this preference, the Editor shall inform the Council of the fact.

See Chap. iv, art. 1; chap. vi, art. 3; chap. x, art. 6; chap. xi, art. 4.

CHAPTER X.

The Council.

Article 1. The Council shall exercise a discreet supervision over all nominations and elections to membership, and in general supervise all the affairs of the Academy not explicitly reserved to the Academy as a whole or entrusted by it or by the Statutes to standing or special committees.

It shall consider all nominations duly sent to it by any Class Committee, and act upon them in accordance with the provisions of Chapter III.

With the consent of the Fellow interested, it shall have power to make transfers between the several Sections, reporting its action to the Academy.

See Chap. iii, art. 2, 3; chap. xi, art. 1.

Article 2. Seven members shall constitute a quorum.

Article 3. It shall establish rules and regulations for the transaction of its business, and provide all printed and engraved blanks and books of record.

Article 4. It shall act upon all resignations of officers, and all resignations and forfeitures of Fellowship; and cause the Statutes to be faithfully executed.

It shall appoint all agents and subordinates not otherwise provided for by the Statutes, prescribe their duties, and fix their compensation. They shall hold their respective positions during the pleasure of the Council.

Article 5. It may appoint, for terms not exceeding one year, and prescribe the functions of, such committees of its number, or of the Fellows of the Academy, as it may deem expedient, to facilitate the administration of the affairs of the Academy or to promote its interests.

Article 6. At its March meeting it shall receive reports from the President, the Secretaries, the Treasurer, and the Standing Committees, on the appropriations severally needed for the ensuing financial year. At the same meeting the Treasurer shall report on the expected income of the various Funds and from all other sources during the same year.

A report from the Council shall be submitted to the Academy, for action, at the March meeting, recommending the appropriation which in the opinion of the Council should be made.

On the recommendation of the Council, special appropriations may be made at any Stated Meeting of the Academy, or at a Special Meeting called for the purpose.

See Chap. xi, art. 3.

ARTICLE 7. After the death of a Fellow or Foreign Honorary Member, it shall appoint a member of the Academy to prepare a biographical notice for publication in the Proceedings.

ARTICLE 8. It shall report at every meeting of the Academy such business as it may deem advisable to present.

See Chap. ii, art. 4, 5, 8; chap. iv, art. 1, 2; chap. vi, art. 1; chap. vii, art. 1; chap. xii, art. 1, 4.

CHAPTER XI.

STANDING COMMITTEES.

ARTICLE 1. The Class Committee of each Class shall consist of the Vice-President, who shall be chairman, and the four Councillors of the Class, together with such other officer or officers annually elected as may belong to the Class. It shall consider nominations to Fellowship in its own Class, and report in writing to the Council such as may receive at a Class Committee Meeting a majority of the votes cast, provided at least three shall have been in the affirmative.

See Chap. iii, art. 2.

ARTICLE 2. At the Annual Meeting the following Standing Committees shall be elected by ballot to serve for the ensuing year:

(i) *The Committee on Finance,* to consist of three Fellows, who, through the Treasurer, shall have full control and management of the funds and trusts of the Academy, with the power of investing the funds and changing the investments thereof in their discretion.

See Chap. iv, art. 3; chap. vii, art. 1, 4; chap. x, art. 6.

(ii) *The Rumford Committee,* to consist of seven Fellows, who shall report to the Academy on all applications and claims for the Rumford Premium. It alone shall authorize the purchase of books, publications and apparatus at the charge of the income from the Rumford Fund, and generally shall see to the proper execution of the trust.

See Chap. iv, art. 3; chap. x, art. 6.

(iii) *The Cyrus Moors Warren Committee,* to consist of seven Fellows, who shall consider all applications for appropriations from the income of the Cyrus Moors Warren Fund, and generally shall see to the proper execution of the trust.

See Chap. iv, art. 3; chap. x, art. 6.

(iv) *The Committee of Publication,* to consist of the Editor, *ex officio,* as Chairman, and three others Fellows, one from each Class, to whom all communications submitted to the Academy for publication shall be referred, and to whom the printing of the Proceedings and the Memoirs shall be entrusted.

It shall fix the price at which volumes of the publications shall be sold; but Fellows may be supplied at half price with volumes which may be needed to complete their sets, but which they are not entitled to receive gratis.

It shall determine when the pressure of material offered for publication makes it necessary to give preference to members of the Academy as compared with non-members, or to give priority to certain members as compared with others, and to what extent this preference or priority shall be applied in each of the three Classes, to the end that a proper balance of the facilities of publication with respect to subject matter and authors may be maintained.

See Chap. iv, art. 3; chap. vi, art. 1, 3; chap. ix; chap. x, art. 6.

(v) *The Committee on the Library,* to consist of the Librarian, *ex officio,* as Chairman, and three other Fellows, one from each Class, who shall examine the Library and make an annual report on its condition and management.

See Chap. iv, art. 3; chap. viii, art. 1, 2; chap. x, art. 6.

(vi) *The House Committee,* to consist of three Fellows, who shall have charge of all expenses connected with the House, including the general expenses of the Academy not specifically assigned to the care of other Committees or Officers.

See Chap. iv, art. 1, 3; chap. x, art. 6.

(vii) *The Committee on Meetings,* to consist of the President, the Recording Secretary, and three other Fellows, who shall have charge of plans for meetings of the Academy.

See Chap. iv, art. 3; chap. x, art. 6.

(viii) *The Auditing Committee,* to consist of two Fellows, who shall audit the accounts of the Treasurer, with power to employ an expert and to approve his bill.

See Chap. iv, art. 3; chap. vii, art. 1; chap. x, art. 6.

ARTICLE 3. The Standing Committees shall report annually to the Council in March on the appropriations severally needed for the ensuing financial year; and all bills incurred on account of these Committees, within the limits of the several appropriations made by the Academy, shall be approved by their respective Chairmen.

In the absence of the Chairman of any Committee, bills may be approved by any member of the Committee whom he shall designate for the purpose.

See Chap. vii, art. 1, 7; chap. x, art. 6.

CHAPTER XII.

MEETINGS, COMMUNICATIONS, AND AMENDMENTS.

ARTICLE 1. There shall be annually eight Stated Meetings of the Academy, namely, on the second Wednesday of October, November, December, January, February, March, April, and May. Only at these meetings, or at adjournments thereof regularly notified, or at

Special Meetings called for the purpose, shall appropriations of money be made or amendments of the Statutes or Standing Votes be effected.

The Stated Meeting in May shall be the Annual Meeting of the Corporation.

Special Meetings shall be called by either of the Secretaries at the request of the President, of a Vice-President, of the Council, or of ten Fellows having the right to vote; and notifications thereof shall state the purpose for which the meeting is called.

A meeting for receiving and discussing literary or scientific communications may be held on the fourth Wednesday of each month, excepting July, August, and September; but no business shall be transacted at said meetings.

ARTICLE 2. Twenty Fellows having the right to vote shall constitute a quorum for the transaction of business at Stated or Special Meetings. Fifteen Fellows shall be sufficient to constitute a meeting for literary or scientific communications and discussions.

ARTICLE 3. Upon the request of the presiding officer or the Recording Secretary, any motion or resolution offered at any meeting shall be submitted in writing.

ARTICLE 4. No report of any paper presented at a meeting of the Academy shall be published by any Fellow without the consent of the author; and no report shall in any case be published by any Fellow in a newspaper as an account of the proceedings of the Academy without the previous consent and approval of the Council. The Council, in its discretion, by a duly recorded vote, may delegate its authority in this regard to one or more of its members.

ARTICLE 5. No Fellow shall introduce a guest at any meeting of the Academy until after the business has been transacted, and especially until after the result of the balloting upon nominations has been declared.

ARTICLE 6. The Academy shall not express its judgment on literary or scientific memoirs or performances submitted to it, or included in its Publications.

ARTICLE 7. All proposed Amendments of the Statutes shall be referred to a committee, and on its report, at a subsequent Stated Meeting or at a Special Meeting called for the purpose, two-thirds of the ballot cast, and not less than twenty, must be affirmative to effect enactment.

ARTICLE 8. Standing Votes may be passed, amended, or rescinded at a Stated Meeting, or at a Special Meeting called for the purpose, by a vote of two-thirds of the members present. They may be suspended by a unanimous vote.

See Chap. ii, art. 5, 8; chap. iii; chap. iv, art. 3, 4, 5; chap. v, art. 1; chap. vi, art. 1, 2; chap. x, art. 8.

STANDING VOTES.

1. Communications of which notice has been given to either of the Secretaries shall take precedence of those not so notified.

2. Fellows may take from the Library six volumes at any one time, and may retain them for three months, and no longer. Upon special application, and for adequate reasons assigned, the Librarian may permit a larger number of volumes, not exceeding twelve, to be drawn from the Library for a limited period.

3. Works published in numbers, when unbound, shall not be taken from the Hall of the Academy without the leave of the Librarian.

4. The Council, under such rules respecting nominations as it may prescribe, may elect as Associates of the Academy a limited number of men of mark in affairs or of distinguished service in the community.

Associates shall be entitled to the same privileges as Fellows, but shall not have the right to vote.

The admission fee and annual dues of Associates shall be the same as those of Fellows residing within fifty miles of Boston.

5. Communications offered for publication in the Proceedings or Memoirs of the Academy shall not be accepted for publication before the author shall have informed the Committee on Meetings of his readiness, either himself or through some agent, to use such time as the Committee may assign him at such meeting as may be convenient both to him and to the Committee, for the purpose of presenting to the Academy a general statement of the nature and significance of the results contained in his communication.

RUMFORD PREMIUM.

In comformity with the terms of the gift of Sir Benjamin Thompson, Count Rumford, of a certain Fund to the American Academy of Arts and Sciences, and with a decree of the Supreme Judicial Court of Massachusetts for carrying into effect the general charitable intent and purpose of Count Rumford, as expressed in his letter of gift, the Academy is empowered to make from the income of the Rumford Fund, as it now exists, at any Annual Meeting, an award of a gold and a silver medal, being together of the intrinsic value of three hundred dollars, as a Premium to the author of any important discovery or useful improvement in light or heat, which shall have been made and published by printing, or in any way made known to the public, in any part of the continent of America, or any of the American islands; preference always being given to such discoveries as, in the opinion of the Academy, shall tend most to promote the good of mankind; and, if the Academy sees fit, to add to such medals, as a further Premium for such discovery and improvement, a sum of money not exceeding three hundred dollars.

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